# THE THEORY OF SPECIAL RELATIVITY

## BASIC PRINCIPLES, LIMITS, POSSIBLE AMENDMENTS AND SUGGESTIONS FOR DECISIVE EXPERIMENTS

A new approach after more than a century

Gerhard W. Borst

# Preface

## Motivation

"Ladies and Gentlemen, as you certainly know, two fundamental theories in physics exist on which the foundations of our science are built. These are the Theories of General Relativity and Quantum Mechanics. Today we know that at least one of these theories cannot be true, because they exclude each other. As a representative of quantum mechanics my statement is that because of many positive and coherent experiments we are sure, that we are on the right side. So, let's get started."

This was the statement of the professor right at the start of the lecture in quantum mechanics during the summer session in 2014 at the University of Bonn (Theoretical Physics III). I never experienced a situation in a lecture, that there was absolute silence. Here it was the case.

The statement of the "competitors" teaching General Relativity during summer 2015 were quite different. Responding to a question from a student concerning this matter it was stated, that at present there are ongoing works to improve the theory to resolve the appearing contradictions.

This situation encouraged me to find my own view concerning this debate. Because of my status as a "senior student" (born 1953) with a lot of freedom compared to others and no obligations to pass examinations it was possible for me to spend some time to study these problems. My main interest went to the Theory of Special Relativity, which is the basis for all further considerations, and I found some interesting results inside this theory which will be presented in the following.

## Short summary

Detailed investigations on the Theory of Special Relativity (SRT) show that the phenomenon of "invariance of phase velocity of light" is of great importance. However, although it is well-known since a long time this concept was not used in a comprehensive way for interpretation up to now. When this phenomenon is applied to classical experiments, where usually interactions between light beams coming and going to mirrors are investigated, the comparison between resulting frequencies makes no sense. In this elaboration relevant experiments are interpreted in a new way and interesting results were found. For the

Michelson-Morley experiment only small corrections are required, the Kennedy-Thorndike-experiment, however, showed a much higher significance of the results than expected before.

Further on considerations about the exchange of signals between moving observers and examinations concerning momentum and energy are made. Using again the invariance of phase velocity it can be shown in any case, that the assumption of the existence of a system at absolute rest as a frame is just a special case inside the infinite possibilities of SRT and not, as sometimes argued, contradictions to experimental findings appear. The existence of the isotropic cosmic background radiation, known since some decades, is a strong indication for the existence of a system at absolute rest. However, although effort was made by many scientists, up to now no unambiguously clear experimental evidence was found inside the classical frame, which could possibly help to decide whether it really exists or not.

This could change, when in addition quantum mechanical tunneling experiments are incorporated. Theoretical considerations show that superluminal transport of information, e.g. by transmission of a simple pulse, is in accordance with a state of absolute rest but is violating the more general concept of SRT. A proposal was made for the set-up of an experiment that could help to decide, which of the different theories is valid. Furthermore, two new experiments are discussed; the most important could finally provide direct experimental evidence about the existence of the "relativity of simultaneously", which is an essential part of the Lorentz equations.

Beside the considerations concerning theory and experiments, for readers with interest in history of science a short overview was added to show in a general way, how, from ancient times to Galileo and finally Einstein, the development of SRT took place.

## Warning Notice

Please be careful, this presentation contains mathematics! Fortunately, it is a small dose; an advanced course at school will be sufficient to understand the fundamental principles, after the first semester in physics everything will go easy. Should it be impossible to understand one of the details, it is not necessary for the understanding of the following chapters and the related part can be skipped. In particular the use of the tensor calculus, which is often utilized, was avoided, because it is not necessary to understand the principle theoretical foundations outlined here. It was decided instead to include specific examples with connected calculations to support the general understanding of important details.

Any external references are marked, and a publication list is added.

Although all considerations were carried out with due care it can be possible, that parts of this presentation include mistakes. In such a case and of cause when a discussion is requested, I would be glad to get feedback.

Alfter, April 2019 Gerhard W. Borst

## Preface to the 3rd Revision

As a result of the discussions in the two years since the book was first published, it has become clear that the results should be summarized even more concisely and clearly. Although the basic message has not changed, the main results have therefore been restated here in line with the wording on the website.

## Results of the Investigations

The main result of this investigation is that the *phase velocity of light* and not *speed of light* must be applied to classical experiments, where light beams going and coming are observed in moving systems. The comparisons usually made by determining interference patterns are remaining incomplete without further considerations. Re-evaluation according to the concept presented here for the Michelson-Morley and Kennedy-Thorndike experiments lead to different results. This change in the point of view has a substantial effect on other main subjects, as it is the case for the Theory of Special Relativity (SRT).

For the formulation of SRT, Einstein chose an approach whose foundations are the "principle of relativity" and the "constancy of the speed of light" and which does not contain any physical formula in its origin. From this "top-down" concept the Lorentz Transformation, and the relationship for the relativistic increase of the kinetic energy, later also called relativistic mass increase, can be derived.

It is surprising that until today there is no uniform formulation of the two central principles. Every author of a publication about SRT chooses his own approach for this. The representations can be divided basically into "objective observation criterion" and "axiom". First, objective criterion means for the principle of relativity:

1. The execution of any physical experiment leads to the same result in all inertial systems.

This approach was also chosen by Einstein. The representation as "axiom" contains the statement, "All inertial systems are equal". In newer publications rather (but not exclusively) the axiomatic concept is used. With exact interpretation, however, this already contains the statement that a system of absolute rest cannot exist, for which there is no experimental proof until today (but also no counterproof). To keep this open, in the following the classical concept for this basic principle is chosen.

If as second criterion the velocity of light is considered, the same is valid here as already shown before; here also the statements "no differences can be determined" and "the velocity of light is always the same" for different inertial frames are in use. As an essential result of the investigations carried out here it shows, however, that with the observation of oscillations of ***one*** light source from any arbitrarily inertial system moved to each other the phase velocity of light is the only reasonable possibility to achieve contradiction-free results. If instead the velocity of light is used - as it is still usual today - different interpretations concerning the number of oscillations from this source arise for the differently moved observers and connected with this also the view on interference patterns.

The proposal for a contradiction-free and unambiguous formulation of the second principle of the SRT reads thus:

2. The phase velocity of light is invariant in all inertial systems and its speed is equal to the value of the velocity of light measurable in every inertial frame.

However, the investigations presented here have also shown that a “bottom-up” approach with an E*xtended Lorentz-Theory* is also possible. Using this concept, the necessary basic physical laws are defined, and the relativity principle can then be derived from them. This approach reads as follows:

1. From the unlimited number of existing inertial systems, one is selected as base system and marked with index 0.
2. In this basic system, measurements of the speed of light show the same value $c$ in all directions.
3. The properties of all other inertial systems are defined by their relative velocity $v$ to the base system, and the following relations are valid for time $t$, displacement $x$ and mass $m$

a) $t = \gamma\left(t_0 - \frac{v}{c^2}x_0\right), \qquad x = \gamma(x_0 - vt_0)$

b) $m = \gamma m_0$

*with*: $$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In this representation, special relativity and the extended Lorentz approach are mathematically completely equivalent. However, the Theory of Special Relativity excludes with usual interpretation the existence of a system of absolute rest, which can be integrated in the extended Lorentz approach by simple choice of the basic system without further assumptions or restrictions. The since some decades known completely uniform cosmic background radiation has already led many times to considerations to reconcile this with the existence of an absolutely resting space and SRT. So far this was not successful and always led to contradictions with experimental findings. It is of great advantage that the approach shown here allows a completely problem-free integration. However, since up to now no experimental proof has succeeded with conventional approaches, a decision cannot be made at present.

This could change if quantum mechanical tunneling experiments are included. Theoretical considerations show that faster-than-light transmissions of signals, e.g. by sending a simple pulse, are compatible with the extended Lorentz theory but not with SRT. An experiment is proposed, which allows an unambiguous decision concerning the different approaches. Furthermore, two other experiments are presented for discussion, of which the most important is the direct proof of the "relativity of simultaneity", which is an integral part of the Lorentz equations.

# Contents

## Notice

This script may be subject to updates. The actual status can be validated at https://www.analyse-srt.de

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# 1. Introduction

In this investigation first the basic principles of the theory of Special Relativity will be presented in detail. In further steps the consequences derived out of the theory and later the existing limits will be discussed. A major contribution for the understanding of the discussions arising during the presentation of the theory is taking a close view on the historical development. To realize this, three important parts of physical science were chosen (classical mechanics, light and its radiation, electromagnetism) and connected with this, important persons are presented, who had major influence on the developments. The presented selection out of numerous researchers is most probably partly unfair but must be limited for obvious reasons because of the almost unlimited number.

## 1.1 General historical preconditions

After the fall of the Roman Empire as a result of the barbarian migration a general loss of transferred knowledge of Greece and Roman origin was observed in Europe. Many old scripts were saved only, because they were translated and interpreted by Arabian scientists who were at that time part of scientific communities with generally much higher standards compared to those in Europe. The situation did not change until the end of the millennial when a warm epoch began, which had a high impact on the development of the society. Until the year 1300 the population tripled, land was reclaimed on a large scale and many new cities were founded.

For the “dawn of mankind” and the connected explosion of knowledge many different reasons are considered to be important (for further studies the very interesting book “The Morning of the World” [1] by Bernd Roeck is strongly recommended). First in the cities with sufficient supply of food and other necessary things for daily life a group was established which we would today call “middle class” and was formed by craftsmen and merchants. This structure can be defined as “horizontal”, because it was not dominated by aristocratic authorities and was therefore able to develop in a free manner [1]. Furthermore, during the 12th century, the first universities were founded (starting in Bologna, followed by Paris and Oxford) and with the appearance of the professor at these universities the class of the intellectual was founded. The skills of the men appointed for this purpose (women were excluded from this profession and also from studying) certainly did not meet our expectations of the quality of a professor today in most cases, but the procedures of discussion and application of logic originating from the Greek/Roman tradition were generally used. In general, it can be stated, that in Europe starting from the foundation of the first universities until the end of the17th century science and the structures for teaching were quite uniform.

Academic studies included – according to the ancient ideal – the seven liberal arts of classic antiquity comprising the Trivium (grammar, dialectic, and rhetoric, finishing with nomination as "bakkalaureus") and further the Quadrivium (arithmetic, geometry, music and astronomy [including astrology], nomination as magister). In a further step the higher faculties (theological, juridical, and medical) could award the degree of a doctor. The language used was generally Latin, which was of great advantage in the linguistic fragmented environment of that time. Knowledge was generally acquired through the study of the Holy Bible and using scripts of ancient origin mainly from Greek philosophers; experimental work as it is established today was generally not common.

Beside the already presented general issues further advantageous developments occurred towards the end of the 13th century. Important inventions were made, which had a great impact on the progress of science and technology. The most important included quite different subjects like the production of paper and gunpowder (both based on ideas imported from Asia), also the invention of the mechanical clock and of spectacles (and connected with this the knowledge to produce glass of sufficient quality). During the following little ice age starting with the beginning of the 14th century and lasting for over 500 years which caused hunger and distress, developments were possible which improved science in an important and positive way.

Paper showed a clear advantage compared to the parchment used before which was produced out of animal skin, and it was possible to produce it at lower costs and with a better quality and higher quantity. Combined with the letterpress printing invented by Gutenberg and the developing postal services an information exchange was possible not imaginable before. In addition, the use of gunpowder had a great influence on the development of metallurgy and metal machining necessary to produce firearms and a first nucleus of a sector later called "heavy industry" appeared.

It is often said that letterpress printing and the use of gun powder are the major facts for the explanation of the developments happening at that time. The progress of science, however, is also connected with the permanent improvement of precision mechanics which led e.g., to the production of clocks with increasing accuracy which are for obvious reasons necessary for quantitative measurements of physical parameters. This long-term development was also witnessed for the production and processing of lenses. In contrast to this at the beginning of the 17th century the knowledge about the inventions of telescope and microscope spread over Europe in a very short time and had a great influence on natural science. Further the first introduction of property rights (copyright, patents) was also responsible for important promotion effects.

With the beginning of the 17th century first scientists questioned the opinion, that knowledge could only be acquired by studying old scripts but that it was also possible to expand it by own considerations and observations. Francis Bacon (1561-1626) was the first to propose an empiric approach for the development of science. He was sure that knowledge of mankind is cumulative (his considerations finally led to the expression: "knowledge is power"). Initiated by René Descartes (1596-1650) mathematical procedures were identified as an important instrument to derive scientific progress. He was the first to use equations which are quite similar to the form we know today. He used, however, a symbol similar to „æ“ (derived from the Latin word „aequalis“), the equality sign “=” was used for the first time by the Welsh mathematician Robert Recorde (1510-1558) It did not spread over Europe before 1700 but finally became the standard for the formulation in scientific publications. Together with the “invention” of the figure zero at the end of the 13th century, which slowly found its way into mathematics, these were no necessary requirements but led to enormous accelerations in the progress of natural science.

The sociologist Robert K. Merton (1910-2003) made further interesting statements concerning the developments of that time [2]. First, he expressed the opinion that changes and progress in natural science were caused by an accumulation of observations, improved experimental techniques and also the development of additional methodic approaches; this concept is apparently corresponding to the thesis of Roeck [1]. In further considerations he is arguing that the revolution in natural science during the 17th and 18th century was mainly promoted by Protestantism, in particular by English puritans and German pietists.
This was not changing before the French Revolution happened and the disempowerment of the Catholic Church was enforced by Napoleon after the conquest of almost complete Europe. This thesis is not without dispute and is for sure partly unfair against many important scientists of that time. It is symptomatic, however, that publications of Descartes and Galilei (after 1633) banned by the Catholic Church could only be printed by the publishing house Elsevier because it was situated in the protestant town of Leiden and was therefore not under the jurisdiction of the Catholic Church.

## 1.2 Classical mechanics

One of the most important founders of modern natural science is Galileo Galilei (1564-1642). From 1609 on he improved the technique of the telescope which was invented a year before by Hans Lipperhey (1570-1619) by own production of better lenses and the use of enhancements in the construction. He was the first to monitor the sky in a systematic way and discovered already in 1610 the moons of Jupiter, which could not be seen before with the naked eye. It was of great influence on the view of the world that beside earth now another planet possessed moons. He also discovered that the Milky Way is formed as a cluster of many stars and is not a shiny band as it was believed to be before and that planets are not point-shaped but show the form of a disk during observation. He calculated the height of the mountains on the moon by the visible shadows and estimated the value to 8000m [3]. Further he performed experiments concerning the free fall of objects. It is sometimes claimed that these were conducted at the leaning tower of Pisa, but this is most probably not true, he presumably used spheres made of different matter and measured their acceleration rolling down a ramp.

It shall be mentioned that Lipperhey was not able to have his invention patented, because in the following months other competitors on their part claimed it as theirs. Obviously, the time was ripe for the invention of the telescope and further for the microscope shortly before and soon a broad distribution of these important instruments took place.

However, the most important finding of Galilei concerning the following discussion was the first definition of the principle of relativity. The easiest way to understand this is to have a look on his book

> Dialogo di Galileo Galilei sopra i due Massimi Sistemi del Mondo Tolemaico e Copernicano (Dialogue of Galileo Galilei about the most important systems of the world, the Ptolemaic and the Copernican), first edition 1632.

In the following the "case Galileo Galilei" shall be discussed briefly. The book was not written in Latin but in Italian language and was supposed to attract a wide educated audience. It was not structured like a typical scientific publication at that time but is arranged as a conversation between three persons.

The names of these persons were Salviati, Salgredo and Simplicio. While Salvati and Salgredo were the names of old friends of Galilei deceased long ago [4a] and had access to wide range of knowledge, Simplicio is acting as the simple-minded. It can be clearly seen, that Salvati, and partly also Simplicio, is taking the role of Galilei while Salgredo is an ordinary but well-educated person [4b]. Salvati is also explaining the relativity principle already mentioned before. Fig. 1.1 shows in an English translation by Thomas Salusbury the relevant passage [5]. It dates to the year 1661 and was one of many translations in different languages written shortly after the first publication by Galilei. It is a prosaic form at its best and surely can be understood without using a single equation.

The scientific conclusions of the book are today generally outdated. For the understanding of the thinking and the state of knowledge at that time a later translation by Erich Strauss shall be recommended, were a comprehensive introduction and interpretations of the intentions and actions of the involved persons are added [4].

The form of a dialogue was chosen because the acting persons could argue in an open way and so it was possible to discuss positions not obeying the official doctrine. Although the publications of Copernicus about the heliocentric world system were banned by the Catholic Church it was allowed to use his calculations for the planetary motion, which were much easier and more precise compared to the equations utilized before, when in a separate statement it was claimed that these were only founded on a hypothetic basis and the Ptolemaic world system with earth in the center was really valid [4]. Galilei believed that he had obeyed this rule when he passed this obligation to Simplicio. As well-known this went wrong in a disastrous way.

Although his book first got the imprimatur by the inquisition, which means that he was officially allowed to print it, Galilei was charged with blasphemy. Main reason for this was most probably the animosity with the Jesuits; this originated because Galilei was in a fierce controversy with a member of this order named Christoph Schreiner (1573-1650) concerning the first observation of sunspots.

After Pope Urban VIII withdrew his grace (allegedly because his vanity was offended by statements made by Galilei) he was eventually put to court. Galilei had to retract his statements and was sentenced to life-long dungeon imprisonment. Shortly later this was changed to house detention, and so he was not allowed to leave his premises until the end of his life even not for medical consultations he asked for later. In addition, after his death a dignified funeral was refused.

Shut your self up with some friend in the grand Cabbin between the decks of some large Ship, and there procure gnats, flies, and such other small winged creatures get also a great tub (or other vessel) full of water, and within it put certain fishes; let also a certain bottle be hung up, which drop by drop letteth forth its water into another bottle placed underneath, having a narrow neck and, the Ship lying still, observe diligently how those small winged animals fly with like velocity towards all parts of the Cabin; how the fishes swim indifferently towards all sides; and how the distilling drops all fall into the bottle placed underneath. And casting any thing towards your friend, you need not throw it with more force one way then another, provided the distances be equal and leaping, as the saying is, with your feet closed, you will reach as far one way as another. Having observed all these particulars, though no man doubteth that so long as the vessel stands still, they ought to succeed in this manner; make the Ship to move with what velocity you please; for (so long as the motion is uniforme, and not fluctuating this way and that way) you shall not discern any the least alteration in all the forenamed effects; nor can you gather by any of them whether the Ship doth move or stand still. In leaping you shall reach as far upon the floor, as before; nor for that the Ship moveth shall you make a greater leap towards the poop than towards the prow; howbeit in the time that you staid in the Air, the floor under your feet shall have run the contrary way to that of your jump; and throwing any thing to your companion you shall not need to cast it with more strength that it may reach him, if he shall be towards the prow, and you towards the poop, then if you stood in a contrary situation; the drops shall all distill as before into the inferiour bottle and not so much as one shall fall towards the poop, albeit whil'st the drop is in the Air, the Ship shall have run many feet; the Fishes in their water shall not swim with more trouble towards the fore-part, than towards the hinder part of the tub; but shall with equal velocity make to the bait placed on any side of the tub; and lastly, the flies and gnats shall continue their flight indifferently towards all parts; nor shall they ever happen to be driven together towards the side of the Cabbin next the prow, as if they were wearied with following the swift course of the Ship, from which through their suspension in the Air, they had been long separated; and if burning a few graines of incense you make a little smoke, you shall see it ascend on high, and there in manner of a cloud suspend it self, and move indifferently, not inclining more to one side than another: and of this correspondence of effects the cause is for that the Ships motion is common to all the things contained in it, and to the Air also; I mean if those things be shut up in the Cabbin but in case those things were above deck in the open Air, and not obliged to follow the course of the Ship, differences more or lesse notable would be observed in some of the fore-named effects, and there is no doubt but that the smoke would stay behind as much as the Air it self; the flies also, and the gnats being hindered by the Air would not be able to follow the motion of the Ship, if they were separated at any distance from it. But keeping neer thereto, because the Ship it self as being an unfractuous Fabrick, carrieth along with it part of its neerest Air, they would follow the said Ship without any pains or difficulty. And for the like reason we see sometimes in riding post, that the troublesome flies and hornets do follow the horses flying sometimes to one, sometimes to another part of the body, but in the falling drops the difference would be very small; and in the salts, and projections of grave bodies altogether imperceptible.

Fig. 1.1 First formulation of the principle of relativity by Galileo Galilei
Translation by Thomas Salusbury [5] dating back to 1661.

Although the verdict did not include an explicit publication-ban his main work finalized later concerning the foundation of kinematics and the science of strength of materials could not be published in Italy but was presented by the publishing house Elsevier in Leiden.

Is not easy to explain the principle of relativity presented by Galilei using "gnats, flies and other small winged creatures" for a presentation based on equations. To maintain the basis of a moving ship, in the following the situation shall be discussed, that this is passing a harbor mole were at the same time a flag is rising with constant velocity and is finally reaching the top at time $t_0$. For an observer at the mole the movement of the flag appears to be vertical (coordinates $x = 0$, $y$ and time $t$ with variable values) whereas in view from the ship, which is moving with the velocity $v$, the flag relative to the coordinates of the ship (connected to the coordinates $x', y', t'$) is falling behind by the factor $v \cdot t_0$ (see Fig. 1.2)

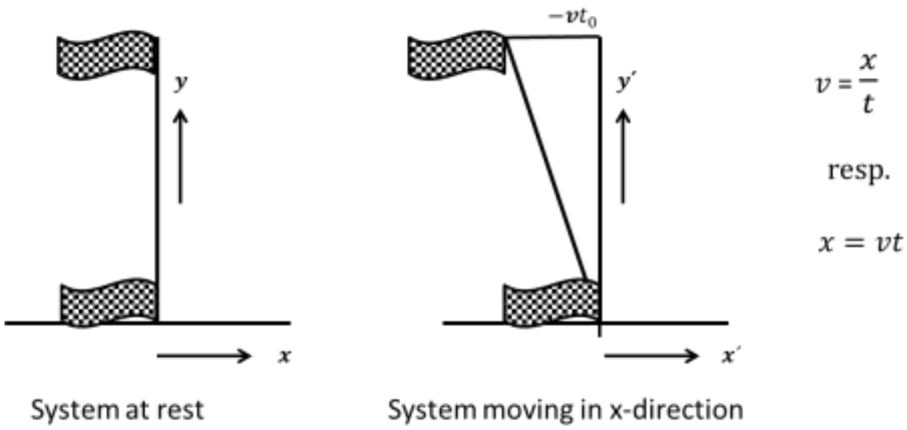



Fig. 1.2: Varying perceptions of the same event observed from different reference systems

It is thus possible to carry out coordinate transformations using the following calculations:

$$x' = x - vt, \quad y' = y, \quad z' = z \quad t' = t \tag{1.01}$$

If on the other hand a flag is rising on the ship the reverse effect will occur and in view of the observer at the mole the flag is moving in $x$-direction

$$x = x' + vt, \quad y = y', \quad z = z' \quad t = t' \tag{1.02}$$

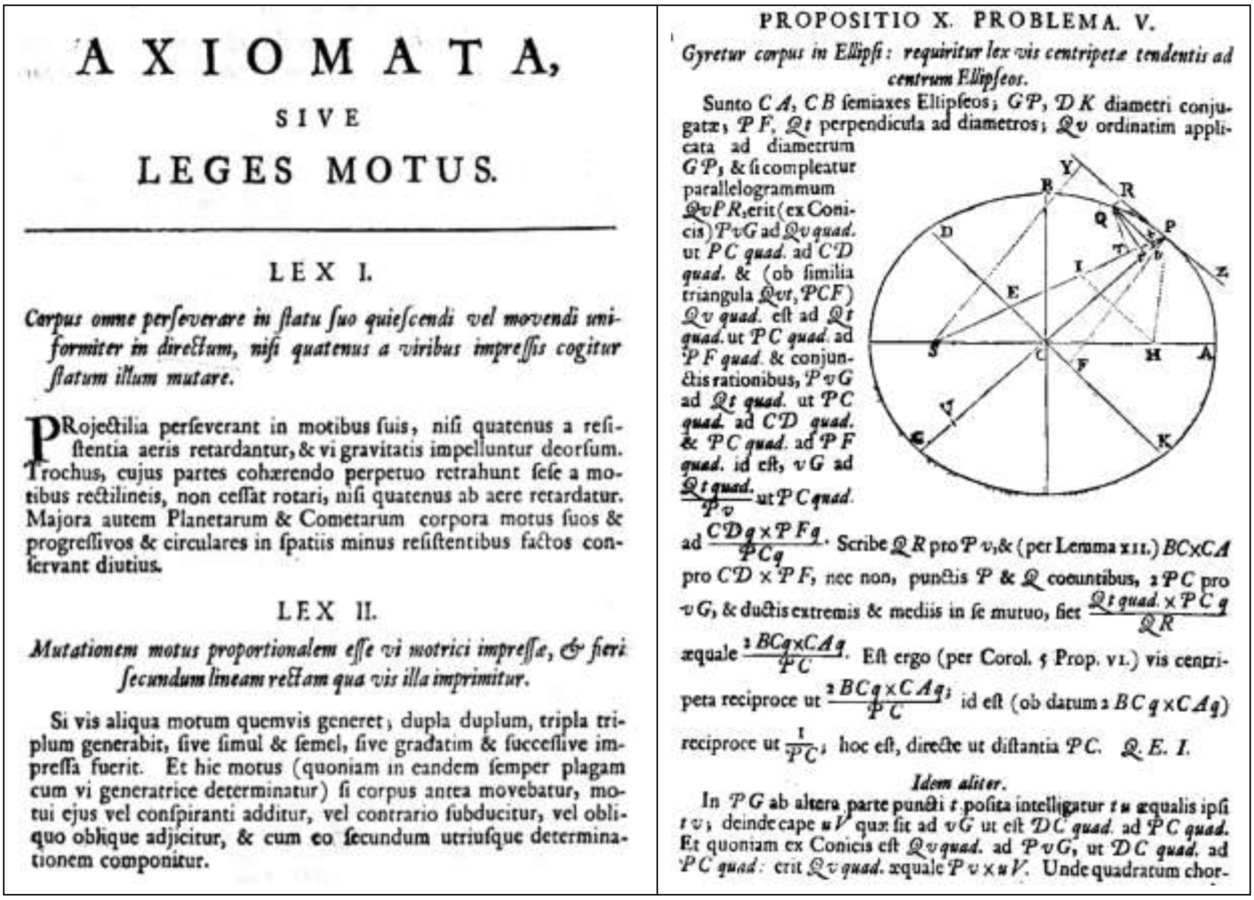
The description requires only a simple conversion of Eq. (1.01). This equation system is called the "Galilei-Transformation" of classical mechanics. It is important that only a variation in the direction of the movement occurs, all other spatial directions are not affected and in addition time is constant for all systems.

This interpretation was taken as a priori valid for centuries because it is conforming to daily experience of human life, and thus was not questioned for a long time. It will be presented later that according to today's knowledge the validity is only (approximately) granted when the velocity of the system (in this case the speed of the ship in $x$-direction) is far lower than the speed of light.

Although an important foundation was created by Galilei the main work to complete classical mechanics was done by another great scientist. In the year 1687 Isaac Newton (1643-1727) published his book

Philosophiae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy)

which is certainly one of the most important books in modern science. It contains the axioms later named after Newton and also many comprehensive calculations and arguments. For the presentation the form of a continuous text was used, and it is hard to understand from today's point of view, not only because it is written in Latin, but also because no equations using the equality sign were used (see Fig. 1.3). The publication is available as original and in several modern transcriptions; a remarkably interesting example is the original book used by Newton with his handwritten remarks which is provided by Cambridge University and is available online.



AXIOMATA,

SIVE

LEGES MOTUS.

LEX I.

*Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*

Projectilia perseverant in motibus suis, nisi quatenus a resistentia aeris retardantur, & vi gravitatis impelluntur deorsum. Trochus, cujus partes cohærendo perpetuo retrahunt sese a motibus rectilineis, non cessat rotari, nisi quatenus ab aere retardatur. Majora autem Planetarum & Cometarum corpora motus suos & progressivos & circulares in spatiis minus resistentibus factos conservant diutius.

LEX II.

*Mutationem motus proportionalem esse vi motrici impressæ, & fieri secundum lineam rectam qua vis illa imprimitur.*

Si vis aliqua motum quemvis generet, dupla duplum, tripla triplum generabit, sive simul & semel, sive gradatim & successive impressa fuerit. Et hic motus (quoniam in eandem semper plagam cum vi generatrice determinatur) si corpus antea movebatur, motui ejus vel conspiranti additur, vel contrario subducitur, vel oblique oblique adjicitur, & cum eo secundum utriusque determinationem componitur.

PROPOSITIO X. PROBLEMA. V.

*Gyretur corpus in Ellipsi: requiritur lex vis centripetæ tendentis ad centrum Ellipseos.*

Sunto $CA$, $CB$ semiaxes Ellipseos; $GP$, $DK$ diametri conjugatæ; $PF$, $Qt$ perpendicula ad diametros; $Qv$ ordinatim applicata ad diametrum $GP$; & si compleatur parallelogrammum $QvPR$, erit (ex Conicis) $PvG$ ad $Qv$ *quad.* ut $PC$ *quad.* ad $CD$ *quad.* & (ob similia triangula $Qvt$, $PCF$) $Qv$ *quad.* est ad $Qt$ *quad.* ut $PC$ *quad.* ad $PF$ *quad.* & conjunctis rationibus, $PvG$ ad $Qt$ *quad.* ut $PC$ *quad.* ad $CD$ *quad.* & $PC$ *quad.* ad $PF$ *quad.* id est, $vG$ ad $\frac{Qt\ quad.}{Pv}$ ut $PC$ *quad.* ad $\frac{CDq \times PFq}{PCq}$. Scribe $QR$ pro $Pv$, & (per Lemma XII.) $BC \times CA$ pro $CD \times PF$, nec non, punctis $P$ & $Q$ coeuntibus, $2PC$ pro $vG$, & ductis extremis & mediis in se mutuo, fiet $\frac{Qt\ quad. \times PCq}{QR}$ æquale $\frac{2BCq \times CAq}{PC}$. Est ergo (per Corol. 5 Prop. VI.) vis centripeta reciproce ut $\frac{2BCq \times CAq}{PC}$; id est (ob datum $2BCq \times CAq$) reciproce ut $\frac{1}{PC}$; hoc est, directe ut distantia $PC$. *Q. E. I.*

*Idem aliter.*

In $PG$ ab altera parte puncti $t$ posita intelligatur $tu$ æqualis ipsi $tv$; deinde cape $uV$ quæ sit ad $vG$ ut est $DC$ *quad.* ad $PC$ *quad.* Et quoniam ex Conicis est $Qv$ *quad.* ad $PvG$, ut $DC$ *quad.* ad $PC$ *quad.*: erit $Qv$ *quad.* æquale $Pv \times uV$. Unde quadratum chor-



Fig. 1.3: Extract of Newton's Philosophiae Naturalis Principia Mathematica
Left: First and second axiom
Right: Typical text with diagram and calculation without using the equality sign "="

In this book for the first time the fundamental laws of classical mechanics were defined which we today call Newton's Axioms. In the following they will be described in detail. Doing this a modern wording is used and in addition the connected equations will be presented using vectors. The definition of physical parameters as vectors, i.e. the combination of magnitude and direction was first used by the German teacher Herrmann Günter Graßmann (1809-1877) and was therefore not established in the 17th century. Although Newton could not know this kind of presentation, it is today's standard and therefore it shall be utilized here.

1. *The Principle of Inertia*

   An object with constant mass either remains in a state at rest or continues to move at a constant velocity, unless acted upon by force.

$$v = const. \quad if \sum_i \vec{F}_i = 0 \tag{1.03}$$

   This determination needs a high degree of abstraction because all motions, that can be observed in daily life, are more or less superimposed by effects like friction or gravitation.

2. *The Basic Principle of Dynamics*

   The rate of change of momentum is directly proportional to a force applied. For constant mass systems, force is mass multiplied by acceleration.

$$\vec{F} = m\vec{a} \tag{1.04}$$

3. *The Principle of Reaction*

   When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

$$\vec{F}_{12} = -\vec{F}_{12} \tag{1.05}$$

   or generally „action is equal to reaction".

There is a further basic principle that can be derived out of the publication, but this was not assessed as an axiom by Newton. It is also particularly important and therefore today often referred to as Newton's 4th axiom.

4. *The Principle of Superposition*

   If several forces interact, they add up like vectors.

$$\vec{F}_{res} = \sum_i \vec{F}_i \tag{1.06}$$

These 4 axioms form the foundation of classical mechanics, where all processes can be referred to.

It is worth mentioning that the imprimatur for the Philosophiae was granted by Samuel Pepys (1633-1703). Newton belonged to his large circle of friends. Different to countries controlled by the Catholic Church, where representatives of the inquisition were responsible for the approval of publications, in England this was his duty as the president of the

Royal Society. Pepys is well known until today for his secret dairies written between 1660 and 1669, which were found shortly after his death and then published. They contain interesting reports e.g. about the Plague 1665 and the great fire in London 1666. Further on the drastic comments on his fellow citizens and the notes about his many extramarital relations are to be mentioned which he described in any detail. He is one of the most important authors of that time and his books are still published today.

Beside his publications Newton also created the first reflecting telescope, which was much valued by the scientific community. Further on he was co-founder of the infinitesimal calculus. This led to a bitter dispute with Gottfried Wilhelm Leibniz (1646-1716) about the first priority of the discovery. He brought him to the court of the Royal Society – whose president he was at that time – accusing him of plagiarism and not surprisingly Leibniz lost the struggle. Newton vaunted himself later that he had broken his heart. Today Newton and Leibniz are considered the independent co-founders of this part of mathematics.

However, beside his epoch-making discoveries Newton’s main passion belonged to alchemy, on which he concentrated a broad part of his research work. A major part of the books belonging to his heritage, now preserved by the Kings College in London, is dealing with themes connected to alchemy. Further, he served as Warden (1696-1700) and Master (1700-1727) of the Royal Mint in London. So, he finally was not able to produce gold or silver, but this appointment brought him into a position to rule money.

Due to his special character Newton carried out his job at the Royal Mint in a very serious way. One of the main problems of this institution at that time was the coining of counterfeit money. The silver coins minted by the Royal Crown were fined down and the produced swarf was remelted and coined into false money. He persecuted the offenders in a rigorous way and brought them to court, what at that time generally meant that they were sentenced to death. This and many other additional occurrences are presented in the very unorthodox book of F. Freistetter (Newton, the way an asshole reinvented the world, in German language [83]).

## 1.3 Light and radiation

Beside classical mechanics further important foundations for the following considerations are the nature of light and the basic physical principles of radiation. Early history shows, dependent on the particular cultural background, that different myths exist to describe the origin of light and corresponding to it the ability for man to see. In Greek mythology goddess Aphrodite created the eyesight out of the four elements earth, water, wind and fire; the main understanding of this divine gift was, that light was leaving the eyes, and, in a reaction, different objects became visible.

About 300 BC the important Greek Philosopher Euclid started examinations concerning the behavior of light and found out, that light beams travel in straight paths and in a further approach he also discovered the laws of reflection. In addition, he concluded that it is not reasonable to adhere to the opinion that light leaves the eye because in such a case no differences between day and night would be possible. Although these observations paved the way for further discoveries and improvements of the theory, it took about 2000 years to take the next steps.

Newton followed the idea, that light is consisting of small corpuscles with different sizes and properties. He carried out experiments with mirrors, lenses, and prisms to verify the laws of reflection and to discover the general nature of light. He was partly successful, but his theory using corpuscles was not able to explain some of the experimental results; especially the nature of interferences caused conflicts to his approach which could not be solved.

In the year 1690 the Dutchmen Christiaan Huygens (1629-1695) developed the first complete wave-theory of light. With this comprehensive theoretical approach, it was possible for the first time to explain the phenomena of reflection and refraction of light without discrepancies. Beside his pioneering work concerning the wave-theory he was also very successful as astronomer; he was the first to discover Titan, the moon of Saturn, and he identified the rings of Saturn. For this purpose, he used an improved telescope, which he had constructed and built co-working with his brother Constantijn. He also developed mathematical basics concerning the figure $\pi$ using arithmetic series, further to the application of logarithms and he is co-founder of the calculus of probabilities.

The wave-theory of light was discussed highly controversial for a long time, especially because the theory using corpuscles was the idea of the great Isaac Newton. One of the main arguments of supporters of Newton's theory was that light is completely shielded by barriers and no wave can be seen behind it, like e.g. visible on a water surface when a wave is passing an obstacle. It was not known at that time that the wavelengths of light are very small (approx. 400-700 nanometers). It was not before the double-slit experiment was performed by Thomas Young (1773-1829) at the beginning of the 19th century, which supported the contention that light is composed of waves, that the discussion ended. Young also solved the problem to explain the effect of polarization, because he interpreted light as a transversal wave. According to our today's vocabulary this means, that the vectors of the electric and magnetic field are perpendicular to each other and also to the propagation direction (see Fig. 1.4). This contrasts with the behavior of a sound wave which is propagating longitudinal; this means that the transporting medium e.g. air or water is oscillating in moving direction and thus no polarization is possible. Linear polarization of light is observed when many superimposing waves show the same orientation.

In the year 1676 Ole Christian Rømer (1644-1710) was the first to provide evidence that the velocity of light is limited. He observed the eclipse of the Jupiter-moon Io, which occurs during perigee (shortest distance to earth) earlier than during apogee (farthest distance). This result was in contradiction to the established understanding of many others, from Aristotle to Descartes, who were convinced that the speed of light was unlimited, so it was only reluctantly accepted. The results found by Rømer, who just measured the time delay, were converted by Huygens 1678 using calculations to a velocity of approx. 212000km/s, which is approximately 70% of the correct value. Evaluated in comparison to the available resources at that time the result was already remarkable exact.

According to the understanding of that time it was presumed that light needs a transportation medium for propagation. This idea was transferred from the knowledge about the conditions valid for the transport of sound, where atoms resp. molecules are forced to oscillate. The center of the oscillation is always constant, which means that atoms or molecules in an observation of the average position are not moving but that just energy is transported by the waves.

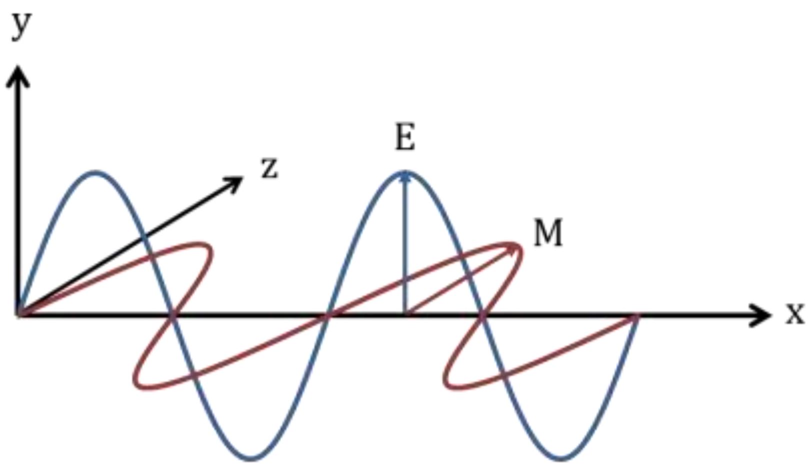



Fig. 1.4: Propagation of an electromagnetic wave with the components of the electric and magnetic field (E and M)

First knowledge concerning this was collected by Otto von Guericke (1602-1686). In the year 1649 he invented the vacuum pump and used it for many experiments. The most spectacular was surely the demonstration of the action of force caused by air pressure. He produced two half spheres made of copper (diam. approx. 42 cm) and during the Reichstag 1657 held in Regensburg he combined these with a sealing and used his pumps for evacuation. In presence of Kaiser Ferdinand III, it was shown that eight harnessed horses at each side were not able to tear the combination apart. This experiment was so impressing to the audience, that Archbishop Johann Philip von Schönborn bought and passed it to his Jesuit College at Würzburg. Beside this spectacular experiments Guericke also performed basic investigations and was able to show that a vacuum is not conducting sound, but that light is passing.

The medium that, according to the knowledge of that time, was needed to transport light was called "luminiferous ether" or just "ether". The word is originating from the Greek myths and is in its genuine sense describing the (blue) sky. In contrast to the four earthly elements (these are earth, wind, water, and fire which are interestingly complementary to the conditions of aggregation solid, liquid, gaseous and ionized), ether was the 5th element, which stood in relation to heaven and therefore in contrast to the others was inalterable [4d].

During the passing centuries, many theories were developed to describe the nature of ether. As its main characteristics it was expected to permeate anything but not to produce any resistance to objects, because in this case it would influence physical laws. It was the general view that light is transported by ether in the same way as sound by air. However, there were two observations from experiments which prevented a distinct determination because they are in fundamental contradiction:

1. The effect of stellar aberration first detected in the year 1725 by James Bradley (1693-1762).

2. The effect observed in moving transparent media (e.g. glass or water) of dragging light in the direction of motion. This effect is dependent on the refraction index of the media.

*Point 1:*

Stellar aberration is a definition used in astronomy to describe a small apparent shifting of the position of stars, when an observer is moving in transverse direction. Earth is travelling around the sun with a speed of about 30km/s; this means that after half a year a measuring effect of 60km/s compared to the position of an unmoved sky will appear. This is causing a misalignment for the incoming light, which was first detected by Bradley with precision measurements using a zenith telescope. This type of telescope is designed to point straight up to or near to the zenith. Bradley installed it in his house along the chimney and spent most of his observation time upon a bench underneath the instrument.

The major precondition for the occurring of an aberration effect is that the speed of light is limited. Bradley was able to measure that the speed of light is 10210-times higher than the velocity of the earth orbiting the sun. He achieved a remarkable precision of 2% compared to the exact value we know today. Furthermore, he concluded that ether could not be affected by mass like that of earth. If earth would drag ether with it, then no aberration effect could be detected.

This effect must not be mixed up with the measurement of the parallax, i.e. the deviation of the angle of a star relatively close to earth depending on the position of earth to the sun during the year. Such a measurement was first successfully completed by Friedrich Wilhelm Bessel (1784-1846) in the year 1838 during observation the star 61 Cygni. Out of the measured angle he calculated a distance of 10.28 lightyears to the sun (today's value is 11.4 lightyears). The parallax effect is approx. 2 orders of magnitude smaller than that of aberration.

Distance determinations are an essential part of cosmology today. However, the measurement of the parallax is possible with earthbound telescopes only up to distances of about 100 light-years. In 1912 Henrietta S. Leavitt (1868-1921) found out by extensive investigations on stars of the Magellanic Clouds that the absolute value of the maximum brightness of periodically changing stars is directly related to their period. Since there are enough variable stars in the near-earth region, a first calibration was possible, and the extent of our galaxy could be determined (100,000 light years) and consequently the distances to the Large and Small Magellanic Cloud (163,000 and 200,000 Lj. respectively) and later by Edwin Hubble (1889-1953) to the Andromeda Galaxy (2.5 million Lj.).

*Point 2:*

In the year 1810 Francois Arago (1786-1853) made an experiment where he used a prism for aberration measurements. The expected alteration effect, however, could not be observed. Already in 1818, a theory was presented by Augustin Jean Fresnel (1788-1827), that light is partly dragged by the medium in moving direction and that the appearing effect is dependent on the refraction index of the media.

In the year 1851 Hyppolyte Fizeau (1819-1896) performed an experiment where he measured the speed of light in running water. He found the result that the speed of light is increasing when the examined beam has an orientation in moving direction of the water and decreasing when the direction is opposite. He also verified the equations first postulated by Fresnel. This result changed the view on ether and the characteristic of a dragging effect by matter was added.

Because of the fundamental importance of the presented experiments these will be discussed in detail. Aberration is presented in chapter 2.1.2 and the dragging effect in moving transparent media in chapter 4.2.

Towards the end of the 19th century due to the inconsistent experimental results many different ether-theories were discussed, who should be able to explain the complex situation. Even Einstein, in his most probably first publication as a youth discussed an approach to the problem. Looking at the situation at that time it can be summarized, that no consensus on the nature of ether could be achieved, but that nobody seriously denied the existence.

## 1.4 Electromagnetism

Phenomena connected to electrostatic effects were already known to Greek philosophers in ancient times. When amber (Greek: electron) is rubbed with a fur or cloth it will show visible effects like e.g. the emission of sparks or attraction of dust and other small particles. Also, magnetism is well known since a long time; in this case the observed phenomena were generally connected to the availability of magnetic iron ore named magnetite. The origin of the word is deriving from the Greek region called Magnesia, where these stones were found already in ancient times. A practical use was solely for application as a compass, which was known in China already in pre-Christian times and in Europe from the beginning of the 13th century on.

This did not change before the electrostatic generator was invented. Otto v. Guericke made experiments using a rotating Sulphur sphere and tried to find evidence for the existence of cosmic forces. The experimental set-up is referred to as the first electrostatic generator; although Guericke found attracting and repelling force, he had most probably no idea about the background of his experiment. Later constructions by successors using glass and leather were able to create quite high voltages. A further progress was made when the "Leiden Jar" was developed. This is the early form of a capacitor and from now on it was possible to generate and to store charges. Although now first experiments were possible and different electrical phenomena became known the invention was mostly used for spectacular presentations to an interested audience. It was e.g. immensely popular to pass electric shocks to a crowd of people who were taking each other by the hand.

However, during the 18th century also some new scientific perceptions were derived, e.g. the frog leg experiment by Luigi Galvani (1737-1798), where he found that a leg of a dead frog is kicking as if alive when it is touched with an electrostatic generator. Further the experiments of Benjamin Franklin (1706-1790) proving that lightning is a form of electricity shall be mentioned. However, because of the limited experimental capabilities these approaches were exceptions, and it is not reasonable to talk about a comprehensive scientific approach concerning this matter.

A turning point was reached when in the year 1799 Alessandro Volta (1745-1827) constructed the first stable electric power supply in form of a battery, which was later called "Volta's pile". For the pile he used elements made of copper and zinc, which were separated by pieces of leather or paper soaked with sulfuric acid and so electrochemical cells were built. The pile was consisting of several cells and so it was possible to produce more than 100 Volt (a physical unit later named after him). It is for sure one of the most important inventions of all time and the public paid high tribute to him. He also drew admiration from Napoleon Bonaparte for his invention and in 1810 he was made a count.

This invention laid the basis for many new experiments and subsequently to further important discoveries. Namely Faraday, Ampère, Heavyside and Lorentz are to be mentioned, who examined the properties of electric charge, electrical current and the relation to magnetism. André-Marie Ampère (1775-1836) was the first to introduce the concept of a field and discovered an electromagnetic relationship, which was of great importance for scientific progress.

Further knowledge was established by theoretical considerations of James Clerk Maxwell (1831-1879) who was able to show, that the existence of electric and magnetic effects is connected. He also used for the first time the expression of electromagnetic fields. Maxwell demonstrated that electric and magnetic fields travel through space as waves moving at the speed of light. He proposed that light is an undulation in the same medium that is the cause of electric and magnetic phenomena; this medium was supposed to be the "luminiferous ether". A further important result of his investigation was that the relations he developed, which later were called "Maxwell-Equations", are not conform to the Galilei-Transformation and so this was in contradiction to classical theories.

The experimental work of Heinrich Hertz (1857-1894) later confirmed that the shining of light can in fact be interpreted as propagation of electromagnetic waves. From 1889 until his death, he was professor for physics at the University of Bonn. To this very day the experiments built by him are working and presented during the lectures of experimental physics. They provide an impressing view at the technical possibilities of that time.

Towards the end of the 19th century knowledge concerning electromagnetic effects had improved significantly. The gathered knowledge both on theoretical and experimental basis made clear for anybody that ether for the transport of electromagnetic waves must exist. This view was generally also expanded to gravitation.

## 1.5 The Michelson-Morley Experiment and first interpretation

Albert A. Michelson (1852-1931) was one of the most important physicists at the end of the 19th century. In the year 1869 he joined the US Naval Academy and graduated in 1873. After 2 years at sea, he became instructor in physics and chemistry at the naval academy until 1879. Then he was posted to the Nautical Almanac Office in Washington and in the following year he obtained leave of absence to continue his studies in Europe (Berlin, Heidelberg, and Paris). In the year 1877 he married the daughter of a wealthy stockbroker and so he achieved financial independency. He was extremely interested in physical experiments, especially in measurements of the speed of light; his special knowledge as a naval officer was very helpful, because during his duty one of his tasks was the measurement of distances by optical means. In the year 1881 he resigned from the navy and started his scientific career. In 1907 he was the first American to receive the Nobel Prize in physics.

The first experiment by Michelson to provide evidence of "luminiferous ether" performed 1881 at the Helmholtz' laboratory in Berlin was not successful, because the vibrations of the city traffic made it impossible. It was repeated at the observatory in Potsdam and there he found a zero result [6]. Due to experimental shortcomings in the execution the result was first generally rejected by most scientists. Together with Eduard W. Morley (1838-1923) the apparatus was improved, and the experiment was repeated in Cleveland in 1887 [7]. It was now detected and verified without doubt, that the measurement of the speed of light led to the same results in every direction, irrespective of the movement of the measuring device in comparison to the supposed ether. Because of the paramount importance of the experiment the set-up of the device and the interpretation of the results will be discussed in detail (see chapter 9.1).

During the following years, the experiment was widely discussed and addressed in many publications, of which the most important shall be mentioned shortly here. George F. FitzGerald proposed already in 1889 the idea, that the length of material bodies is contracting at velocities close to the speed of light [8]. He expected this contraction to be dependent on the square of the ratio of their velocities. The same issue was also predicted independently by Hendrik A. Lorentz (1853-1928) three years later [9]. Because of further contradictions Lorentz and also Henri Poincaré (1854-1912) introduced in the year 1900 the concept of "local time" [10]. This means, that in view of an observer at rest the clocks of other moved observers show different times during a synchronization process depending on their distance. It was now possible to perform calculations between systems with different velocities. The basic equations were converted into their modern appearance by H. Poincaré, who also created the name "Lorentz-Transformation [11]. It was shown that contraction of space and dilatation of time is covered by the same factor (Poincaré named it $k$, Einstein $\beta$ today usually the Symbol $\gamma$ is used).

The transformation equations are

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \tag{1.07}$$

$$x' = \gamma(x - vt) \tag{1.08}$$

with

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.09}$$

In these equations $x$ and $t$ are the coordinates of a reference system and $x'$ and $t'$ the coordinates of another system moving constantly relative to this, the coordinates in $y$- and $z$-direction are not changing. These relations today are normally called Lorentz-Transformation (LT) or "Lorentz-boost". Although the term "boost" implies the existence of an accelerated system this is not the case. In contradiction to this the equations describe relations between systems, which are constantly moving relative to each other and are not subject to acceleration or rotation. Furthermore, these equations show similar characteristics compared to the Maxwell equations which are valid for the interpretation of electromagnetic fields.

A detailed derivation of the equations will be presented later. It must be mentioned further, that at velocities $v \ll c$ the factor $\gamma$ is approaching 1 and the equations are merging with the Galilei-Transformation in Eq. (1.01).

## 1.6 Einstein's Theory of Special Relativity

In the year 1905 Albert Einstein published his famous paper "On the electrodynamics of moving bodies" and presented a main contribution to the theory of relativity (later called "Special Relativity" or SRT). For an exact representation it is necessary first to introduce the concept of an inertial system. Inertial systems are defined by the fact that they are moving in arbitrary speed to each other but are not accelerated or show a rotational motion.

Fundamentals of SRT are the principle of relativity and the principle of constancy of the speed of light. In the original version Einstein has chosen the following formulation [12]:

> "Principle of Relativity: The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems in uniform translatory motion relative to each other.
>
> Principle of constancy of the speed of light: Every light ray moves in the "resting" coordinate system with a certain speed V, independent of whether this light ray is emitted by a resting or a moving body. Here is
>
> $$\text{velocity} = \frac{\text{lightpath}}{\text{time period}}$$
>
> where "time period" is to be understood in the sense of the definition of § 1."

The interpretation is not easy, also because Einstein speaks here of a "resting" system. But the meaning, especially of the 2nd paragraph, is clear, when it is considered that the procedure chosen in the further text, especially the application of the synchronization procedure (today: Einstein synchronization, see chapters 3.4 and 12.2). Because of the complexity, details will be discussed later in this paper.

Important here is the radical break with the previous approach to the establishment of a physical theory. While Lorentz and Poincaré interpreted the available experimental results, derived the transformation equations from them and then found the principle of relativity, Einstein put this first and was able to derive the equations in a relatively simple way. Generally speaking, these are the principles bottom-up (Lorentz, Poincaré) and top-down (Einstein).

Lorentz in 1892 first assumed that there must be an absolutely resting fundamental system [9]; then in 1910 he was of the opinion that it would never be possible to distinguish between the two approaches [13]. Independently, however, he welcomed Einstein's formulation of relativity and became its advocate [14,15], especially because of the "boldness" of the approach [14].

At the time of development, it was not foreseeable that a metrological verification of the theory would ever be possible. In the following decades, however, new experiments were added, the most important of which are those of Kennedy-Thorndike [16] and Ives-Stilwell [17,18], which will be discussed in detail later. In addition, the measurement accuracies were improved more and more; modern measurements with very high precision showed among other things the validity of the time dilation formulated by Lorentz impressively [19,20,21]. On the other hand, however, the Theory of Special Relativity in its general form cannot be proved in principle. Every positive experiment strengthens the theory, but a single unambiguous counterexample would lead to the fact that it must be considered as disproved.

In the first part of his publication, Einstein derived the transformation equations from the principles already mentioned. However, since these had already been discovered by Lorentz before, they are generally called "Lorentz equations" today. Einstein's publication does not contain any literature references and thus a parallel development to Lorentz can be concluded. Moreover, it is clearly the merit of Einstein to have combined the photoelectric effect with these relations and thus to have been able to break completely with the ether concept.

In further considerations of the principle of relativity, Einstein also predicted already in 1905 the effect that the kinetic energy of a moving mass at higher velocities according to the formula

$$E_{kin} = m_0 c^2 (\gamma - 1) \tag{1.10}$$

must increase [22]. This effect has been experimentally confirmed and is now commonly referred to as relativistic mass increase. It is important to see here that the designations are different. Lorentz chose $x, t$ for the reference system, while Einstein used $m_0$. In Einstein's probably best-known formula

$$E = mc^2 \tag{1.11}$$

the total mass $m$ includes the part of the kinetic energy defined in Eq. (1.10). Also, the mass increases with higher velocity by the factor $\gamma$. Both representations are used in parallel until today.

| | Lorentz-equations | Relativistic mass increase |
|---|---|---|
| Equation | $t' = \gamma\left(t - \frac{v}{c^2}x\right)$<br>$x' = \gamma(x - vt)$ | $m = \gamma m_0$<br>$\{E = mc^2 = \gamma m_0 c^2\}$ |
| Reference system | $x,\ t$ | $m_0$ |
| Moving system | $x',\ t'$ | $m$ |

These relations together form the basis for the Theory of Special Relativity.

For the description of the principles postulated by Einstein, today often called Einstein axioms, there is no uniform definition, and it is chosen differently in every publication. In some cases, the description for both axioms are descriptive ("no differences can be found in measurements"), in others the properties are put in the foreground ("the speed of light is the same in all inertial frames", "all inertial frames are equivalent"). Although these expressions are identical at first sight, there are important differences which have to be discussed in more detail in the following. The already mentioned relativistic increase of mass is not mentioned in the axioms, but without this effect the statement concerning the principle of relativity would not be possible.

However, the principle of relativity formulated by Einstein also requires a precise interpretation. First, this can be divided into the following detailed statements:

a) If identical experiments are carried out by different observers in reference systems moving uniformly relative to each other, the results will be the same.
b) An observer can describe results of any experiment in another inertial system that shows a constant relative movement using only the Lorentz transformation equations and the relativistic increase of mass. In particular, the observation of the time sequence of events is the same in all cases.
c) All systems moving uniformly relative to each other are equivalent and there is no absolute "system at rest".

The statement a) will now be defined as “principle of identity”, b) as “principle of equivalent observations” and c) as “principle of complete equivalence of all inertial systems”. While points a) and b) are backed up by multiple test results, this must be considered in a differentiated manner for point c). Although there is a wide consensus about the validity of the SRT within the physical research community, there are still many theoretical and experimental attempts to refute individual points. This concerns in particular measurements concerning minor violations of the Lorentz equations, which have been predicted by theoretical considerations concerning a general, unified theory of all laws occurring in nature. Furthermore, a possibility to integrate a state of absolute rest is still searched for.

Finally, some interesting historical questions should be addressed. Einstein became involved with physical topics at an early age. At the age of 16, he wrote a letter to his uncle in which he outlined possible experiments to prove the existence of ether [99]. In 1901, around 6 years later, he already had more far-reaching ideas and wrote about himself and his future wife Mileva Marić, whom he met while studying physics and mathematics at the ETH in Zurich: " How happy and proud will I be, when we both together have brought our work on the theory of relativity victoriously to an end". She was the only woman in this field of natural science, which was clearly dominated by men at the time. However, her contribution to the development of the theory is unclear, and it is also doubted whether the ether theory had already been overcome at this time [85]. In the epilogue to his work, Einstein expressively thanked his friend and fellow M. Besso that he was faithfully standing at his side during the work and that he owes him valuable suggestions; his wife was not mentioned at all [12].

Although there is no clear evidence, it seems very plausible that Einstein had the extensive support of his wife in 1905, the year in which he submitted his dissertation and wrote another 4 publications in addition to his work at the patent office. In 2005, Mileva Marić was officially honored as a co-founder of the theory of relativity by the university ETH Zurich [84]. However, there are a large number of publications on this topic and also dissenting opinions (e.g., [85]). In 2003, television stations in the USA broadcast the documentary “Einstein's wife”. During and after the broadcast, viewers were asked online for their opinion and 75% of viewers were convinced that his wife had indeed collaborated with him. However, “history is not a matter for democratic voting” [85]. Due to the lack of sources, it must be stated today that we simply do not know the details.

This also applies to information about her first child. Mileva Marić gave birth to a girl in 1902, before her wedding (which took place in 1903). For this purpose, she returned alone to her parents in Novi Sad (today Serbia, then Austrian Monarchy); it is not clear whether the child died there or was given up for adoption. Even though Einstein was a public figure as the most famous scientist of his time, there are mysteries about this early period that will probably never be disclosed.

## 1.7 Current discussions

Already at the beginning of the second half of the last century it became clear that the background radiation of the Big Bang, which was discovered at that time, runs completely isotropic and constant in all space directions. This has made it possible to measure a velocity relative to this background radiation. Recent measurements with extreme accuracy have shown that our sun moves with $369.1 \pm 0.9$ km/s relative to it [23]. It should be noted here that the sun is orbiting the galactic center at a speed of approx. 220 km/s, and that the velocity is directed almost opposite to it. This means, that our galaxy is moving with a speed of approximately 600 km/s relative to the detected background radiation [19].

In particular because of these observations there have been considerations to bring special relativity in accordance with a state of absolute rest (i.e. “relativity without relativity” [24]). None of these theories were able to show results without severe discrepancies to experimental findings. Details are summarized in chapter 12.1.

Moreover, a problem has recently arisen from the measurements of velocities faster than that of light. Experiments carried out by different research groups for several years already show that such velocities can be measured in connection with tunnelling experiments. However, there are great differences in the interpretation of these results. While some researchers are convinced that despite of observed superluminal velocities no information can be transmitted with this speed, others expect this to be the case. If the latter is true, this is basically not compatible with the theory of special relativity. The effects will be discussed in detail.

Further theoretical considerations disclose a severe problem, which is a fascinating part of today’s discussion within physics: It is broad agreement that the fundamental physical theories of our time, the theory of (general) relativity and quantum mechanics are in contradiction [20]. The problems which occur are presented in a very comprehensive way by T. Müller [25].

Generally, it can be stated, that after more than 110 years since the first presentation of Special Relativity many questions are still open. It is the aim of this presentation to develop proposals for a modification.

## 1.8 Contents of this presentation

Today, the Theory of Special Relativity (SRT) represents a fundamental standard within physics. There is an almost unmanageable number of books, literature, and lecture notes on this subject. This paper is intended as a supplement to other books on this subject, in particular the excellent work of Max Born (1882 -1970), a contemporary and friend of Einstein [26]. The book was first published in 1920 and is still reprinted today with some necessary additions. In addition to the theoretical part, which is deliberately kept simple for training purposes, the developments in physics that took place in the 19th century are also very accurately reproduced here. This also applies to the important subject of electromagnetism, which is only briefly touched upon here.

Usually, papers on special relativity follow the scheme that first the results of classical experiments are presented and based on them the theory is formulated. In the present case, however, the theory shall be chosen as axiomatic framework and then the consequences resulting from it shall be discussed. As will be shown, this systematic approach also captures effects that otherwise are not in focus but are of great importance. The resulting calculations partly require the use of numerical methods. Their execution is described in detail in an appendix (A to D).

The central approach of the presented investigations is the following: First, all investigated phenomena are presented from the point of view of an observer at rest. Based on this, it will be evaluated how the same facts arise for a moving observer; for this, exclusively the formalism of the Lorentz transformation and the relativistic mass increase will be used. It will be shown for a large number of investigated relations that the same results are obtained for both observers and that no counter example exists.

In the following, first an exact representation of the connections within special relativity is given. This begins with investigations to the signal exchange between two observers moved relatively to each other. Afterwards the Lorentz transformation is derived from the basics of Special Relativity (equivalence of all inertial systems and constancy of the speed of light).

In addition, the important item of the synchronization of events is considered in more detail. This is done first on the basis of synchronization by means of signal exchange, later also by exchange of clocks. Subsequently, the relations between several moving observers are the subject of considerations. In addition, the relations of signal exchange in moving transparent media are also investigated. In all examples it can be stated that the validity of the equations developed by Lorentz is guaranteed without restrictions.

The synchronization with slow clock transport presented in detail in chapter 5 contains some new approaches for the unambiguous proof of a zero result.

In chapters 6 and 7 considerations of relativistic influences on mass, momentum, force, and energy are made. Further the situation of observers exchanging signals with others during acceleration and afterwards will be investigated. For this purpose, the conditions during elastic relativistic collisions are investigated, and the relationship for a relativistic rocket equation is derived from this. It is also shown here in all cases that there are no differences in the considerations for an observer assumed to be at rest or to be moving.

Further investigations on the conditions during the exchange of light signals with constant frequency show new aspects for the interpretation of classical experiments (chapter 8). It will be shown that at the transition between systems with a movement relative to each other not the speed of light, but the phase velocity of light is the relevant parameter. As a consequence, classical experiments like the Michelson-Morley experiment and also the Kennedy-Thorndike experiment have to be re-evaluated, although their basic statements remain the same.

Furthermore, the case is discussed, when superluminal velocities occur, which are observed in connection with tunnelling experiments. If it is possible to transfer information in this case faster than light, contradictions will occur between identity and equivalence principle.

A proposal is developed, how these contradictions can be eliminated. In contrast to the basic idea developed by Einstein, a top-down concept with given principles, a different approach is chosen. Instead, the Lorentz equations are used as a basis and, in addition, the concept of relativistic mass increase with increasing relative velocities derived by Einstein from the principle of relativity. Their combination into an "Extended Lorentz Theory" allows to describe all phenomena occurring in nature in the same way as the Theory of Special Relativity. Absolute precondition is that information is transmitted at the speed of light. If a transport should ever be possible with superluminal velocity, then SRT is proofed to be false, for the Extended Lorentz Theory then the opportunity would arise to determine the position of a system of absolute rest.

Finally, on basis of these considerations, different experiments will be proposed. With their help, clear statements on the validity of the proposed theory could be made.

# 2. Relations between two moving observers

It was already mentioned before in the introduction of this presentation, that in the following the Theory of Special Relativity (SRT) will be placed first in an axiomatic way to discuss general physical relationships. Using this basis, different combinations for the exchange of signals between two observers will be examined first. This will start with point-shaped observers before they will be looked at as containing an extended space. Subsequently the relations of angles between moving observers during the exchange of signals will be investigated.

The consequences derived will be discussed and compared with observations and calculations presented in the literature. It will be shown that the results do not contain any contradictions. Furthermore, additional considerations concerning the calculations of angles will be derived. These are based on geometric calculations and lead first to the expected result that a defined contraction of space must exist. It will also be shown that the contraction must be considered as symmetric in moving direction and opposite to it. This will become important later for the examination of alternative theories, which will be discussed in chapter 11.1.

Following the historical development, the participating observers performing experiments will first be specified as “at rest” and “moved”. In further considerations it will become clear, that these definitions in general can be replaced by “relatively moving against each other”. This approach is not used very often today, but sometimes it still can be found in new literature [21].

## 2.1 Exchange of signals between point-shaped observers

Although the first considerations and deductions presented here will be trivial at first sight, these simple approaches are already providing clear evidence of the limits of classical mechanics. To avoid discrepancies, it is even necessary for simple constellations, like these are valid for the exchange of signals between two point-shaped observers, to implement the calculations of the Lorentz-Transformation.

In the following this will be shown for some simple examples before more complex considerations will be discussed in detail.

### 2.1.1 Movement decreasing or increasing the distance

When two observers A and B decrease or increase their distance without acceleration, the transmission of light signals periodically emitted is of general interest. Following the classical theory according to Newton it is apparent, that the moved observer will detect a larger interval compared to the observer at rest, although the period of emission is the same for both (see Fig. 2.1).

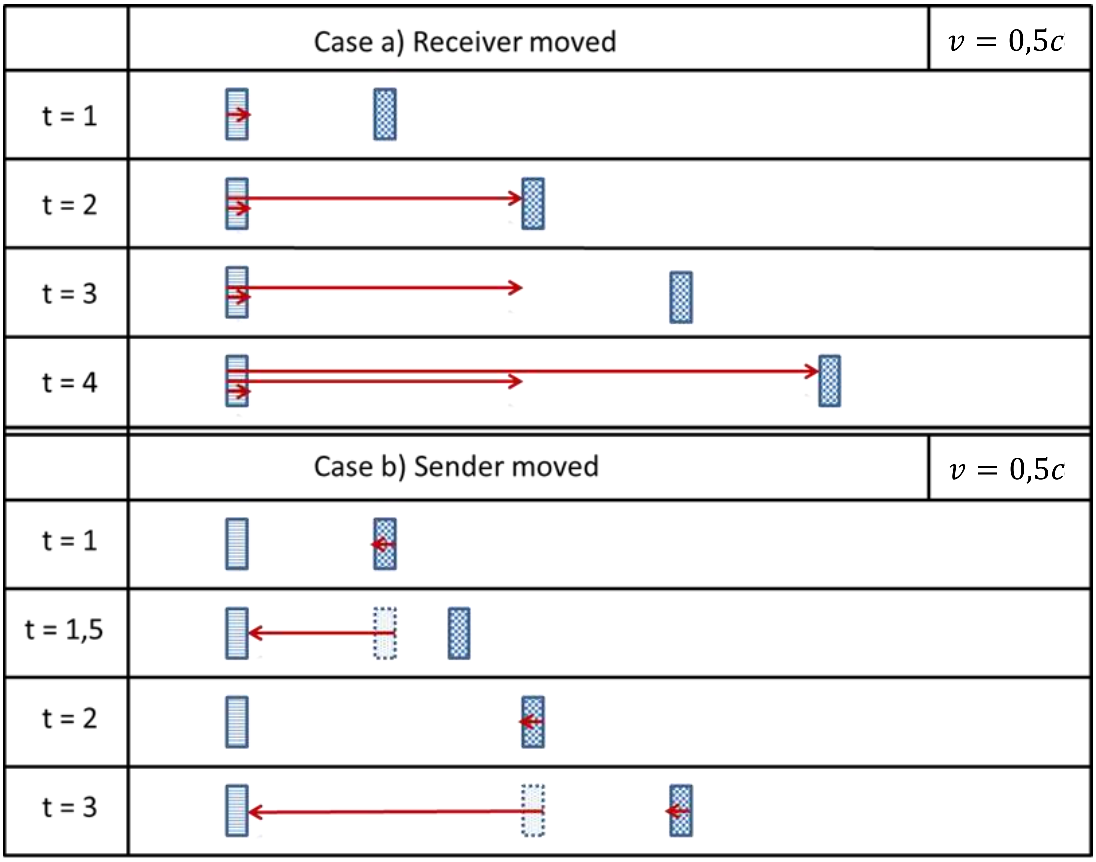



Fig. 2.1: Differences in the intervals of detected light signals by an observer at rest and a moving observer according to classical theory.
Observers have contact at $t = 0$,
Signal interval $\Delta t = 1\ TU$ (time unit),
Example for $v = 0{,}5c$

In this example with $v = 0{,}5\ c$ the moving observer would detect a signal every 2 time units ($TU$), whereas the observer at rest would find a difference of 1,5 $TU$. According to these considerations both observers would be able to calculate their velocity by the measurement of the signals from the partner. This is in clear contradiction to the experimental observation, that the results of trials like these are always independent of the state of motion.

In Fig. 2.2 the possibilities for the state of motion between a moved observer and an observer at rest are put together. Furthermore, in Tab 2.1 the fundamental relations are presented.

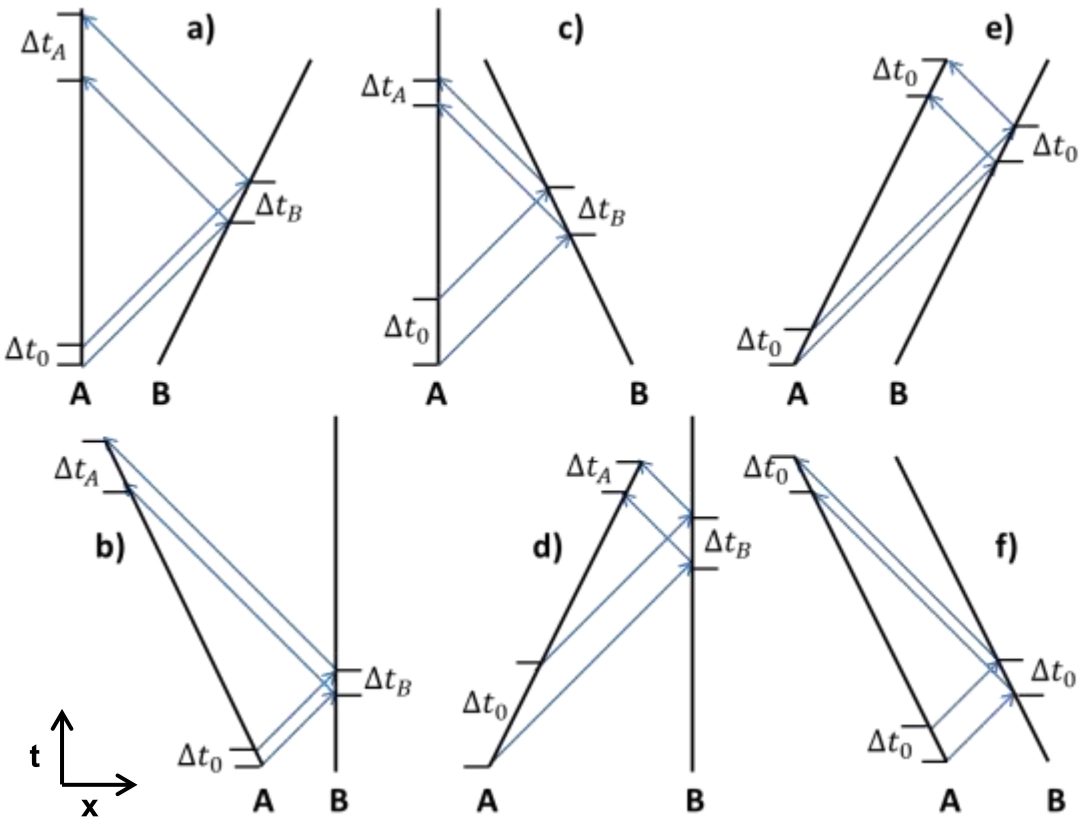



Fig. 2.2 Space-time diagrams for possibilities of light signal exchange

| | | | | | |
|---|---|---|---|---|---|
| a) | $\Delta t_B = \Delta t_0 \frac{1}{1-\frac{v}{c}}$ | c) | $\Delta t_B = \Delta t_0 \frac{1}{1+\frac{v}{c}}$ | e) | $\Delta t_B = \Delta t_0$ |
| | $\Delta t_A = \Delta t_0 \frac{1+\frac{v}{c}}{1-\frac{v}{c}}$ | | $\Delta t_A = \Delta t_0 \frac{1-\frac{v}{c}}{1+\frac{v}{c}}$ | | $\Delta t_A = \Delta t_0$ |
| b) | $\Delta t_B = \Delta t_0 \left(1+\frac{v}{c}\right)$ | d) | $\Delta t_B = \Delta t_0 \left(1-\frac{v}{c}\right)$ | f) | $\Delta t_B = \Delta t_0$ |
| | $\Delta t_A = \Delta t_0 \frac{1+\frac{v}{c}}{1-\frac{v}{c}}$ | | $\Delta t_A = \Delta t_0 \frac{1-\frac{v}{c}}{1+\frac{v}{c}}$ | | $\Delta t_A = \Delta t_0$ |

Tab. 2.1 Time intervals for the signal exchanges presented in Fig. 2.2

In the following the conditions for an exchange of light signals from A to B and vice versa according to Fig. 2.1 shall be presented in a simple space-time-diagram (see Fig 2.3). To realize this, the variations a) and b) from Fig. 2.2 will be combined.

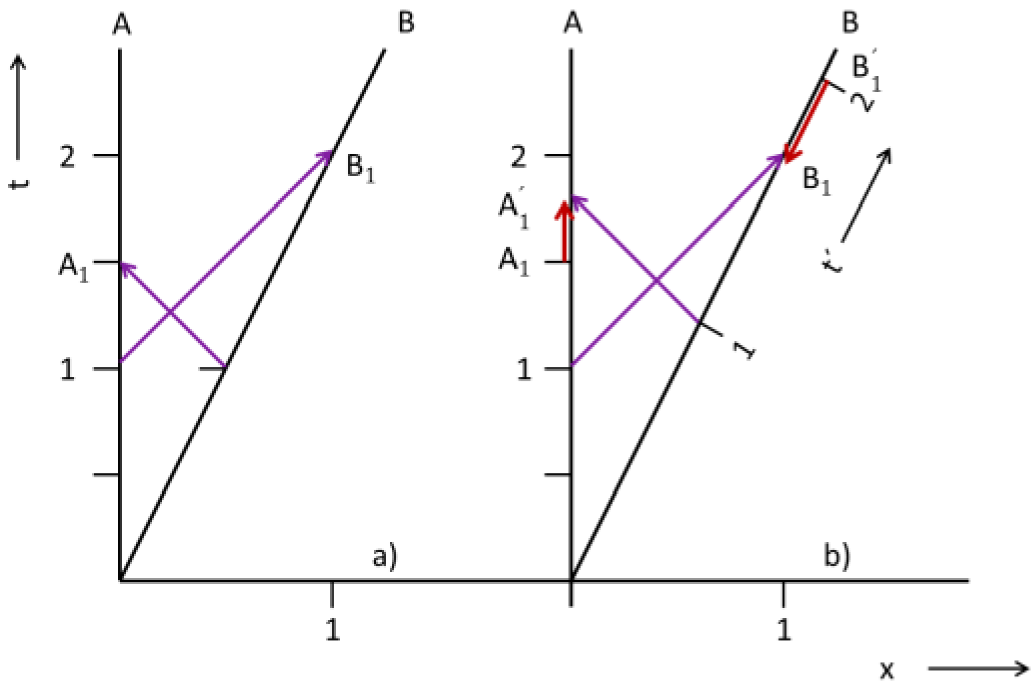



Fig. 2.3: Space-time-diagram for a signal exchange between observers A (at rest) and B (increasing the distance), Example for $v = 0{,}5c$
a) conventional (acc. to Galilei/Newton)
b) relativistic (acc. to Lorentz)

In case a) the conventional situation (acc. to Galilei/Newton) is presented. Both observers are emitting their signals at time $t = 1TU$ and these are detected at $A_1$ resp. $B_1$ by the partner. This diagram is valid e.g. for the exchange of acoustic waves, when A is at rest against a medium (i. e. air or water). But it was already mentioned before that this could not be detected by any experiments conducted using light signals.

Already at the end of the 19th century a solution for this (inside classical mechanics acc. to Newton existing) problem was presented by H. A. Lorentz. To realize this, it is necessary to assume, that at higher velocities an effect of time dilatation will be present. This means that time is running slower for the moved observer. This effect is integrated in part b) of the diagram. For observer B the time is running slower and therefore B is sending his signal later; this will arrive at the partner at $A_1'$. Because of the time dilatation the additional effect occurs that B is subjectively detecting the signal sent from A earlier. This effect is presented in the diagram by the transition from $B_1$ to $B_1'$.

The exact parameter of the time dilatation can be calculated in an easy way according to Fig. 2.2, cases a) and b). For the transition from a system at rest to a moved observer for $\Delta t_0$ the relation is valid

$$\Delta t_{AB} = \Delta t_0 \frac{1}{1 - \frac{v}{c}} \tag{2.01}$$

In opposite direction it is

$$\Delta t_{BA} = \Delta t_0 \left(1 + \frac{v}{c}\right) \tag{2.02}$$

To match $\Delta t_{AB}$ and $\Delta t_{BA}$ it is necessary to expand the equations (2.01) and (2.02) by the parameter $\gamma$ (where $\Delta t_{AB}$ will be smaller and $\Delta t_{BA}$ will be larger) and the equations develop to

$$\frac{1}{\gamma} \cdot \frac{1}{1-\frac{v}{c}} = (1+\frac{v}{c}) \cdot \gamma \tag{2.03}$$

with

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{2.04}$$

The parameter $\gamma$ calculated here is the same as the Lorentz-Factor of Eq. (1.03).

It is therefore not possible for observers A and B to decide, whether they are moving or at rest. This implies that observer B also has the impression, that the time is running slow for A compared to his perception.

The example presented here for observers who increase their distance can also easily transformed to the view of observers which are approaching each other (see. Fig. 2.4, larger scale compared to Fig. 2.3).

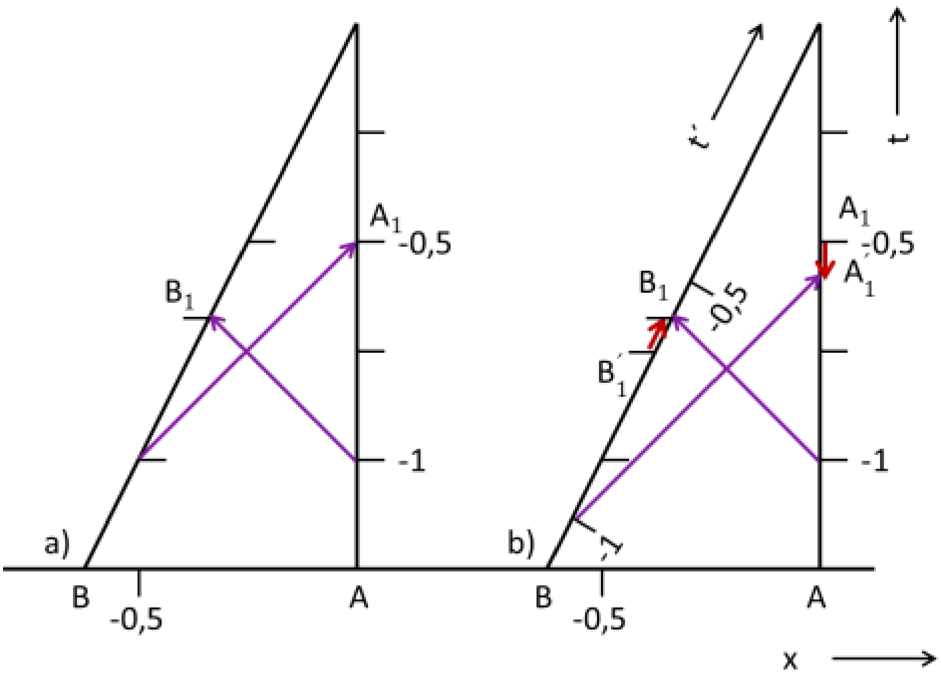



Fig. 2.4: Space-time-diagram for a signal exchange between observers A (at rest) and B (approaching), Example for $v = 0{,}5c$
a) Conventional (acc. to Newton)
b) Relativistic (acc. to Lorentz)

For the transition from a system at rest to a moving observer the time $\Delta t_0$ is according to case c) and d) presented in Fig. 2.2

$$\Delta t_{AB} = \Delta t_0 \frac{1}{1+\frac{v}{c}} \tag{2.05}$$

and in opposite direction

$$\Delta t_{BA} = \Delta t_0 \left(1 - \frac{v}{c}\right) \tag{2.06}$$

The equations (2.05) and (2.06) must again be expanded by the parameter $\gamma$ ($\Delta t_{AB}$ smaller and $\Delta t_{BA}$ larger) and it follows

$$\frac{1}{\gamma} \cdot \frac{1}{1 + \frac{v}{c}} = \left(1 - \frac{v}{c}\right) \cdot \gamma \tag{2.07}$$

with the same result for $\gamma$ as shown in Eq. (2.04).

It shall be stated again that the time dilatation of the moving observer is necessary to avoid discrepancies. Without this effect it would always be possible to distinguish a moving observer from an observer at rest by simple experiments.

### 2.1.2 Movement in arbitrary directions

It was established so far, that it is not possible for two observers increasing their distance or approaching each other to decide by measurements regarding the exchange of light signals whether they are moving or at rest. When the velocity vectors of the observers are not parallel, and they are passing by with the minimum distance $a$ the situation changes, and more effort is necessary to verify that the observations of all participants are equivalent.

The following examination set-up shall be chosen:

1. Both observers will send out signals, the (subjective) interval is $\Delta t$.
2. For an incoming signal the angle referring to the direction of the sending observer is determined.
3. If the incoming signal is exact transverse to the moving direction of the sender a response signal with a special designation will be sent.
4. The signals are coded in a defined way to realize a final evaluation at the end of the trial. After the exchange of all data it is possible to find out, at what time the signals were sent which were detected as coming in exactly from the transverse direction.

First a moving observer B is considered, which is passing the observer at rest (A) in a minimum distance $a$ with the speed $v$. In this case A will detect the signals sent from B in a (subjective) interval $\gamma\Delta t$. Compared to this observer B has a completely different view. Caused by the aberration effect B will measure the angle of the signal according to the equation

$$\delta = arcsin\left(\frac{v}{c}\right) = arctan\left(\gamma \cdot \frac{v}{c}\right) \tag{2 .10}$$

as coming in from the transverse direction (see Fig. 2.5). Here $v = 0{,}5c$ is chosen and the measured angle is $\delta = 30°$. Further discussions concerning the measurements of angles different to the transverse direction require additional geometric considerations which are presented in detail in chapter 2.3.

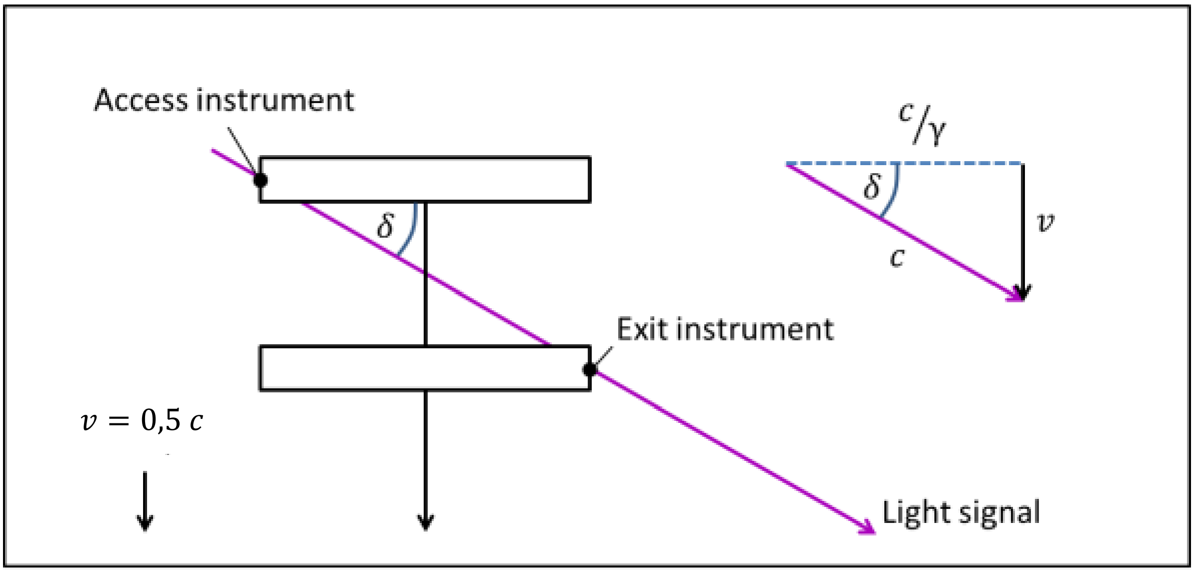



Fig. 2.5: Aberration effect: Measurement of angle $\delta$ caused by the movement of the receiver of a signal.

In the following it will be discussed, which values will be measured for the interval $\Delta t$ and other relevant time measurements according to the situation presented in Fig. 2.6 for the moving observer and a system at rest.

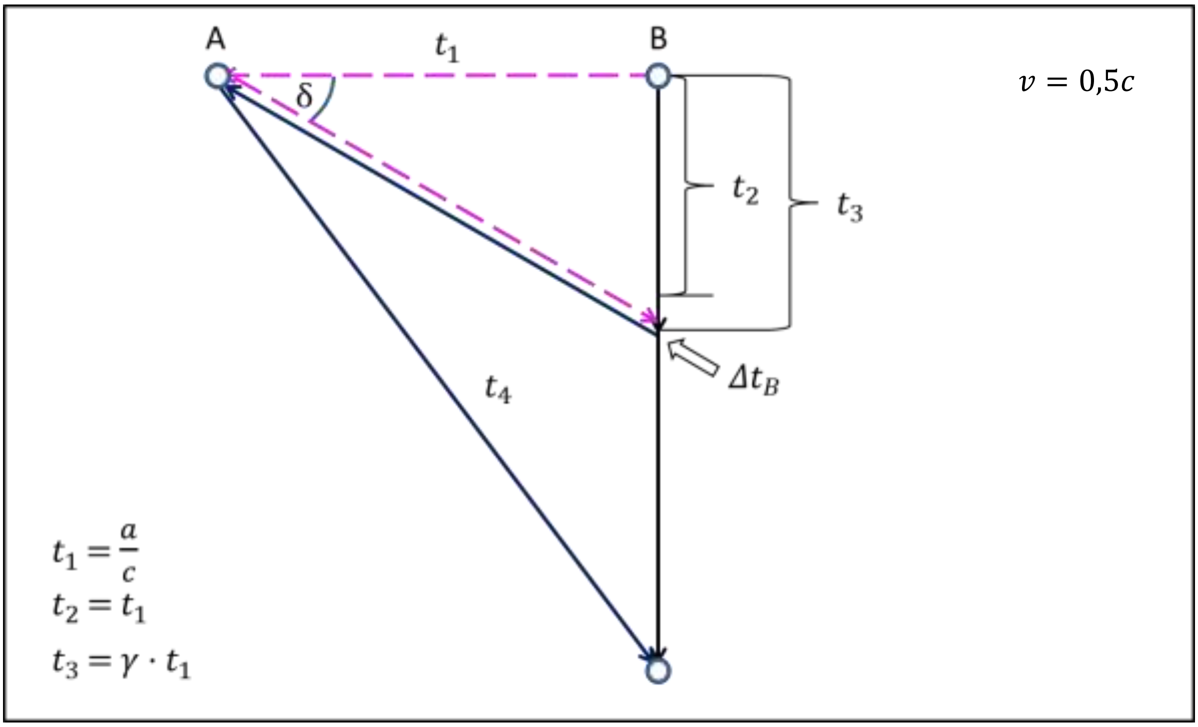



Fig. 2.6: Exchange of signals between A and B, example for $v = 0,5c$, $\delta = 30°$
Details for signal $\Delta t_B$: see Fig. 2.7; Total running time: Fig 2.8

*a) Measurement of signal interval*

As already shown the intervals between the signals emitted by the moving observer B will be measured by observer A at rest as $\Delta t_A = \gamma \Delta t$. This is caused by the effect of time dilatation valid for B.

The value $\Delta t_B$ measured by B can be calculated using an approximation calculation according to the scheme presented in Fig. 2.7. At the beginning a signal is sent by A and this is received at point $B_0$, the next signal is following after time $\Delta t_0$. When it arrives at point $B_0$, then the observer has already moved on to point $B_1$ and the additional time for the extended way must be added. If it is presumed that $\Delta t_0 \ll t_1$ then it is possible for the calculation to shift the signals sent by A parallel in direction of $B_1$ without changing the value of $\delta$. When the signal arrives at point $B_1$ then an additional movement to $B_2$ took place and the calculation must be repeated accordingly.

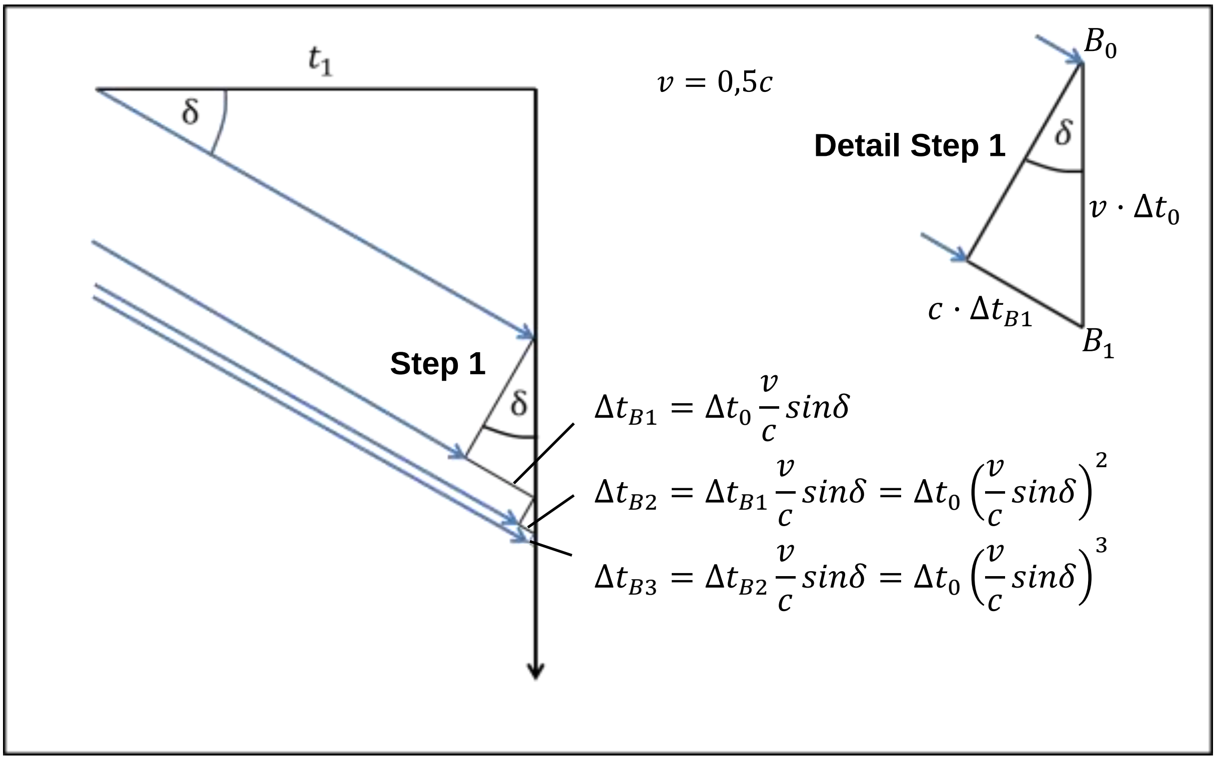



Fig. 2.7: Scheme for calculation of signal interval $\Delta t_B$ (for $\Delta t_0 \ll t_1$). Presentation of the first 3 steps.

The single values can be summarized

$$\Delta t_B = \Delta t_0 + \sum_{i=1}^{\infty} \Delta t_{i-1} \frac{v}{c} sin\delta = \Delta t_0 \sum_{i=0}^{\infty} \left(\frac{v}{c} sin\delta\right)^i \tag{2.11}$$

In this case a geometrical series of the form

$$S_n = \sum_{i=0}^{n} q^i \tag{2.12}$$

is derived, where $S_n$ is the limit value and

$$q = \frac{v}{c} sin\delta \tag{2.13}$$

With $n \to \infty$ and $q < 1$ it follows

$$S_\infty = \frac{1}{1-q} \tag{2.14}$$

Because B is subjectively realizing that the signal is arriving from the transverse direction Eq. (2.10) is valid

$$sin\delta = \frac{v}{c} \tag{2.15}$$

Hence

$$S_\infty = \frac{1}{1-\frac{v^2}{c^2}} = \gamma^2 \tag{2.16}$$

The combination with (2.11) reveals

$$\Delta t_B = \gamma^2 \cdot \Delta t_0 \tag{2.17}$$

The calculation shows that the moving observer B will measure (subjective) a value of $\gamma\Delta t$, because he is subject to time dilatation himself. Thus, it is verified that observers A and B are measuring the same values for the intervals of incoming signals.

*b) Measuring of total running time of signals*

The running time of a signal emitted by A and identified by B as transverse to his moving direction is $\gamma t_1$ (see Fig. 2.6).

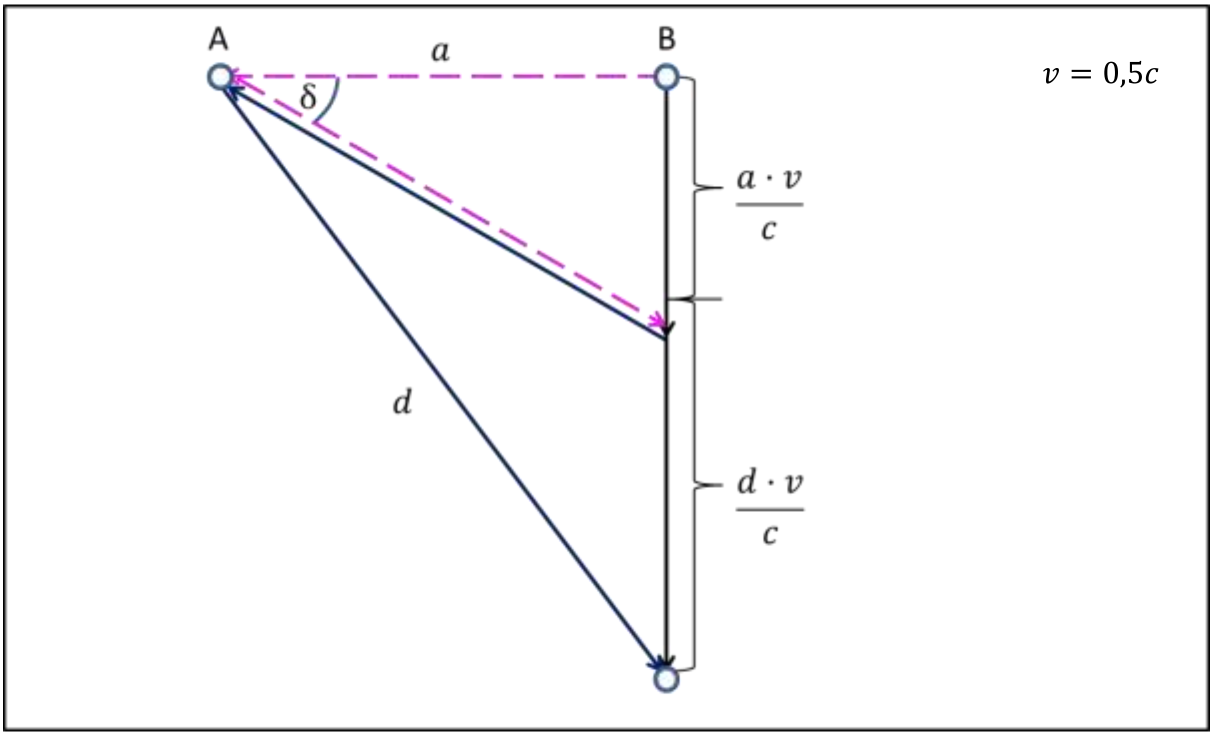



Fig. 2.8: Signal path B → A → B and definition of distances travelled.

Because B is sending the signal back the same way the total running time is $2\gamma t_1$. For B the first value is $t_1$ (see Fig. 2.6), the way back $t_4$ must be calculated. To do this some important definitions are necessary (see Fig. 2.8).

The distance $d$ (corresponding to the time $t_4$) is derived by

$$a^2 + \left(\frac{v}{c}a + \frac{v}{c}d\right)^2 = d^2 \tag{2.18}$$

Completing the square shows

$$a = d\left(-\frac{v^2}{c^2\left(1+\frac{v^2}{c^2}\right)} \pm \sqrt{\left(\frac{v^2}{c^2\left(1+\frac{v^2}{c^2}\right)}\right)^2 + \frac{1-\frac{v^2}{c^2}}{1+\frac{v^2}{c^2}}}\right) \tag{2.19}$$

Considering only positive values, it is achieved after simplification

$$a = d\left(-\frac{v^2}{c^2\left(1+\frac{v^2}{c^2}\right)} + \sqrt{\frac{v^4}{c^4} + \left(1+\frac{v^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}\right) \tag{2.20}$$

$$= d\left(-\frac{v^2}{c^2\left(1+\frac{v^2}{c^2}\right)} + 1\right) = d\left(\frac{1-\frac{v^2}{c^2}}{1+\frac{v^2}{c^2}}\right) \tag{2.21}$$

and

$$d = a\left(\frac{1+\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}}\right) \tag{2.22}$$

For calculation of the total distance the value of $a$ is added

$$d + a = a\left(\frac{1+\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} + 1\right) = a\left(\frac{1+\frac{v^2}{c^2}+1-\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}}\right) = 2a\gamma^2 \tag{2.23}$$

The calculations lead to a total time of $2\gamma^2 t_1$ and therefore to a difference of factor $\gamma$ between observers A and B which is compensating the time dilatation for the moving observer B. It is shown again that identical subjective measurements are valid.

## 2.2 Exchange of signals inside moving bodies

The considerations taken so far illustrate the fundamental relations during experiments concerning an exchange of signals between observers at different speed. Doing this, the conditions are, however, not fully described without discrepancies. If for example an observer at rest could directly monitor measurements of the speed of light between two moving observers, he would find differences between his results compared to the results of the other observers without further modification. This would cause a violation of the fact, that measurement of the speed of light show the same results in any inertial system. It has to be mentioned that here differences for the results in moving direction and in other arbitrary directions occur; in the following these cases will be treated separately.

In the following only the exchange of light pulses will be part of the calculations. The discussion of light as a wave and the special characteristics connected with this feature require special considerations and will be presented in chapter 8.

### 2.2.1 Exchange of signals in moving direction

For the presentation of this situation the time for the exchange of signals between observers A and B shall be investigated.

While the time in a system at rest for going and coming is

$$t_{AB} + t_{BA} = 2t_0 \tag{2.30}$$

it is different for moving objects for observations from a system at rest (see Eq. 2.01 and 2.05)

$$t_{AB} + t_{BA} = t_0 \frac{1}{1 - \frac{v}{c}} + t_0 \frac{1}{1 + \frac{v}{c}} \tag{2.31}$$

with

$$t_{AB} + t_{BA} = t_0 \left[ \frac{\left(1 + \frac{v}{c}\right) + \left(1 - \frac{v}{c}\right)}{\left(1 + \frac{v}{c}\right)\left(1 - \frac{v}{c}\right)} \right] = 2\gamma^2 t_0 \tag{2.32}$$

It was already mentioned before that the time for moved observers is enlarged by the parameter $\gamma$. During the above-mentioned calculation, the spatial extension is reaching, however, the factor $\gamma^2$. To overcome this contradiction, it is necessary to reduce in addition the distance between the two observers by the factor $\gamma$. This reduction is generally named "space contraction".

When the effects of time dilatation and space contraction are considered together all discrepancies disappear. It is worth mentioning, that the times for travelling the distances between A → B and B → A are different in view of a system at rest, but that the summation of the times (when time dilatation is considered) is leading to the same result compared to a system at rest.

These correlations are not only valid for the observer at rest. The moved observer also will find during the evaluation of own measurements concerning the distances in the system at rest that these are contracted by the factor $\gamma$. Time dilatation and space contraction are thus depending on each other to create a physical frame without discrepancies.

A simple example shall demonstrate the results. A case shall be monitored where observers A and B are placed in a system with a constant distance $a$. At time 0 observer A is sending out a signal to B which is immediately reflected to A. When A and B are viewed as at rest, the distances of going and coming and the connected times for the transport of the signal are equal in both directions. If both observers are moving constantly in relation to a different inertial system, however, the situation is completely different. This shall be demonstrated in a space-time-diagram (Fig. 2.9). For simplification of the presentation the values are normalized. This means that $a = 1$, in addition the time $t$ is converted to $ct$ and is – as valid for the space values $x$ – standardized to a value of 1. (The use of $ct$ instead of $t$ is frequently used; in this case the dimensions of $x$ and $ct$ are identical and it is easily possible to take direct readings out of the diagram).

Calculations analog Eq. (2.32) lead to

$$x_T = x_1 - x_2 = \frac{a}{\gamma\left(1 - \frac{v}{c}\right)} + \frac{a}{\gamma\left(1 + \frac{v}{c}\right)} = \frac{2\gamma v a}{c} \tag{2.33}$$

$$t_T = t_1 + t_2 = \frac{a}{c\gamma\left(1 - \frac{v}{c}\right)} + \frac{a}{c\gamma\left(1 + \frac{v}{c}\right)} = \frac{2\gamma a}{c} \tag{2.34}$$

Inserting these values into the Lorentz-Equations Eq. (1.07) and (1.08) the results $x' = 0$ and $t' = 2\,a/c$ will appear which are the expected findings for observers at rest. At this stage of the discussion it is not clear, how the Lorentz-Equations can be derived; in chapter 3.3 different methods will be presented in which way this is possible.

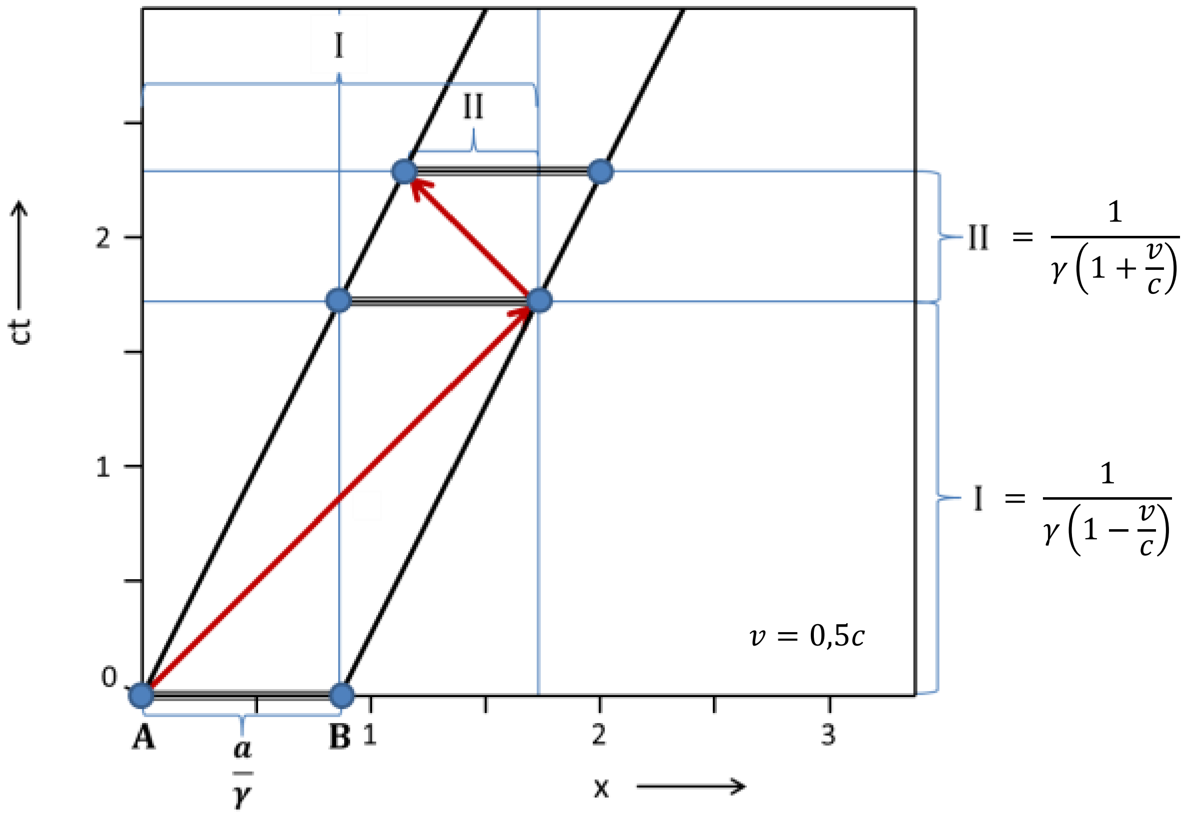



Fig. 2.9: Exchange of signals between observers A and B (marked using red arrows) in a moving system. Example for $v = 0{,}5c$

### 2.2.2 Exchange of signals during passing of two observers

When a more complex approach for the observations is considered, like it is the case for measurements between identical laboratories, which are passing in a close distance and exchanging light signals between front and back end, also no deviations will occur. An example shall be discussed in detail.

The experimental set-up is the following:

1. Two identical laboratories with observers A, B, C and A′, B′, C′ shall be prepared. The orientation is presented in Fig. 2.10. The positions of C and C′ are situated exactly in the middle of the laboratories.

2. The laboratory with A′, B′, C′ is moved relative to A, B, C according to the presentation in the diagram.
3. The moved laboratory is passing the observers at rest in a minimum distance to keep aberration effects as small as possible.
4. As soon as the observers of both systems pass each other signals to C resp. C′ will be sent. C resp. C′ are reflecting the signals to the sender and are recording the relevant periods.

At first observers A′ and A are passing. For small velocities (compared with the speed of light) the passing of B′ and A plus also A′ and B will happen simultaneously. When relativistic velocities are used, however, this will not be the case. Here the moved system will show a contraction in moving direction and the contacts between the observers will happen at different times. At the end B′ and B will pass. In total there are 4 different situations for contacts, which are presented in Fig. 2.11 in a space-time-diagram.

After the end of the experiment the corresponding time records between all observers shall be compared. For the selected example with the velocity $v = 0{,}5\ c$ the coordinates for C and C′ are presented in table 2.2. In addition to the values from the experiment the calculated results determined by the Lorentz-Transformation are also presented in this table. The space and time coordinates will be discussed in the following to allow an exact comparison between the different situations.

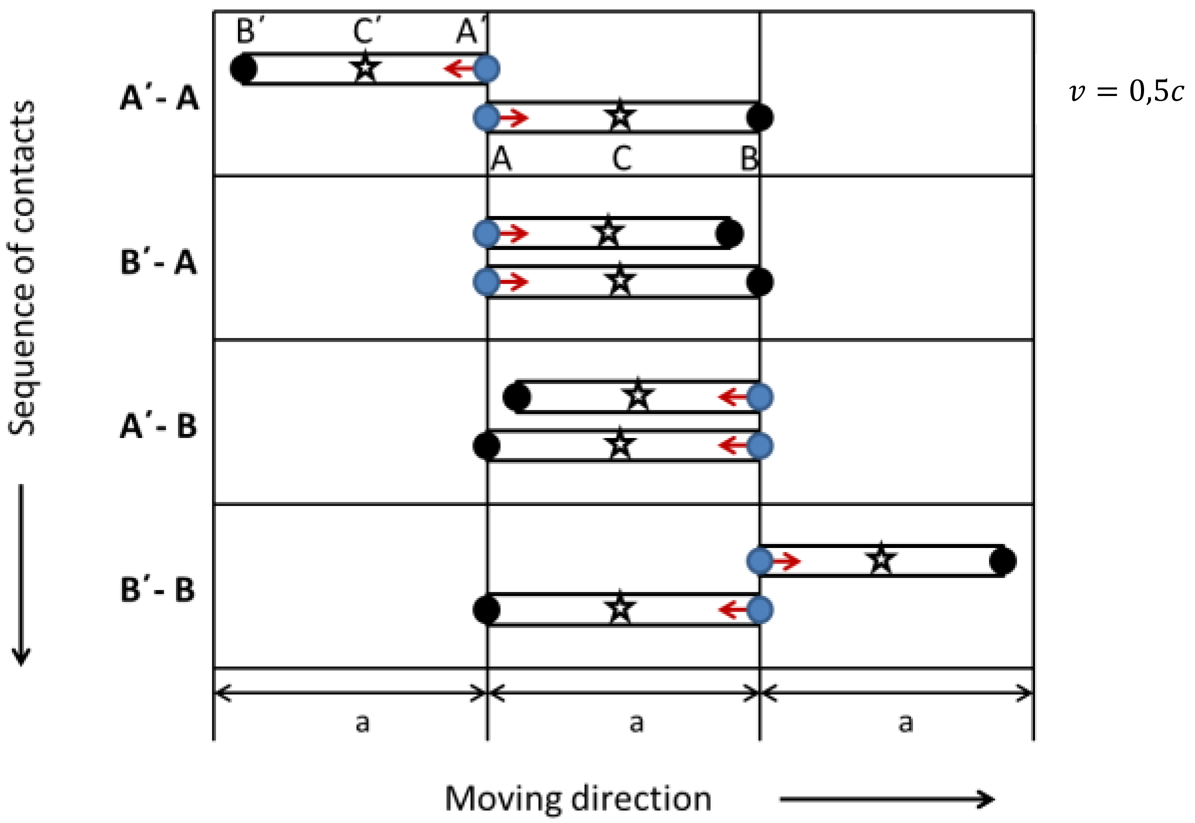



Fig.:2.10: Laboratory with observers A and B to transmit signals and C to receive. An identical laboratory with observers A′, B′ and C′ is passing with the velocity $v = 0{,}5\ c$. During all contacts of A and B with A′ and B′ a signal is transmitted and received by C and C′.

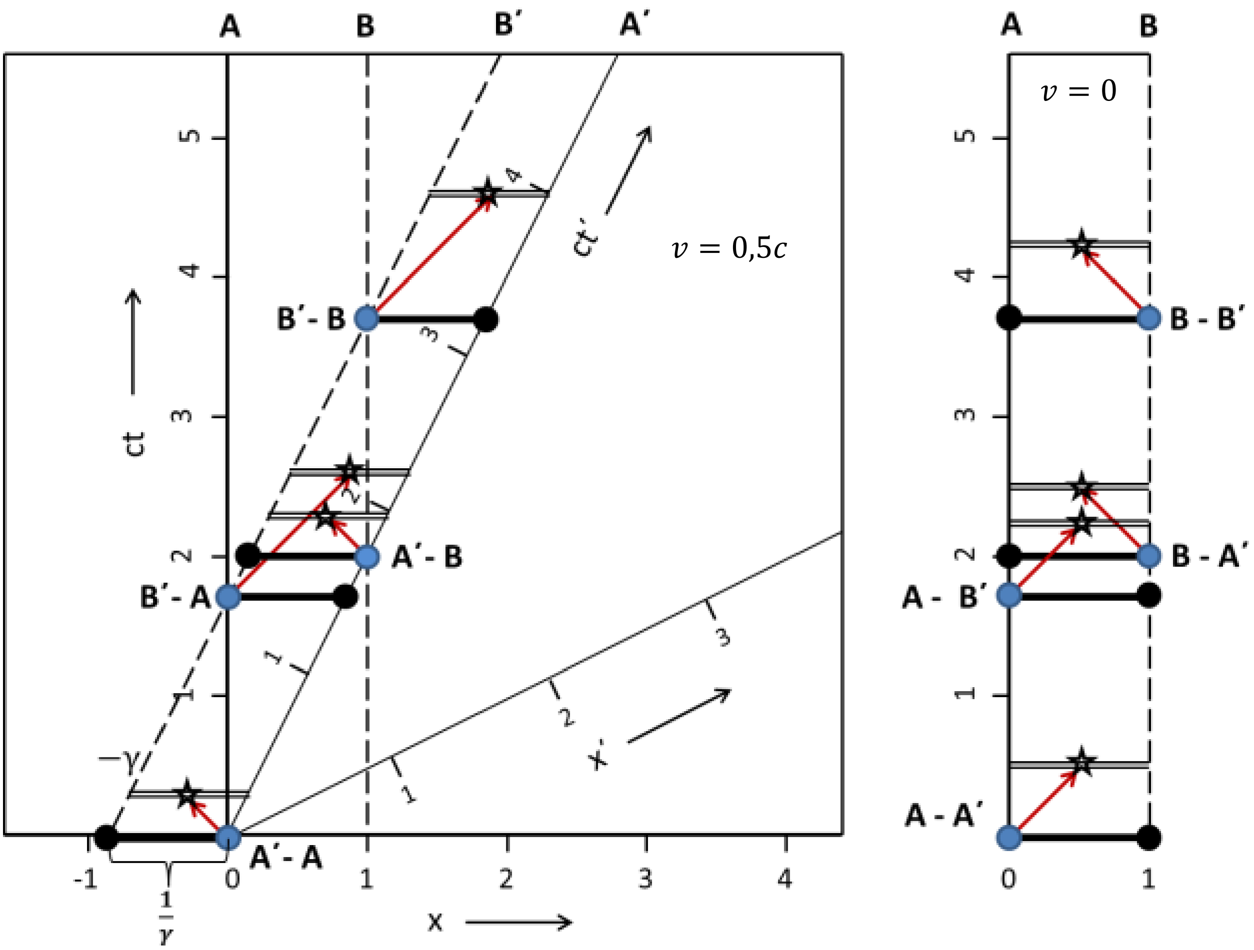



Fig. 2.11: Time sequence of received signals in the middle of two identical laboratories; signals are transmitted when passing.
Left: Moving laboratory
Right: Laboratory at rest

| Case | Observer | A/A′ | A/B′ | B/A′ | B/B′ |
|---|---|---|---|---|---|
| 1 | C | [0,5 ; 0,5] | [0,5 ; 2,232] | [0,5 ; 2,5] | [0,5 ; 4,232] |
| 2 | C | [-0,289 ; 0,289] | [0,866 ; 2,598] | [0,711 ; 2,289] | [1,866 ; 4,598] |
| 3 | C′ | [-0,5 ; 0,5] | [-0,5 ; 2,5] | [-0,5 ; 2,232] | [-0,5 ; 4,232] |

Tab. 2.2: Coordinates for space [bracket left] and time [bracket right] for the experiment according to Fig. 2.11.
Line 1: Values for the observer at rest
Line 2: Observation by the observer at rest regarding the moving system
Line 3: Calculated values for the moved observer according to the Lorentz-Transformation

*Coordinates of space*

It is clear at first sight that the coordinates of space in the first line must be constant. The chosen parameters lead to a value of 0.5.

For the moved system, the parameters vary depending on the geometrical relations according to line 2. The values of the coordinates of space derived by calculations using the Lorentz-Transformations are equal to those of the system at rest with the only difference that the algebraic sign is negative. This means, that the observers at rest and in the moved system are measuring the same values.

*Coordinates of time*

The coordinates of time show a similar effect. In this case the situation is different, however, because for C and C′ the values of A/B′ and B′/A are exchanged. It is obvious, that the principle of relativity requires, that C resp. C′ must receive the signal of "their" observer A resp. A′ first. It is important, that for the observer at rest the change in the values of time is necessary to show a proper sequence of contacts between A′ and B′ to C′. So, this short summary provides clear evidence that no differences between measurements of all observers taking part will appear.

### 2.2.3 Exchange of signals in arbitrary directions

In the following the situation shall be discussed, that a signal is transmitted and reflected transverse to the moving direction (i. e. $y$-direction). The time dilatation occurring for the moving observer, which travels the distance of $d = vT$ when the signal reaches the reflector, is exactly compensated by the longer path of the signal $D' = cT$ (Fig. 2.12). This means that it is not possible for the moving observer to find a difference compared to the situation at rest and so again no violation of the principle of relativity can be found.

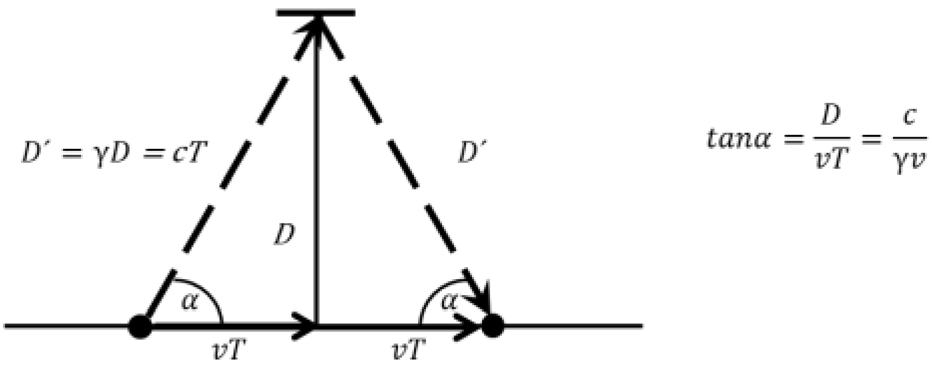



Fig. 2.12: Signal exchange transverse to the moving direction

In contradiction to the effects of a longitudinal signal exchange this means, that in the view of an observer at rest in transverse direction there is a change in the transmission angle because of aberration. The value can be calculated as presented in Fig. 2.12 using the tangent value (see also Eq. (2.11) with $\alpha = 90° - \delta$).

Whereas the situation concerning the exchange of signals in direction of a moving observer was discussed first, the behavior in transverse direction is described here. No discrepancies to the expected circumstances for the observer in motion appear and the principle of relativity is respected in any case.

To start with the next step discussing the observations during signal exchange in any arbitrary spatial direction it is necessary first, to start with basic considerations concerning the dependencies between the angles of incoming and outgoing signal due to aberration for moved observers in view of a reference system at rest. This will be presented in the following; afterwards, using these derivations, it will be shown that no differences appear between the subjective measurements in a system at rest and for a moved observer. This issue will be discussed in chapter 2.4 and the validity will be proven by calculations of an example using a spere where light signals start from the center and return after reflection.

## 2.3 Exchange of signals and correlation of angles

In the following it shall be investigated, which correlations appear when emitted and received signals have different directions compared to a moving body. This effect is commonly referred to as aberration (see Fig. 2.5).

As already discussed in detail, the relativistic approach to calculations of a moved observer requires the consideration of the effect, that the body will be contracted in moving direction. Up to now this effect was only treated as a summation of going and coming of the signal and first nothing is known about the splitting into the single trips. Out of the principle of relativity it can be deduced, however, that this contraction must be symmetric to the middle axis of the moved body according to Fig. 2.13. It makes no difference in which direction the movement will take place.

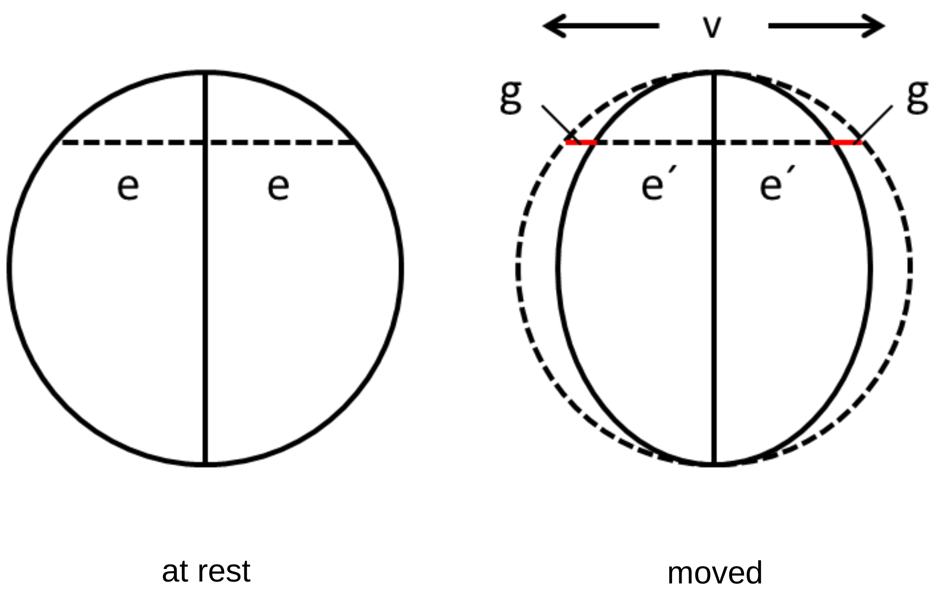



Fig. 2.13 Contraction of a moved body

In this case the distance $e'$ in the moved system is equal to $e - g$ or $e/\gamma$.

### 2.3.1 Reception in a moving body

In the following the values for the reception in a moving body will be investigated. First it is necessary to define the exact conditions for the analysis. The following set-up shall be used:

A sphere with the radius $a$ contains holes in the circumference in adequate quantity where adjusted light beams can enter (i. e. at point $P_1$, see Fig. 2.14). When such a beam is touching the center ($P_2$), then the observer can define the corresponding angle using geometric evaluations. Any of these holes relates to an angle of $\alpha'$ resp. $\beta'$ because of the geometrical definitions of the exact position and the radius $a$.

If the observer receiving the signal is moving, then an observer at rest will find different angles for the incoming signal and his measurements will be $\alpha$ resp. $\beta$. In his view the signal will travel a distance $d$ inside the system. For the calculations it has to be considered that, as already stated before, the sphere will be deformed in moving direction (see Fig. 2.13). In this case for the incoming signals the geometric dependencies are defined according to Fig. 2.14. The incoming direction from behind (part a) leads to the following dependencies

$$d^2 = f^2 + (e + b - g)^2 \tag{2.40}$$

and

$$f = d \cdot sin\alpha \qquad f = a \cdot sin\alpha' \tag{2.41}$$

Further

$$e = a \cdot cos\alpha' \tag{2.42}$$

$$\frac{b}{v} = \frac{d}{c} \tag{2.43}$$

$$e - g = \frac{e}{\gamma} \tag{2.44}$$

The first calculation yields

$$a = d \cdot \frac{sin\alpha}{sin\alpha'} \tag{2.45}$$

Eq. (2.40) is developing to

$$d^2 = (d \cdot sin\alpha)^2 + \left(d\frac{v}{c} + d\frac{cos\alpha' \cdot sin\alpha}{\gamma \cdot sin\alpha'}\right)^2 \tag{2.46}$$

$$1 - sin^2\alpha = cos^2\alpha = \left(\frac{v}{c} + \frac{sin\alpha}{\gamma \cdot tan\alpha'}\right)^2 \tag{2.47}$$

$$tan\alpha' = \frac{sin\alpha}{\gamma\left(\pm\, cos\alpha - \frac{v}{c}\right)} \tag{2.48}$$

where because of geometrical considerations only positive values for $cos\alpha$ are valid. If the signal is approaching from the front (Fig. 2.14b) the relations are

$$d^2 = f^2 + (e - b - g)^2 \tag{2.49}$$

After the same calculation as presented before this leads to

$$tan\beta' = \frac{sin\beta}{\gamma\left(cos\beta + \frac{v}{c}\right)} \tag{2.50}$$

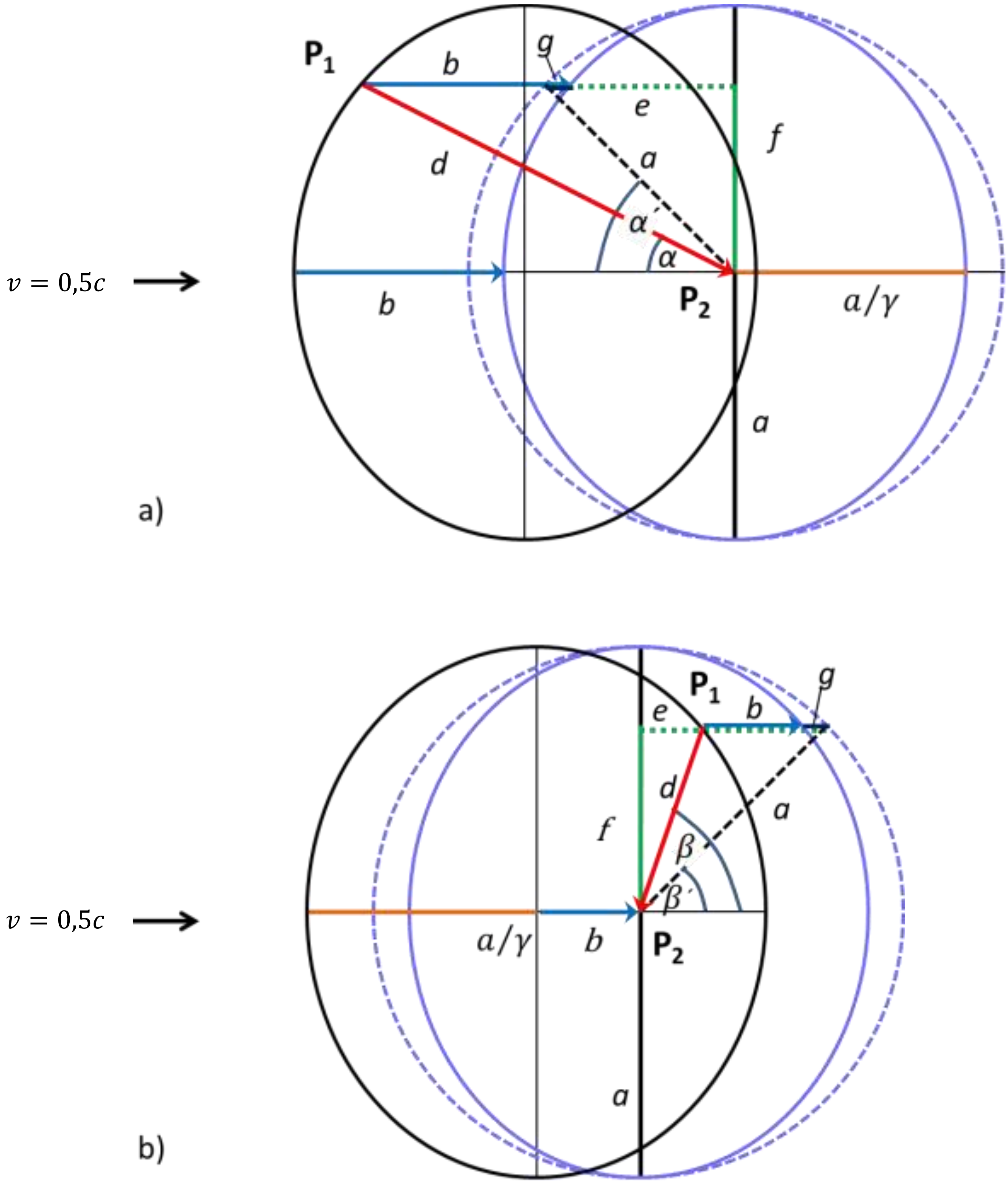



Fig. 2.14: Definition of parameters to determine the angle of incoming beams for a moved observer (examples for $v = 0{,}5c$ and $\alpha', \beta' = 45°$)
a) Signal approaching from behind, b) Signal approaching from the front

Before reviewing the results, the opposite situation with an outgoing light beam shall be discussed first.

### 2.3.2 Outgoing signals of moving bodies

For outgoing signals similar correlations apply. The relevant parameters are presented in Fig. 2.15. In this case the signal will be emitted from the center ($P_1$) and is passing a hole in the circumference of the sphere ($P_2$). In this case the space contraction of the moving body has also to be considered.

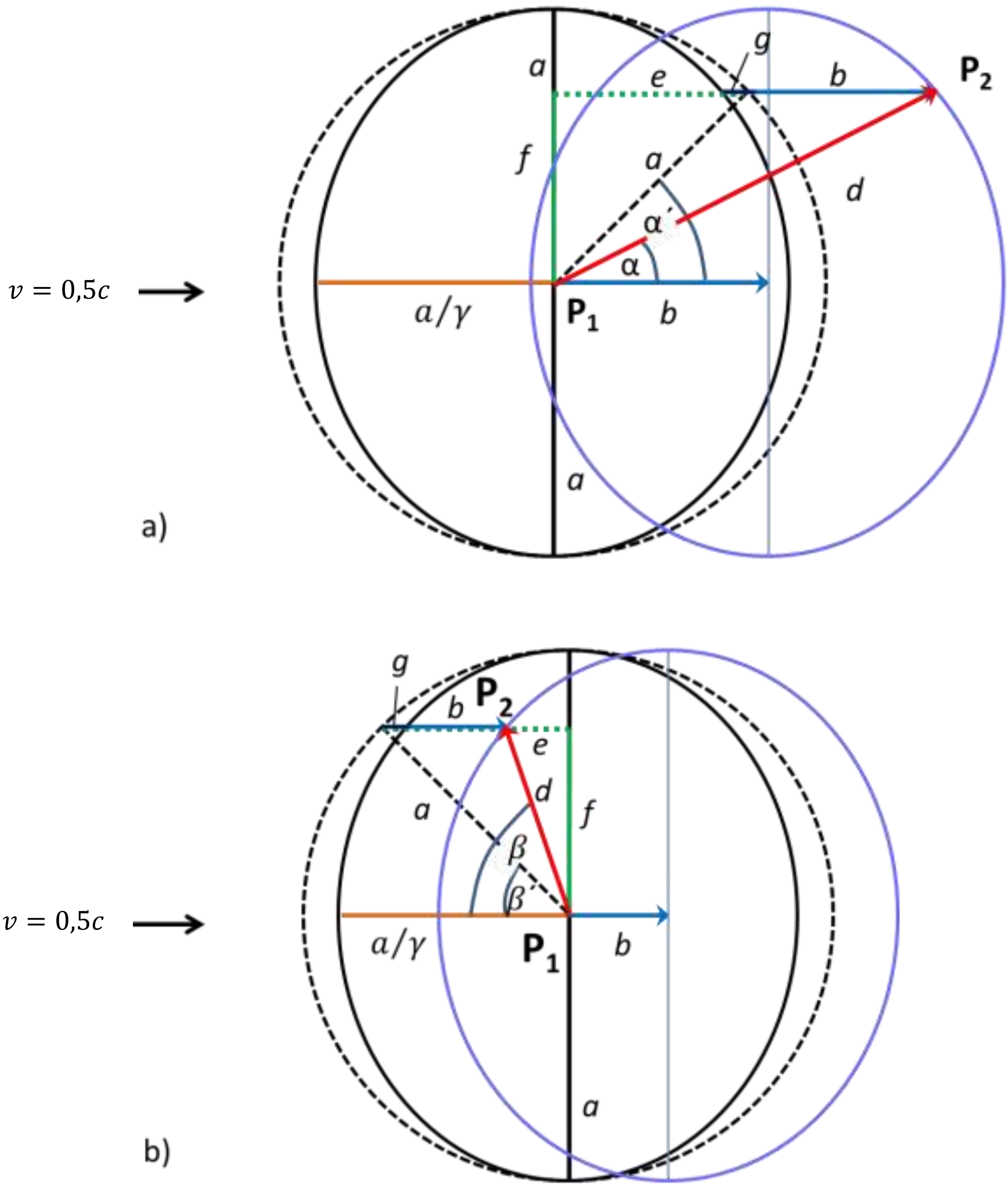



Fig. 2.15: Definition of parameters to determine the angle of outgoing beams for a moving observer (examples for $v = 0{,}5c$ and $\alpha', \beta' = 45°$)
a) Signal emitted in moving direction, b) Signal emitted backwards

For outgoing signals in moving direction (Fig. 2.15a) the results are exactly the same compared to incoming signals approaching from behind, which are covered by the equations presented from Eq. (2.40) to Eq. (2.48). For outgoing signals emitted backwards (Fig. 2.15b) the opposite combination occurs, and the result is Eq. (2.50) corresponding to the signal approaching from the front end.

### 2.3.3 Results of calculations of angles

At first it shall be demonstrated for the example discussed in chapter 2.1.2, that the results for a moved observer and a system at rest are exactly the same. To realize this, the propagation of the signals and the connected angles will be investigated. In view of the observer at rest (marked as "A") the process will start sending the signal 1 to observer B, following this, the signal 2 will be detected and returned, at the end the reflection of signal 1 is arriving. The angles of outgoing signals are marked with $\varepsilon$, whereas incoming signals carry the letter $\delta$.

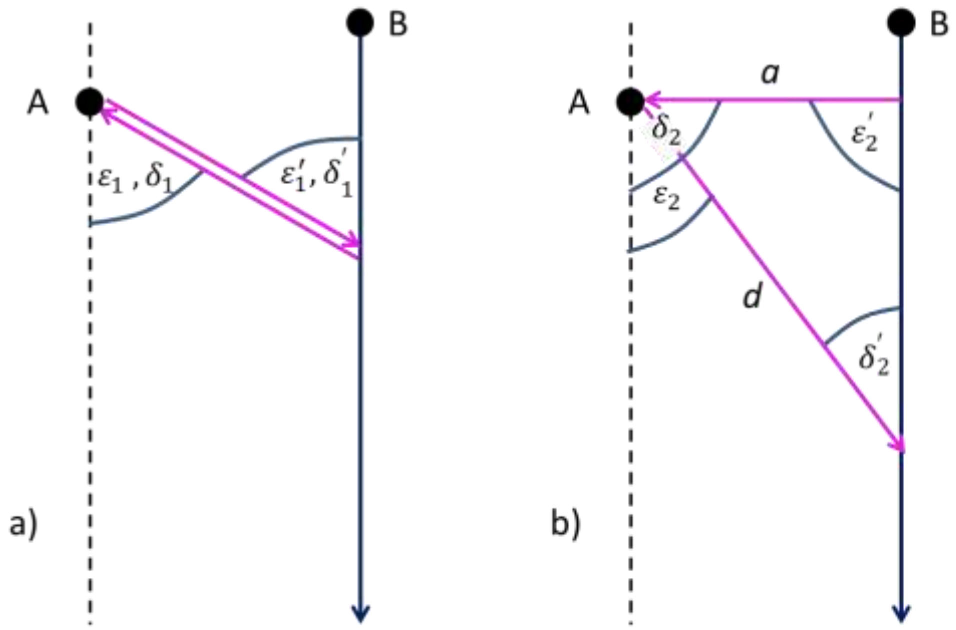



Fig. 2.16: Signal propagation according to situation in chapter. 2.1 with corresponding angles, example for $v = 0{,}5c$

Due to the chosen conditions the following situation is defined:

- The angles for incoming signals $\delta_2$ and $\delta_1'$ are 90°.
- The values for incoming signal $\delta_1$ and outgoing signal $\varepsilon_1$ are equal.
- The outgoing signal $\varepsilon_2$ can be calculated using Eq. (2.23) as

$$\varepsilon_2 = arcsin\left(\frac{a}{d}\right) = arcsin\left(\frac{1 - \frac{v^2}{c^2}}{1 + \frac{v^2}{c^2}}\right) = 36{,}87° \qquad (2.51)$$

Calculations for the chosen speed of $v = 0{,}5c$ show the following results:

| | Initial value | Calculation | Result |
|---|---|---|---|
| 1 | $\delta_1' = 90°$ | $tan\delta_1' = \frac{sin\varepsilon_1}{\gamma\left(cos\varepsilon_1 + \frac{v}{c}\right)}$ | $\varepsilon_1 = 60°$ |
| 2 | $\delta_2 = 90°$ | $tan\varepsilon_2' = \frac{sin\delta_2}{\gamma\left(cos\delta_2 + \frac{v}{c}\right)}$ | $\varepsilon_2' = 60°$ |
| 3 | $\varepsilon_2 = 36.87°$ | $tan\delta_2' = \frac{sin\varepsilon_2}{\gamma\left(cos\varepsilon_2 - \frac{v}{c}\right)}$ | $\delta_2' = 60°$ |
| 4 | $\delta_1 = 60°$ | $tan\varepsilon_1' = \frac{sin\delta_1}{\gamma\left(cos\delta_1 + \frac{v}{c}\right)}$ | $\varepsilon_1' = 36.87$ |

Tab. 2.3: Calculation of angles for the situation corresponding to Fig. 2.16

It is shown here that A and B find the same values for outgoing (60°; 36.87°) and incoming signals (90°; 60°). It is thus demonstrated that the principle of relativity is also valid for measurements of angles and that the spatial contraction must be symmetric to the middle axis of the moved body in moving direction and vice versa.

### 2.3.4 Literature review and evaluation

The following simple derivation of the aberration formula for relativistic velocities was presented by D. Giulini [19]. Here the emission of a light pulse from an observer with the coordinates $x_0$ and $y_0$ in a system at rest resp. $x_0'$ and $y_0'$ for a system moving with the velocity $v$ is investigated in relation to their relative point of origin. In this case $\delta$ and $\delta'$ are the angles to the $x$-axis. At the time $t = t_0 = t_0'$ the systems meet in their respective points of origin. In this case the component $u_x$ in the system at rest can be calculated using

$$u_x = -c \cdot cos\delta \tag{2.60}$$

and in the moving system

$$u_x' = -c \cdot cos\delta' \tag{2.61}$$

Integrated in the equation of relativistic addition of velocities

$$u_x' = \frac{u_x + v}{1 + \frac{u_x \cdot v}{c^2}} \tag{2.62}$$

the calculation yields

$$cos\delta' = \frac{cos\delta - \frac{v}{c}}{1 - \frac{v}{c} \cdot cos\delta} \tag{2.63}$$

Further comprehensive derivations of the calculations are leading to the same results (e. g. presented by R. K. Pathria [27]). Other investigations, however, show additional derivations, e. g. [28,89a]

$$sin\delta' = \frac{sin\delta}{\gamma\left(1-\frac{v}{c}\cdot cos\delta\right)} = \frac{\left(1-\frac{v^2}{c^2}\right)^{1/2} sin\delta}{1-\frac{v}{c}\cdot cos\delta} \tag{2.64}$$

A particularly useful formula is derived using the general valid formula for the tangent [19,28] yielding

$$tan\left(\frac{\theta}{2}\right) = \frac{sin\theta}{1+cos\theta} \tag{2.65}$$

Inserting equations Eq. (2.63) and Eq. (2.64) the transformation leads to

$$tan\left(\frac{\delta'}{2}\right) = \frac{sin\delta}{\gamma\left(1+\frac{v}{c}\right)(1+cos\delta)} \tag{2.66}$$

$$tan\left(\frac{\delta'}{2}\right) = \left(\frac{c-v}{c+v}\right)^{1/2} tan\left(\frac{\delta}{2}\right) \tag{2.67}$$

Using this equation, it is possible to determine in an easy way the value of $\delta$ depending on $\delta'$. In the following, some selected results for all equations are calculated and compared. It must be considered that inverse functions (arc) for values between 0 and 180° are not exactly defined in cases where a sinus is present. The reason is, that in contrast to the cosine, which is monotonously decreasing in this interval, the sine wave shows a maximum at 90° and therefore the inverse function contains two possible solutions. This is the reason why for angles $> 90°$ the standard result must be converted as presented in tables 2.4 and 2.5. (The tangent is monotonously increasing between 0 and 90°, which is sufficient acc. to Eq. (2.67), because when taking $\delta/2$ as argument the necessary interval is halved).

| | |
|---|---|
| **1**: $\alpha' = arctan\left(\frac{sin\alpha}{\gamma\left(cos\alpha-\frac{v}{c}\right)}\right)$ | **2**: $\alpha' = arccos\left(\frac{cos\alpha-\frac{v}{c}}{1-\frac{v}{c}\cdot cos\alpha}\right)$ |
| **3**: $\alpha' = arcsin\left(\frac{sin\alpha}{\gamma\left(1-\frac{v}{c}\cdot cos\alpha\right)}\right)$ | **4**: $\alpha' = 2\cdot arctan\left[\left(\frac{c+v}{c-v}\right)^{1/2} tan\left(\frac{\alpha}{2}\right)\right]$ |

| $\alpha$ | | 1 | | 2 | | 3 | | 4 | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0,523599 | 30 | 0,869038 | 49,79 | 0,869038 | 49,79 | 0,869038 | 49,79 | 0,869038 | 49,79 |
| 0,785398 | 45 | 1,244669 | 71,31 | 1,244669 | 71,31 | 1,244669 | 71,31 | 1,244669 | 71,31 |
| 1,047198 | 60 | 1,570796 | 90,00 | 1,570796 | 90,00 | 1,570796 | 90,00 | 1,570796 | 90,00 |
| 1,570796 | 90 | -1,047198 | 120,00 | 2,094395 | 120,00 | 1,047198 | 120,00 | 2,094395 | 120,00 |
| 2,094395 | 120 | -0,643501 | 143,13 | 2,498092 | 143,13 | 0,643501 | 143,13 | 2,498092 | 143,13 |
| 2,356194 | 135 | -0,469475 | 153,10 | 2,672117 | 153,10 | 0,469475 | 153,10 | 2,672117 | 153,10 |
| 2,617994 | 150 | -0,306968 | 162,41 | 2,834625 | 162,41 | 0,306968 | 162,41 | 2,834625 | 162,41 |
| 3,141593 | 180 | 0 | 180,00 | 3,141593 | 180,00 | 0 | 180,00 | 3,141593 | 180,00 |

Tab. 2.4: Values for $\alpha'$ depending on $\alpha$ according to equations 1 to 4, $v = 0{,}5c$
Results presented as radian and in degrees [°] (marked grey).
Values with frame: 180°+ angle (Eq. 1) and 180°- angle (Eq. 3)

| | |
|---|---|
| **5**: $\beta' = arctan\left(\dfrac{sin\beta}{\gamma\left(cos\beta + \frac{v}{c}\right)}\right)$ | **6**: $\beta' = arccos\left(\dfrac{cos\beta + \frac{v}{c}}{1 + \frac{v}{c} \cdot cos\beta}\right)$ |
| **7**: $\beta' = arcsin\left(\dfrac{sin\beta}{\gamma\left(1 + \frac{v}{c} \cdot cos\beta\right)}\right)$ | **8**: $\beta' = 2 \cdot arctan\left[\left(\dfrac{c-v}{c+v}\right)^{1/2} tan\left(\dfrac{\beta}{2}\right)\right]$ |

| β | | 5 | | 6 | | 7 | | 8 | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0,523599 | 30 | 0,306968 | 17,59 | 0,306968 | 17,59 | 0,306968 | 17,59 | 0,306968 | 17,59 |
| 0,785398 | 45 | 0,469475 | 26,90 | 0,469475 | 26,90 | 0,469475 | 26,90 | 0,469475 | 26,90 |
| 1,047198 | 60 | 0,643501 | 36,87 | 0,643501 | 36,87 | 0,643501 | 36,87 | 0,643501 | 36,87 |
| 1,570796 | 90 | 1,047198 | 60,00 | 1,047198 | 60,00 | 1,047198 | 60,00 | 1,047198 | 60,00 |
| 2,094395 | 120 | 1,570796 | 90,00 | 1,570796 | 90,00 | 1,570796 | 90,00 | 1,570796 | 90,00 |
| 2,356194 | 135 | -1,244669 | 108,69 | 1,896924 | 108,69 | 1,244669 | 108,69 | 1,896924 | 108,69 |
| 2,617994 | 150 | -0,869038 | 130,21 | 2,272555 | 130,21 | 0,869038 | 130,21 | 2,272555 | 130,21 |
| 3,141593 | 180 | 0 | 180 | 3,141593 | 180 | 0 | 180 | 3,141593 | 180 |

Tab. 2.5: Values for $\beta'$ depending on $\beta$ according to equations 5 to 8, $v = 0{,}5c$
Results presented as radian and in degrees [°] (marked grey).
Values with frame: 180°+ angle (Eq. 5) and 180°- angle (Eq. 7)

The considerations of equations 1 to 8 discussed so far were solely directed on the radiation angle for a light pulse, which could be measured by an observer at rest and was subsequently calculated for a moving system. In this case the angles measured in moving direction cover per definition the designation $\alpha$ (for the system at rest) and $\alpha'$ (moving) whereas $\beta$ and $\beta'$ are situated in opposite direction.

It was already demonstrated in chapter 2.3.2 that the investigation of the case, where the positions are changed and the moving observer is calculating values for the observer at rest, the angles evaluated by the moving observer will reveal exactly the opposite results. This means that measurements in moving direction following angle $\alpha$ will show the formal result of angle $\beta'$ and that it will also be the same case for $\beta$ and $\alpha'$.

The evaluation presented so far is only valid for the equation 1. The same result will appear, however, when equation 4 is converted in a suitable way to show the value of $\alpha$.
Whereas calculations for incoming signals are discussed quite often in the literature, only few solutions for outgoing signals can be found. R. Göhring [47] used the equations for outgoing signals and made a transformation to $\alpha'$; this showed that the results were in accordance with the results described in the following. In the presentation by W. Rindler [28] it is defined, that the values for the velocity $c$ shall be replaced by $-c$ and then the relevant calculations will appear. When this is done for all presented variants then it can be shown that this statement is valid for all calculations investigated here.

The results can be summarized as follows:

| | |
|---|---|
| **1**: $\alpha = arctan\left(\frac{sin\alpha'}{\gamma\left(cos\alpha' + \frac{v}{c}\right)}\right)$ | **2**: $\alpha = arccos\left(\frac{cos\alpha' + \frac{v}{c}}{1 + \frac{v}{c} \cdot cos\alpha'}\right)$ |
| **3**: $\alpha = arcsin\left(\frac{sin\alpha'}{\gamma\left(1 + \frac{v}{c} \cdot cos\alpha'\right)}\right)$ | **4**: $\alpha = 2 \cdot arctan\left[\left(\frac{c - v}{c + v}\right)^{1/2} tan\left(\frac{\alpha'}{2}\right)\right]$ |

The same conversion is possible for the opposite case:

| | |
|---|---|
| **5**: $\beta = arctan\left(\frac{sin\beta'}{\gamma\left(cos\beta' - \frac{v}{c}\right)}\right)$ | **6**: $\beta = arccos\left(\frac{cos\beta' - \frac{v}{c}}{1 - \frac{v}{c} \cdot cos\beta'}\right)$ |
| **7**: $\beta = arcsin\left(\frac{sin\beta'}{\gamma\left(1 - \frac{v}{c} \cdot cos\beta'\right)}\right)$ | **8**: $\beta = 2 \cdot arctan\left[\left(\frac{c + v}{c - v}\right)^{1/2} tan\left(\frac{\beta'}{2}\right)\right]$ |

Finally, it can be stated, that all presented equations are suitable for the calculation of relativistic aberration of moving observers connected to systems at rest and vice versa. The results of the aberration angles are the same for all involved participants and thus the principle of relativity is not violated. Precondition is that the effect of spatial contraction is symmetric to the middle axis of the moved body in moving direction and opposite to it.

For practical use equations 2 or 4 resp. 6 or 8 shall be preferred because they show no sinus in the formula and so no interpretation of the result is necessary for values $> 90°$. The real advantage of the geometric derivation presented here (this means equations 1 and 5) will become apparent later, when subluminal velocities of moving bodies instead of light signals will be discussed. In this case equation 1 (or 5) can be modified using a simple replacement of $c$ by the velocity $v$ of the second moving object, which is not possible for the other calculations. This will be especially important for discussions of questions concerning the momentum, which will be a major topic in chapter 7.

## 2.4 Exchange of signals in any arbitrary spatial direction

After discussion of the basic relations concerning the path of a signal in any arbitrary spatial direction, it is now possible to verify that for a signal in a moved system (here with the shape of a sphere with a standard-radius of $a = 1$) from the center to the outer shell and back, subjectively the same time will be measured compared to the system at rest. The following conditions shall be defined:

An angle $\alpha'$ (related to the moving direction) shall be chosen for the moved system, from which the light signal will be emitted to the outer shell. Then the following values are calculated:

1. The related angle $\alpha_1$ viewed by the observer at rest,
2. The length $d_1$ to the outer shell,
3. The angle $\alpha_2$ for the way back referring to the same angle $\alpha'$,

4. The length $d_2$ from the shell to the center,
5. The calculation of $d_T = d_1 + d_2$. The value of $d_T$ must be exactly $2a\gamma$ to verify that the measurements in both systems (moving and at rest) are subjectively identical.

For the calculation, the equations (2.67) and (2.45) shall be used and the following relations appear:

| | |
|---|---|
| **2**: $\alpha_1 = 2 \cdot arctan\left[\left(\frac{c-v}{c+v}\right)^{1/2} tan\left(\frac{\alpha'}{2}\right)\right]$ | **3**: $d_1 = \frac{sin\alpha'}{sin\alpha_1}$ |
| **4**: $\alpha_2 = 2 \cdot arctan\left[\left(\frac{c+v}{c-v}\right)^{1/2} tan\left(\frac{\alpha'}{2}\right)\right]$ | **5**: $d_2 = \frac{sin\alpha'}{sin\alpha_2}$ |

In table 2.6 calculations for an example $v = 0{,}5c$ are presented. For the values $\alpha' \to 0°$ and 180° with respect to $\alpha_1$ and $\alpha_2$ a division of 0 by 0 would appear and it would be necessary to extrapolate, for simplification only values between 1° to 179° were selected. The values directly in moving direction and opposite to it (0° and 180°) were already determined before in chapter 2.1.

For all calculated values of $d_T$ the result of $2\gamma$ (in this case $v = 0.5c \Rightarrow 2\gamma = 2{,}309401..$) appear. This means that in view of the observer at rest the distance travelled by the light pulse and the time needed is exactly longer by this value. All values show impressively that no deviations between the subjective measurements of the moved observer and a system at rest will appear. The time in the moving system is running slower by the calculated factor and the principle of relativity, as in all cases discussed before, will not be violated.

| $\alpha'$ [°] | $\alpha'$ | $\alpha_1$ | $\alpha_1$ [°] | $d_1$ | $\alpha_2$ | $\alpha_2$ [°] | $d_2$ | $d_T$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0,017453 | 0,010077 | 0,577360 | 1,731963 | 0,030228 | 1,731963 | 0,577438 | 2,309401 |
| 15 | 0,261799 | 0,151727 | 8,693343 | 1,712378 | 0,448391 | 25,69090 | 0,597023 | 2,309401 |
| 30 | 0,523599 | 0,306968 | 17,58795 | 1,654701 | 0,869038 | 49,79218 | 0,654701 | 2,309401 |
| 45 | 0,785398 | 0,469475 | 26,89895 | 1,562949 | 1,244669 | 71,31426 | 0,746452 | 2,309401 |
| 60 | 1,047198 | 0,643501 | 36,86990 | 1,443376 | 1,570796 | 90 | 0,866025 | 2,309401 |
| 75 | 1,308997 | 0,834062 | 47,78826 | 1,304130 | 1,851500 | 106,0831 | 1,005271 | 2,309401 |
| 90 | 1,570796 | 1,047198 | 60 | 1,154701 | 2,094395 | 120 | 1,154701 | 2,309401 |
| 105 | 1,832596 | 1,290093 | 73,91689 | 1,005271 | 2,307530 | 132,2117 | 1,304130 | 2,309401 |
| 120 | 2,094395 | 1,570796 | 90 | 0,866025 | 2,498092 | 143,1301 | 1,443376 | 2,309401 |
| 135 | 2,356194 | 1,896924 | 108,6857 | 0,746452 | 2,672117 | 153,1010 | 1,562949 | 2,309401 |
| 150 | 2,617994 | 2,272555 | 130,2078 | 0,654701 | 2,834625 | 162,4120 | 1,654701 | 2,309401 |
| 165 | 2,879793 | 2,693202 | 154,3091 | 0,597023 | 2,989865 | 171,3067 | 1,712378 | 2,309401 |
| 179 | 3,124139 | 3,111364 | 178,2680 | 0,577438 | 3,131516 | 179,4226 | 1,731963 | 2,309401 |

Tab. 2.6: Calculation of values $d_T = d_1 + d_2$ according to equations 2 to 5, $v = 0.5c$
All results reveal exactly $2\gamma = 2{,}309401$

# 3. Lorentz-Transformation and synchronization

The calculations concerning coordinates of space and time presented so far are not sufficient for the complete understanding of the relativistic transformation procedure. Already in the year 1900 the essential additional principle of "local time" and the consequences connected with it were investigated by H. Poincaré [10]. Later A. Einstein implemented the general statement, that the local time of moving observers must always be connected by synchronization processes [12].

Inside Special Relativity the synchronization of incidents between moved observers is of paramount importance. It is part of any comprehensive lecture concerning Special Relativity, further a multitude of publications exists of which only a small part can be discussed here.

Generally, the issue can be divided in two categories:

1. The synchronization of incidents by exchanging signals,
2. The synchronization of incidents by the exchange of clocks.

The results do not correspond to the intuitive human understanding of simultaneity and are therefore not easy to understand. This is due to the fact that an exchange of signals between two observers always occurs at the speed of light, and this must be included in the considerations. In the following the connections with the synchronization of events by using signal exchange are considered first, the synchronization by means of the exchange of clocks is treated in chapter 5.

## 3.1 Local time and synchronization using the exchange of signals

An experimental set-up shall be discussed, where a laboratory with length $a$ is considered as at rest and is passed by a small body with the velocity $v$ (Fig. 3.1). On both ends named A and E of the laboratory a clock is fixed. At the first contact of the moved body at A (case a) the clock is set to the value

$$t = -\frac{a}{v} \tag{3.01}$$

When the moving body has contact at point E (case b) the clock at point A shows the value of zero. Using this procedure, the synchronization of both observers is realized. At the point zero both emitters at A and E shall send simultaneously a signal that will arrive at time

$$t = \frac{a}{c} \tag{3.02}$$

at their partners (case c).

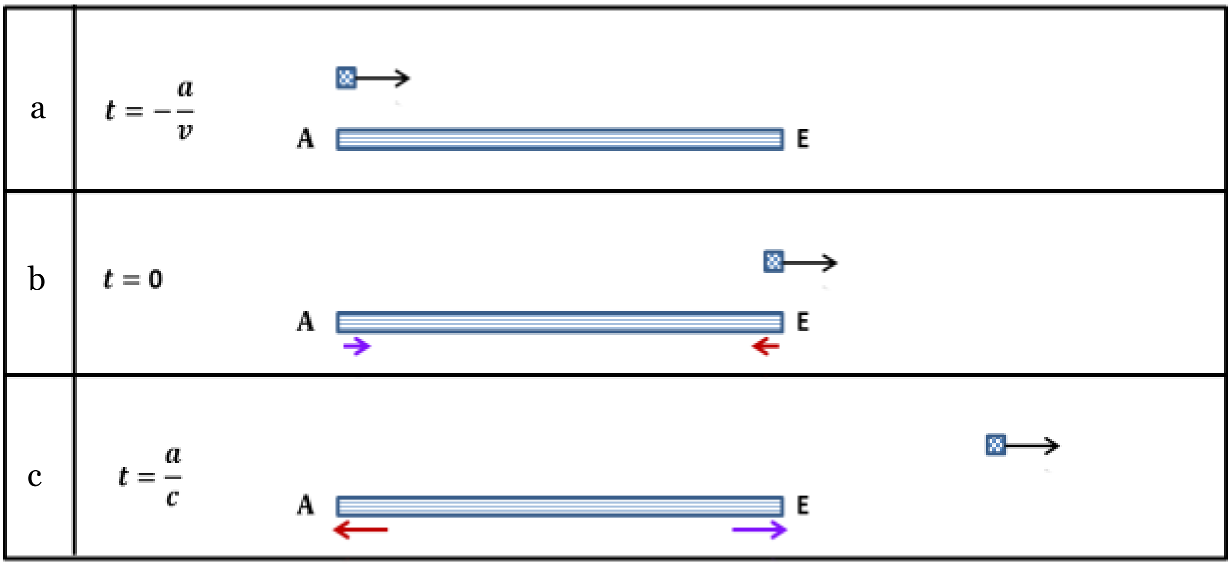



Fig. 3.1: Experimental set-up for the synchronization of an observer at rest using clocks at the ends A and E

According to the principle of relativity all participants of the experiment must find the same results, when instead of the laboratory the moving body in Fig 3.1 is considered as at rest. When these conditions are recorded a completely different diagram will appear. In Fig. 3.2 the space-time-diagram covering the new issue with the changing of the point of view is presented.

First the clock at A is passing the body at rest (presented as point $A_0$). Now the waiting time is starting; for the observer at rest the time dilatation must be considered. The clock in the position E is passing the body at rest at $E_1$ (the presentation is respecting the fact, that the moving laboratory is shortened by the factor $\gamma$ because of its movement). At that point a signal is send to A which will be received there at time $A_4$. After the end of the waiting time A will send at time $A_2$ also a signal to E which will be received there at time $E_3$.

It is clearly visible, that from the point of view of the observer at rest the times for the moved laboratory at A and E are not identical to his observations. In this case the time zero is depending on the distance to the observer at rest and follows a line which is marked as $x'$ in the diagram.

Generally, this is one of the most important features of Special Relativity. This effect is commonly called “Relativity of Simultaneity”.

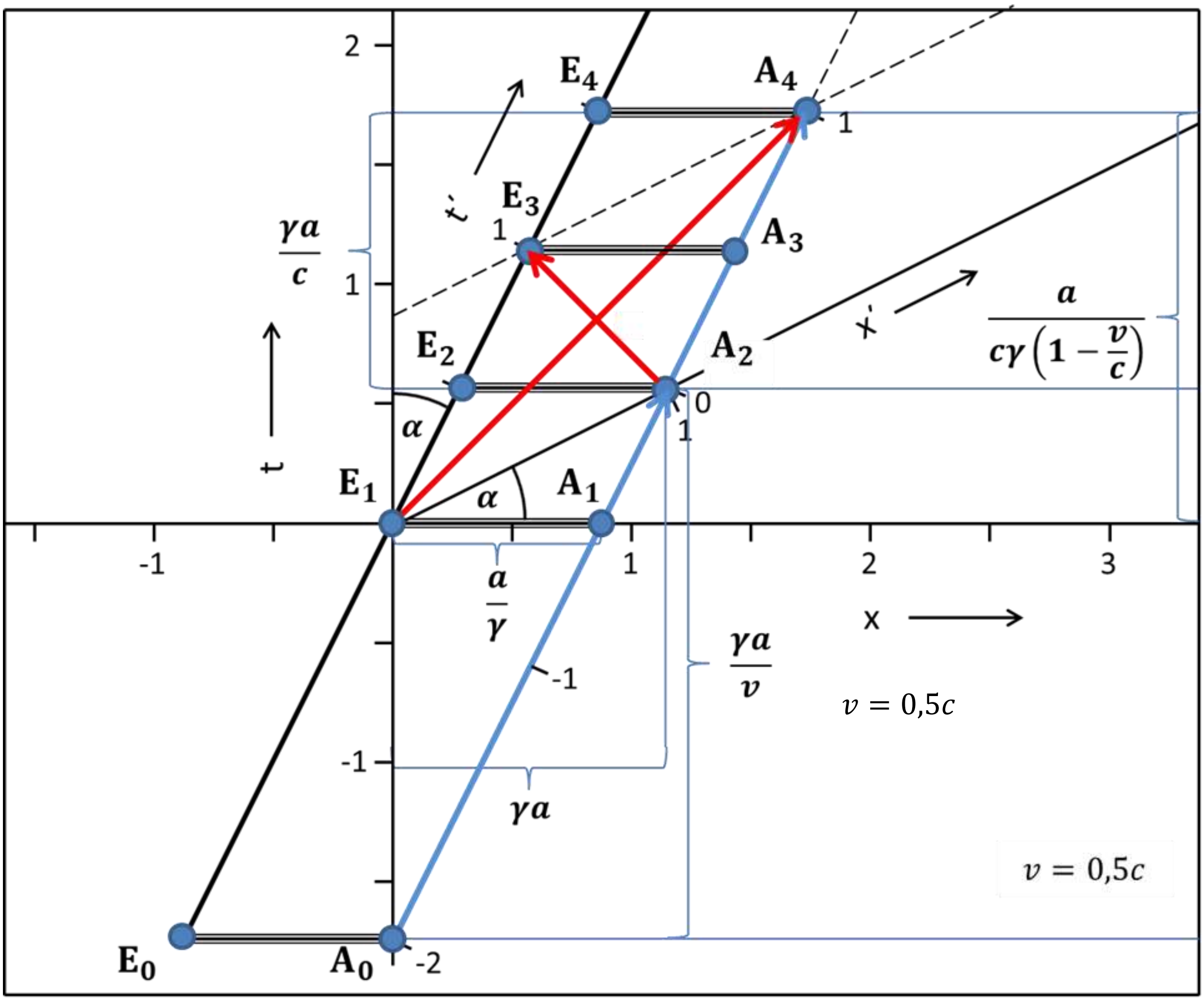



Fig. 3.2 Experimental set-up for the synchronization of a moving observer using clocks at the ends A and E.

The synchronization difference $\Delta t_S$ can be determined easily using

$$\Delta t_S = \frac{a}{c\gamma\left(1 - \frac{v}{c}\right)} - \frac{\gamma a}{c} = \frac{\gamma a}{c}\left(1 + \frac{v}{c}\right) - \frac{\gamma a}{c} \tag{3.03}$$

$$\Delta t_S = \frac{\gamma a v}{c^2} \tag{3.04}$$

The angle between the $x'$- and the $x$-axis is calculated from the synchronization difference divided by $\gamma a$

$$\tan \alpha = \frac{c \cdot \Delta t_S}{\gamma a} = \frac{v}{c} \tag{3.05}$$

and is thus identical with the angle between the $ct'$- and $ct$-axes.

The diagram developed here has interesting features, which will be discussed in the following.

## 3.2 Minkowski-diagram

The diagram presented above was introduced into Special Relativity by Hermann Minkowski (1864-1909) who, among many important scientific contributions, developed this presentation later named after him [15c].

Minkowski diagrams show several peculiarities. First of all, usually not the representation of $t$ but of $ct$ over $x$ is chosen. This gives both axes the same dimension (length) and direct derivations can be made from them. After normalization, the appearance shown in Fig. 3.3 is obtained. In this form, the diagram shows a mirror symmetry with respect to the 45° axis passing through the origin.

It is possible to determine directly from these diagrams the coordinates which result for the stationary $(x, ct)$ and for the moving observer $(x', ct')$ for the same circumstances. In the diagram Fig. 3.4 the point $P_{x,ct}$ with the coordinates $x = 3$ and $ct = 2$ is shown as an example. This is the value, at which a moving observer from the view of the stationary system is at a distance of 3 length units (LU) after 2 time units (TU) referred to the origin.

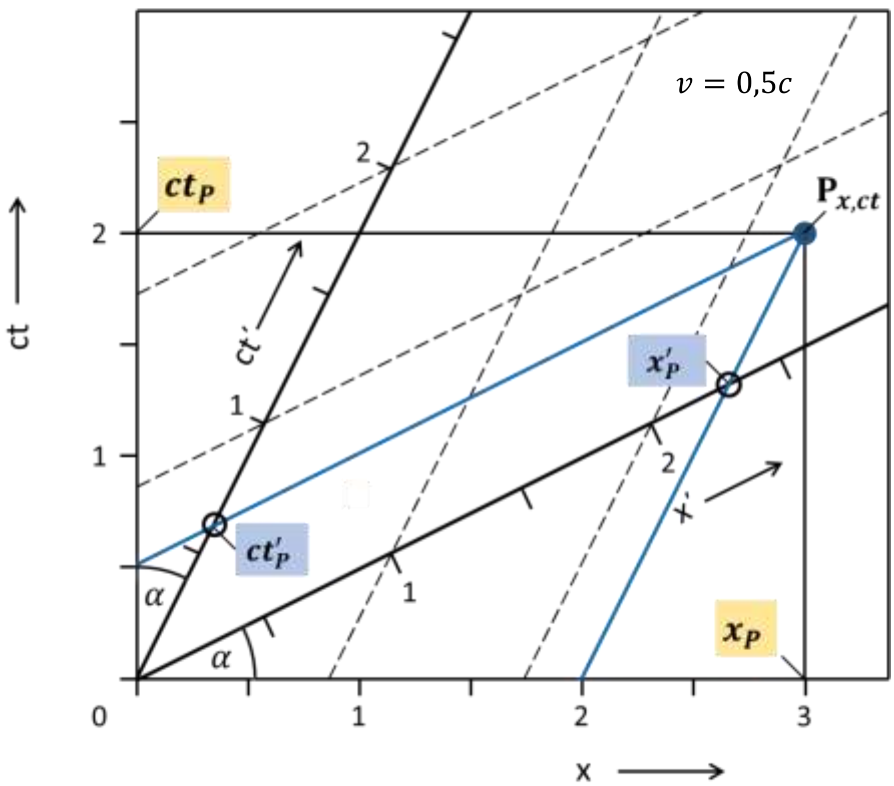



Fig. 3.3: Minkowski diagram: Example with point $x = 3$ and $ct = 2$.
Graphical determination of the coordinates in the moving system $(x', ct')$ '.

The $x', ct' -$ coordinate system is not rectangular but has angles $\alpha$ to the system $x, ct$. Therefore the coordinates are also read under this angle. Parallels to the $x'$ and $ct'$ axis are formed. The values for $x'_P$ and $ct'_P$ can then be read from the intersections with the axes $ct' = 0$ and $x' = 0$ respectively as shown.

It will be shown in the next chapter that a purely graphical/geometric derivation leads in consequence to the Lorentz transformation equations. This is absolutely necessary, because otherwise there would be contradictions within the theory.

## 3.3 Lorentz-Transformation

For the derivation of the Lorentz transformation there is a multiplicity of approaches, which can be mentioned here only exemplarily. According to the classification introduced by M. Born [26] and still used today [47], there is basically the graphical and the algebraic approach. While the graphical derivation is rarely used [e.g. 26a], there is a multitude of variants for the algebraic approach. These range from the classical representation [12,29] to the "fastest" derivation [30], conventional approaches [31,32] and to the use of the tensor calculus [27,28,33]. Moreover, parts of the graphical and algebraic derivation can also be combined [19]. Since the Lorentz transformation is one of the most important elements of Special Relativity, its derivation will be shown here with selected examples for both basic approaches.

In principle, the present relations must be linear. If there were e.g. quadratic terms, then derivations after space or time would depend on the space or the time itself. All physical laws, which contain derivations after place or time (e.g. velocity, accelerations) would then depend on the zero point of the corresponding space or time scale in case of non-linear relations. In such a case, however, this could be the subject of direct measurements and thus contradicts the general idea of the homogeneity of space and time. A further point is that the relations to be determined in the limit case of small velocities must pass over into the Galilei transformation of the classical mechanics.

In the following, first a graphical (and geometric) derivation of the Lorentz transformation from the Minkowski diagram is presented. In contrast to the approach of M. Born [26a], which works with proportion relations and the Pythagorean theorem, angular functions and geometrical approaches are used here and a particularly clear representation appears. Subsequently, a selected algebraic approach is presented.

At this stage, an important point shall be briefly discussed. According to the principle of the constancy of the speed of light in all inertial systems, measurements of the speed of light will lead to the same result for the reference system ("resting") and for an observer moving relative to it (chapter 1.6). This is subjectively correct. However, the derivations discussed in the following are based *exclusively* on the speed of light of the *reference system* and thus describe the observations made from this, from which finally the Lorentz transformations are resulting.

### 3.3.1 Derivation of the Lorentz-Transformation using the Minkowski diagram

As was already explained, the representation of the Minkowski diagram can be derived exclusively using time dilation, space contraction and synchronization difference. Beyond that only the assumption of the isotropy of time and space as well as the constancy of the speed of light (in the system at rest) is necessary. In the following it will be shown that at the transition between the represented systems of this diagram, relations corresponding to the Lorentz transformation must inevitably result.

When an arbitrary point $P_{x,ct}$ is considered in this diagram (Fig. 3.4), the coordinates can be calculated with the help of the values marked in yellow.

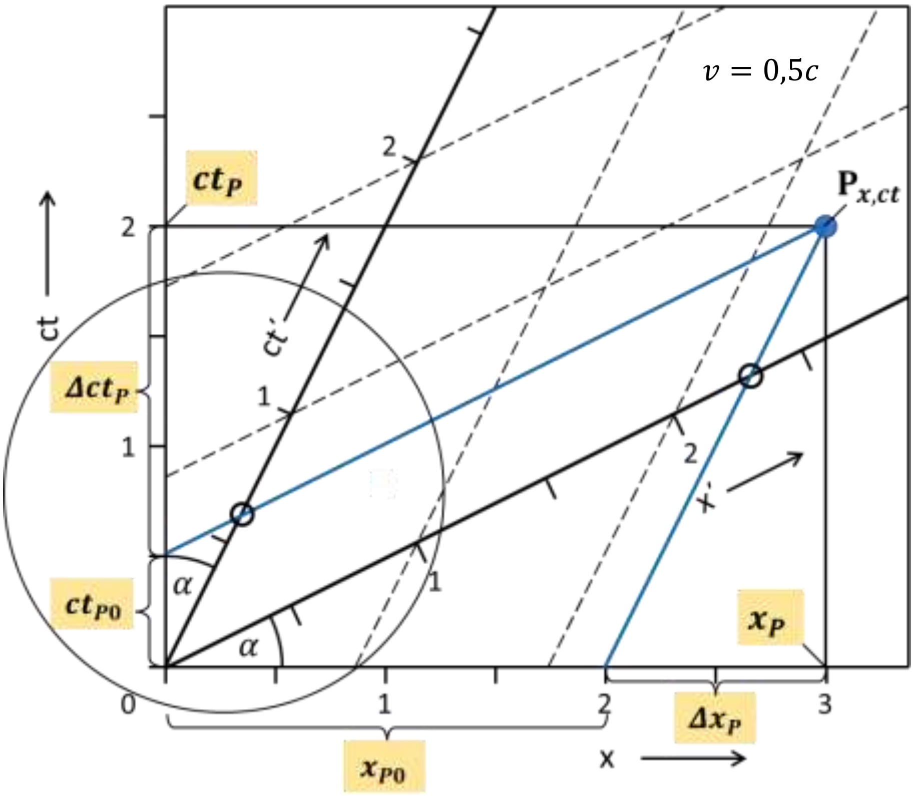



Fig. 3.4: Minkowski diagram with coordinate determination of point $P_{x,ct}$ in the moving system. Quantities relevant for the calculation are colored yellow.

First, parallels to the $x'$ and $ct'$ axes are formed and their intersections with the $ct/x$-coordinate system are determined. The resulting values $ct_{P0}$ and $x_{P0}$ can be converted into $x'_P$ and $ct'_P$. For this purpose, an intermediate calculation is required in the range around 1. For this purpose, a circle is drawn in Fig. 3.4, the contents of which are shown in higher resolution in Fig. 3.5.

In this diagram Fig. 3.5 all values are normalized to 1. In the case shown, no change of location occurs within the moving laboratory, i.e. the movement takes place on the $ct'$-axis. Then, as already shown in chapter 2, the dependence $d = \gamma \cdot ct_1$ applies for the case $ct = 1$. It follows

$$\tan\alpha = \frac{v}{c} = \frac{b}{d} = \frac{e}{b} \tag{3.10}$$

and from this

$$e = d\frac{v^2}{c^2} \tag{3.11}$$

Because of $f = d - e$, it follows after substituting eq. (3.11)

$$f = d - d\frac{v^2}{c^2} = d\left(1 - \frac{v^2}{c^2}\right) = \frac{d}{\gamma^2} = \frac{ct_1}{\gamma} \tag{3.12}$$

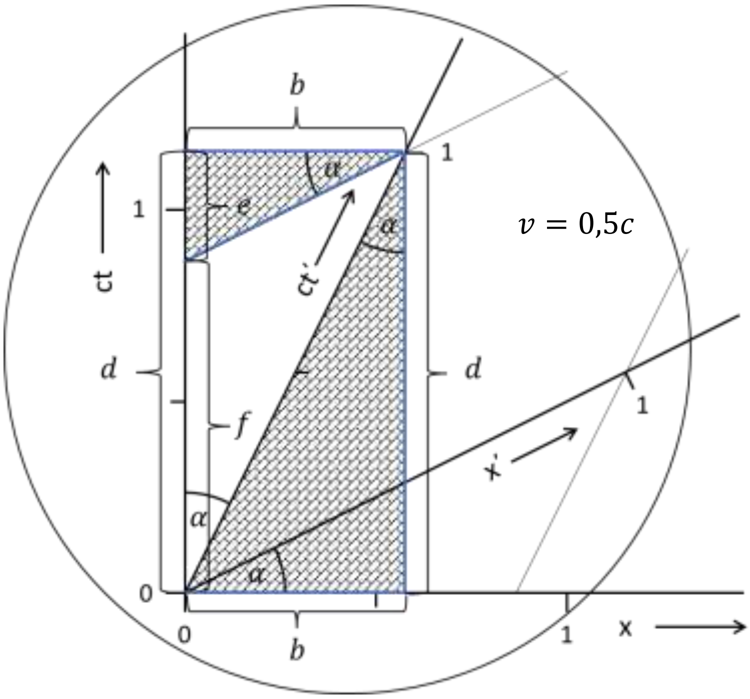



Fig. 3.5: Detail from Fig. 3.4, determination of $f$ corresponding to $ct_{P0}$ from Fig. 3.4.

For the $x'$-axis, the same relationship applies for symmetry reasons. It follows first for the value $ct'_P$:

$$ct'_P = \gamma \cdot ct_{P0} \tag{3.13}$$

From the geometrical conditions in Fig. 3.4, we get

$$ct'_P = \gamma\,(ct_P - \Delta ct_P) \tag{3.14}$$

Because of

$$\tan\alpha = \frac{\Delta ct_P}{x_P} = \frac{v}{c} \tag{3.15}$$

then finally appears

$$t'_P = \gamma\left(t_P - \frac{v}{c^2}x_P\right) \tag{3.16}$$

For $x'_P$ we obtain in the same way

$$x'_P = \gamma \cdot x_{P0} \tag{3.17}$$

$$x'_P = \gamma\,(x_P - \Delta x_P) \tag{3.18}$$

$$\tan\alpha = \frac{\Delta x_P}{ct_P} = \frac{v}{c} \tag{3.19}$$

$$x'_P = \gamma\,(x_P - v\,t_P) \tag{3.20}$$

The calculation results in the following values

| System at rest | Moved System |
|---|---|
| $x_P = 3$ | $x'_P = 2{,}309$ |
| $ct_P = 2$ | $ct'_P = 0{,}577$ |

The equations (3.16) and (3.20) correspond exactly to the relations of the Lorentz transformation as they were already presented in Eq. (1.01) and (1.02). Thus it is shown that these equations can be derived from a Minkowski diagram by establishing simple geometrical correlations.

### 3.3.2 Algebraic concept for the derivation of the Lorentz-Transformation

To complete the considerations concerning the Lorentz-Transformation in addition a "classic" approach, which means a typical derivation of the equations used in the literature, shall be discussed. To show this concept in detail the presentation of H. J. Lüdde and T. Rühl [34] was chosen, because it has a basic approach and does not need assumptions during the derivation, which show later that they are reasonable. A similar derivation was also used by A. Einstein in the year 1905, although his only comment was "after easy calculation" without showing any details [12b].

Using this concept, two systems shall be looked at which are moving against each other. It is generally required that these are inertial systems, which means acceleration and rotation is not permitted. The position of any point in these systems is characterized by three coordinates for the space and one for the time. For the system S these are $x, y, z, t$ and S′ with $x', y, 'z', t'$. It is assumed, that the systems move against each other with a speed of $v$ concerning the $x$- coordinate and that in $y$- and $z$- direction no motion exists.

First the situation is discussed that the point of origin (where space and time are defined as zero) of both systems get in contact at the time

$$t = t' = 0 \tag{3.40}$$

In this case the correlations between the coordinates are, because of the required linearity

$$x' = Ax + Bt, \qquad y' = y, \qquad z' = z, \qquad t' = Cx + Dt \tag{3.41}$$

This means that $t$ is no longer invariant concerning space and furthermore $x$ is not invariant concerning time. Thus, for an arbitrary sphere with a light emitter in the center the following equations will apply:

$$S: \quad x^2 + y^2 + z^2 = c^2 t^2 \tag{3.42}$$

$$S': \quad x'^2 + y'^2 + z'^2 = c^2 t'^2 \tag{3.43}$$

Hence

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2 \tag{3.44}$$

For the solution of the equations first the system-velocities are considered. In view of system $S'$ the velocity of $S$ is

$$v = \frac{x}{t} \tag{3.45}$$

When the situation is discussed that both systems have contact in the point of origin Eq. (3.41) develops to

$$0 = Avt + Bt \tag{3.46}$$

or

$$B = -Av \tag{3.47}$$

The use of Eq. (3.44) leads to

$$(Ax + Bt)^2 - c^2(Cx + Dt)^2 = x^2 - c^2 t^2 \tag{3.48}$$

and

$$x^2(A^2 - c^2 D^2 - 1) + 2xt(AB - c^2 CD) + t^2(B^2 - c^2 D^2 + c^2) = 0 \tag{3.49}$$

Because the relations (3.48) and (3.49) are valid for arbitrary values of space and time the following equations apply:

$$A^2 - c^2 C^2 - 1 = 0 \tag{3.50}$$

$$AB - c^2 CD = 0 \tag{3.51}$$

$$B^2 - c^2 D^2 + c^2 = 0 \tag{3.52}$$

The solution of this system with 4 equations and 4 unknown factors [Eq. (3.47) and also Eq. (3.50) - (3.52)] leads to the following relations

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \tag{3.53}$$

$$x' = \gamma(x - vt) \tag{3.54}$$

The $y$- and $z$- coordinates remain unchanged.

The results of the derivation presented here are in full agreement with the Lorentz-Transformation already discussed before several times. The requirements concerning time dilatation, space contraction and local time (with asynchronous characteristics) can be derived out of subsequent calculations. This contrasts with the calculations presented before, where the equations were derived using a graphic approach; in this case time dilation and length contraction were preconditions and not the results of calculations.

Finally the question remains, what significance the result has for the interpretation of the conditions. In chapter 2.2 it was already presented in detail that it is impossible for an observer at rest or in a moving system using the exchange of signals to decide about the state of movement. This is caused by the simultaneously appearing effects of dilatation of time and contraction of space.

However, it is by no means the case that an observer at rest is determining a different speed of light in the moving system; in his view the speed of light of his system will be valid for all investigations instead. The fact that the moving observer will find the same results in comparison to the system at rest is exclusively caused by differences in the synchronization procedures between the two systems. This question will be taken up again in chapter 11.

## 3.4 Einstein-synchronization

The synchronization procedure later named after Albert Einstein was first mentioned in his pioneering publication in the year 1905 [12]. To illustrate this point further, an extract of the original work is presented in Fig. 3.5, which was part of the derivation of the Lorentz-Transformation. The following equation is of special interest

$$\frac{1}{2}(\tau_0 + \tau_2) = \tau_1 \tag{3.60}$$

Einstein used Greek letters for the time in a moving system, for which today generally $t'$ is taken (further he used the letter $V$, not $c$ for the speed of light); today the equation is generally presented in a different form like

$$\frac{1}{2}(t_0' + t_2') = t_1' \tag{3.61}$$

It is a special characteristic of this equation, that the synchronization is solely depending on the exchange of signals between the participants.

The synchronization procedure following this specification can generally be characterized as follows:

> Clock U(0) is situated in the coordinate origin of an arbitrary inertial system. An identical clock U(x) is located at a different point with the distance $x$. When U(0) is showing time $t_0$ a light signal is emitted from here to point $x$ and from there immediately reflected to the coordinate origin. At arrival U(0) is showing time $t_2$. U(x) is synchronized with U(0) when U(x) at the time of reflection is showing time $t_1$ following the relation:

$$t_1 = t_0 + \frac{1}{2}(t_2 - t_0) \tag{3.62}$$

Equation Eq. (3.62) is identical to Eq (3.60) resp. (3.61). This is independent from the situation, whether the clocks are at rest or shall be moved (which means the use of $t$ or $t'$ is possible).

To any system of values $x, y, z, t$, which completely defines the place and time of an event in a stationary system, a system of values $\xi, \eta, \zeta, \tau$, determining that event relatively to the system $k$ belongs to it, and the task is now to find a system of equations connecting these variables.

First it is clear that the equations must be linear on account of the properties of homogeneity which we attribute to space and time.

If we set $x' = x - vt$, it is clear that a point at rest in the system $k$ must belong to a system of values $x', y, z$, independent of time. We first determine $\tau$ as a function of $x', y, z$, and $t$. To do this we have to express in equations that $\tau$ is nothing else than the summation of the reading of clocks at rest in system $k$, which have been synchronized according to the rules given in § 1.

From the origin of system $k$ let a ray be emitted at time $\tau_0$ along the $X$-axis to $x'$, and at time $\tau_1$ be reflected to the origin of the coordinates, arriving there at time $\tau_2$, then we will find

$$\frac{1}{2}(\tau_0 + \tau_2) = \tau_1$$

or, by inserting the arguments of the function $\tau$ and applying the principle of the constancy of the speed of light in the stationary system:

$$\frac{1}{2}\left[\tau(0,0,0,t) + \tau\left(0,0,0,\left\{t + \frac{x'}{V-x} + \frac{x'}{V+x}\right\}\right)\right]$$
$$= \tau\left(x', 0,0, t + \frac{x'}{V-x}\right)$$

Hence, if $x'$ is chosen infinitesimally small

$$\frac{1}{2}\left(\frac{1}{V-x} + \frac{1}{V+x}\right)\frac{\partial\tau}{\partial t} = \frac{\partial\tau}{\partial x'} + \frac{1}{V-v}\frac{\partial\tau}{\partial t}$$

or

$$\frac{\partial\tau}{\partial x'} + \frac{v}{V^2 - v^2}\frac{\partial\tau}{\partial t} = 0$$

It shall be noted that it is possible to choose any other point of origin for the coordinates of the ray, and the equation just obtained is therefore valid for all values of $x', y, z$.

Fig. 3.5: Extract from original publication of Albert Einstein [12a], translated

The definition used in these equations is not giving information, whether synchronization is still valid at a later point in time or not. In principle the following situations are possible:

a) U(x) remains stationary in relation to U(0),

b) U(x) is passing U(0) in short distance to be synchronized and then moving away,

c) U(x) is passing U(0) in a long distance without direct contact.

It is immediately clear for situation a) that the factor $\gamma$ is always identical for both clocks and so the synchronization can be repeated without difference at any time. Situations b) and c) were dealt with in chapters 2.1.1 resp. 2.1.2. In both cases it was shown, that independent from the distance of objects no differences of their observations are detectable. The only precondition is, that the Lorentz-Transformation is taken as a basis.

Exact interpretation of the situation makes clear, that when using hypothetical superluminal velocities sending information to an observer, differences would appear. However, according to the assumptions made, this is not possible and so synchronization differences cannot occur. As already discussed, the appearing situation is called "Relativity of Simultaneity".

Current concepts for derivation of the Lorentz-Equations generally avoid using the form Einstein selected in the year 1905. In a normal case a presentation using equations Eq. (3.42) and Eq. (3.43) is taken (which was used as a basis for calculation in chapter 3.3.2)

$$S: \quad x^2 + y^2 + z^2 = c^2 t^2 \tag{3.42}$$

$$S': \quad x'^2 + y'^2 + z'^2 = c^2 t'^2 \tag{3.43}$$

The equation system can be interpreted in a way, that the transition from Eq. (3.42) to Eq. (3.43) is in accordance with Einstein synchronization and this relation is implicitly included. Einstein himself in his book about the theory of relativity written as a "simple version" [29], first edited in the year 1916, also used a similar approach. Obviously, he also shared the opinion that this would be easier to understand.

The Einstein-synchronization, connected with Eq. (3.62), is a definition, not an observation. The Einstein synchronization is of paramount importance for the Theory of Special Relativity and is widely discussed until today [19,20,35]. After the presentation of additional important aspects, it will be discussed again in more detail in this investigation (chapter 11.2).

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# 4. Additional considerations for moving observers

The relations discussed so far can easily be extended from two to several observers. Doing this, first the addition of velocities must be derived, because the relativistic case shows not the simple summation which could be expected according to the laws of the Galilei-Transformation. Further special relations exist in connection with velocities lower than the speed of light, which are observed e.g. concerning light in transparent media or connected with the transport of sound in solid bodies. These relations are also valid during acceleration of observers because material objects cannot be considered as absolute rigid.

In addition the case is discussed, when the transport of a signal inside a moving body is not only taking place in the direction of the movement but also transverse to it.

## 4.1 Relativistic addition of velocities

The theorem for the addition of velocities in the relativistic case was derived by A. Einstein already in the year 1905 [12]. It is assumed that in a system S′, which is moving with the speed $v$ in direction of the $x$-axis in relation to the reference system S, an observer is moving according to the relations

$$x' = w'_x t' \tag{4.01}$$

$$y' = w'_y t' \tag{4.02}$$

$$z' = 0 \tag{4.03}$$

where $w'_x$ and $w'_y$ are the components of the velocity in $x'$ resp. $y'$-direction. The aim is to find a relation referring to the reference system S. The coordinate system is selected in a way that all points are situated in the $x - y$ plane and so the coordinate $z'$ can remain unconsidered.

Thus, the Lorentz equations read

$$x' = \gamma(x - vt) \tag{4.04}$$

$$y' = y \tag{4.05}$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \tag{4.06}$$

*Behavior in $x$-direction*

When Eq. (4.04) and Eq. (4.06) are inserted in Eq. (4.01) this yields

$$\gamma(x - vt) = w_x' \cdot \gamma\left(t - \frac{v}{c^2}x\right) \tag{4.07}$$

with

$$x = \frac{w_x' + v}{1 + \frac{vw_x'}{c^2}} \cdot t \tag{4.08}$$

*Behavior in $y$-direction*

For the determination equations Eq. (4.02), (4.06) and (4.08) are successively inserted in Eq. (4.05)

$$y = y' = w_y'\gamma\left(t - \frac{v}{c^2}x\right) \tag{4.09}$$

$$y = w_y'\gamma\left(t - \frac{v}{c^2} \cdot \frac{w_x' + v}{1 + \frac{vw_x'}{c^2}} \cdot t\right) \tag{4.10}$$

following

$$y = w_y'\gamma \frac{1 + \frac{vw_x'}{c^2} - \frac{vw_x'}{c^2} - \frac{v^2}{c^2}}{1 + \frac{vw_x'}{c^2}} \cdot t \tag{4.11}$$

$$y = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{vw_x'}{c^2}} w_y' t \tag{4.12}$$

Because of the linearity of the relations the velocities can be derived out of Eq. (4.08) and (4.12) in a simple way as

$$\frac{dx}{dt} = w_x = \frac{w_x' + v}{1 + \frac{vw_x'}{c^2}} \tag{4.13}$$

$$\frac{dy}{dt} = w_y = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{vw_x'}{c^2}} w_y' \tag{4.14}$$

In a final step the angles of the velocity-components in relation to the $x$-axis are inserted which yields

$$w'_x = w \cdot cos\,\alpha \tag{4.15}$$

$$w'_y = w \cdot sin\,\alpha \tag{4.16}$$

and by using

$$v_T = \sqrt{w_x^2 + w_y^2} \tag{4.17}$$

these are added as vectors

$$\sqrt{\left(\frac{w\,cos\,\alpha + v}{1 + \frac{v\,w\,cos\,\alpha}{c^2}}\right)^2 + \left(\frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v\,w\,cos\,\alpha}{c^2}}\,w\,sin\,\alpha\right)^2} \tag{4.18}$$

For the total velocity $v_T$ in system S and after transformation and using the general relation

$$cos^2\alpha + sin^2\alpha = 1 \tag{4.19}$$

the final solution is

$$v_T = \frac{\sqrt{v^2 + w^2 + 2\,v\,w\,cos\alpha - \left(\frac{v\,w\,sin\alpha}{c}\right)^2}}{1 + \frac{v\,w\,cos\alpha}{c^2}} \tag{4.20}$$

If the velocities $v$ and $w$ are situated unidirectional, which means angle $\alpha = 0$, then Eq. (4.20) is simplified to

$$v_T = \frac{v + w}{1 + \frac{v\,w}{c^2}} \tag{4.21}$$

When this situation concerning emitted signals and their reception is presented in a space-time diagram then the configuration in Fig. 4.1 is achieved. On the left side of this chart the situation is presented, that the emitter in the middle is belonging to a system at rest. The receivers of the signals, which are in addition reflecting the incoming signals immediately, are increasing their distance with equal speed (in this case: $v = w = 0{,}5c$). On the right-hand side, it is illustrated how the same situation develops from the view of an observer which was considered as in motion before (in this case: B). One of the observers is increasing the distance with the same speed of $v = 0{,}5c$, the third shows a speed of $v = 0{,}8c$ according to equation (4.21). A reverse situation develops when observer C is considered as at rest.

To illustrate the exact circumstances, the situation for times $t = 1$ TU and $t = 2$ TU are marked with different shades in the space-time diagram (Fig. 4.1). In this presentation it is clearly visible, that irrespective of the velocity of an observer always the same results will be achieved.

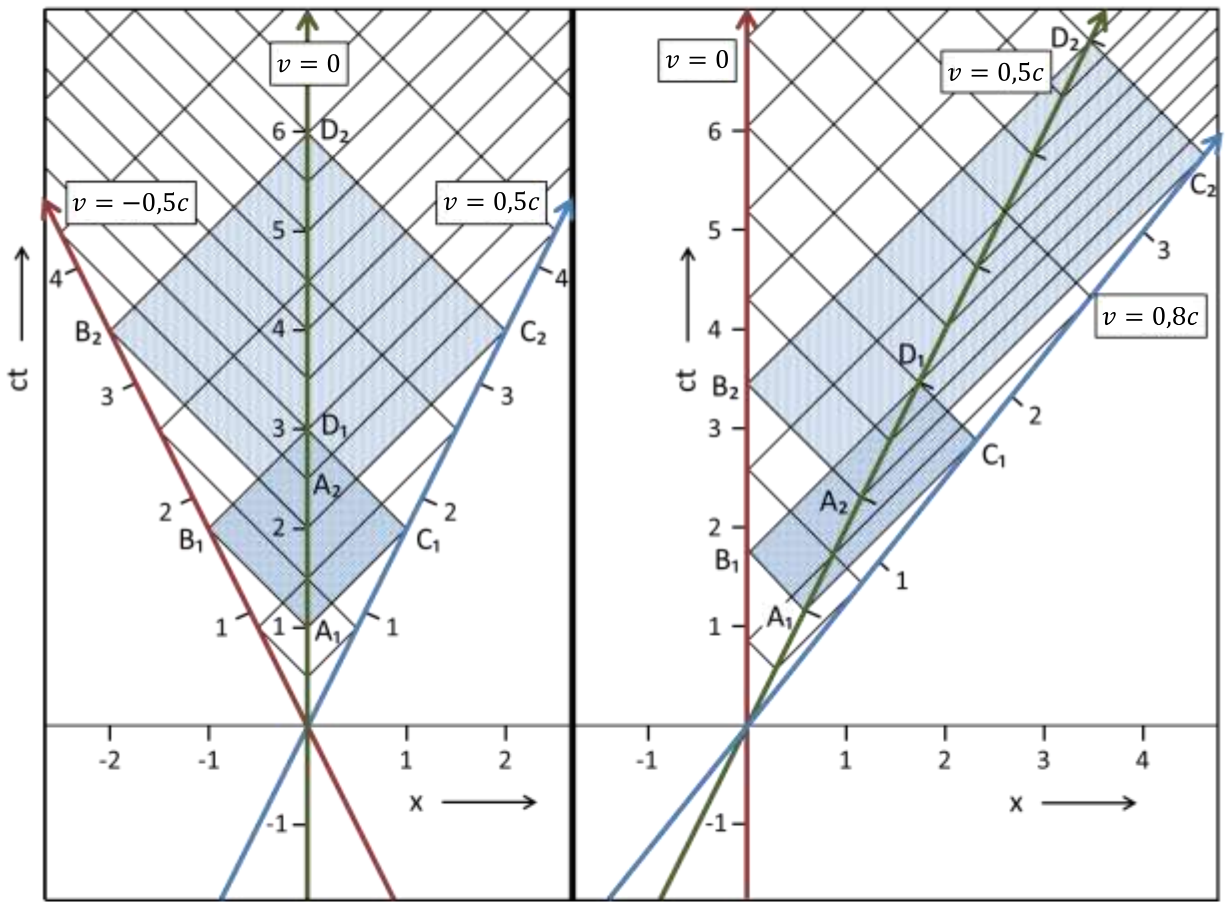



Fig. 4.1: Space-time diagram for observers at rest and in motion

## 4.2 Experiments with transparent media in motion

In the following a further alternative of the case discussed in chapter 2.2.2 will be looked at. Instead of a light pulse a second observer shall be shifted inside the body in moving direction and opposite to it. In conjunction with the exchange of light pulses the following combinations are possible:

A: Light pulse going and coming,

B: Observer in motion (in moving direction), light pulse comes back,

C: Light pulse going, observer in motion (opposite to moving direction),

D: Observer in motion (in moving direction and opposite).

In Fig. 4.2 possible combinations for the velocity of bodies in motion with $v = 0,5\ c$ are presented. As already shown, the velocities in the relativistic range are calculated according to Eq. (4.21). In this case of a system velocity of $v_1 = 0,5\ c$ and an additional velocity of a body in motion of also $v_2 = 0,5\ c$ was assumed and the result is $v_T = 0,8\ c$.

The figure shows clearly that the cases B and C, i.e. the combination of light pulse and body in motion, are leading to the same results.

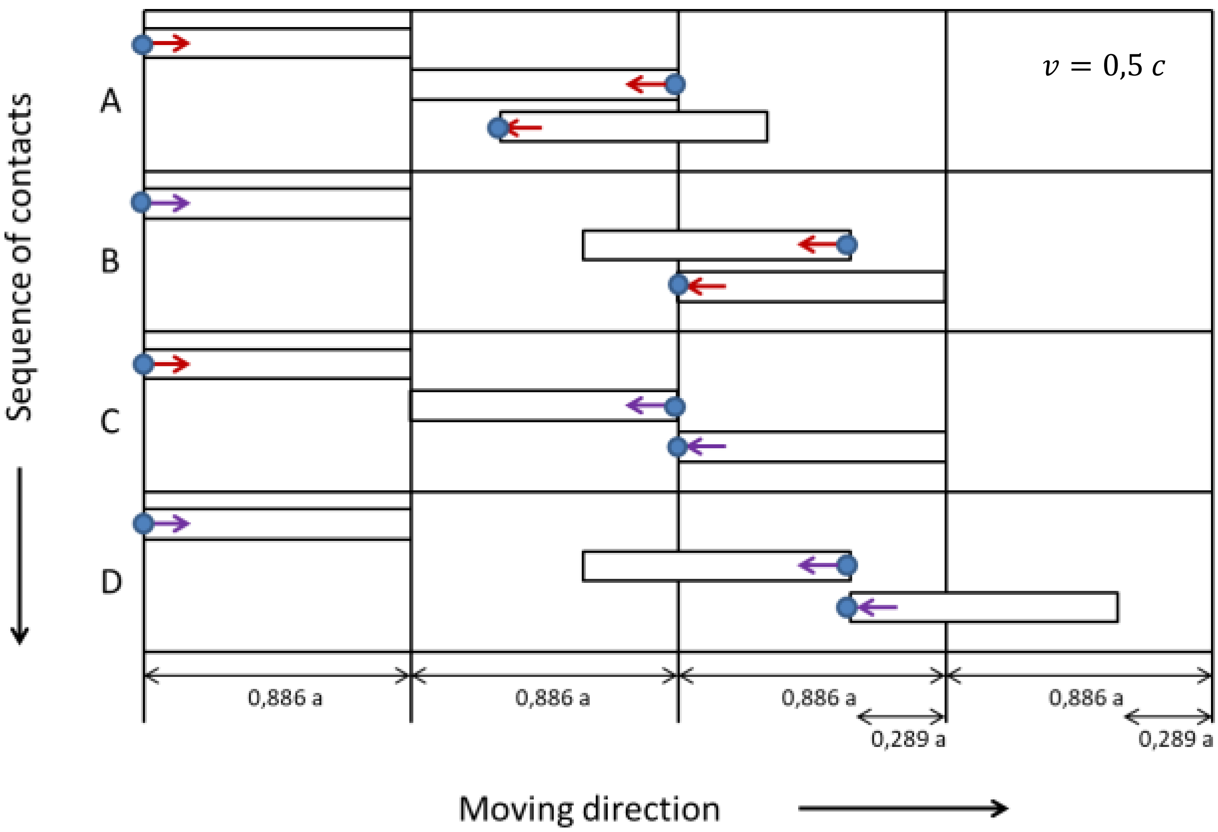



Fig. 4.2: Exchange of signals and bodies in motion in a moved system
A: Light pulse going and coming,
B: Body in motion (in moving direction), light pulse comes back,
C: Light pulse going, body in motion (opposite to moving direction),
D: Body in motion (in moving direction and opposite).

An experimental proof of these cases with bodies in motion is, however, only possible with extreme restrictions because of the high velocities needed. An experimental assessment is yet possible by an examination using optical tools. The speed of light $c_n$ in media is defined as

$$c_n = \frac{c}{n} \tag{4.30}$$

with $n$ as refractive index. It was already in the year 1812 that Augustin Jean Fresnel (1788-1827) developed the hypothesis, that the speed of light in moved media can be calculated by using a dragging coefficient (which was later named after him). According to this the speed of light in a moving system for an observer at rest is

$$c_T = \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right) \tag{4.31}$$

This theory was verified in the year 1851 by Hippolyte Fizeau (1819-1896) with an experiment where he measured the speed of light in water which was flowing with different velocities. After the full development of the Lorentz-equations it was possible to show, that the addition of velocities of moved media and the light propagation $c_n$ inside can be calculated using the addition of relativistic speed [36].

For calculation Eq. (4.21) is used

$$v_T = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \tag{4.32}$$

and with $v_1 = c_n$ this yield

$$v_T = \frac{\frac{c}{n} + v_2}{1 + \frac{v_2}{nc}} = \frac{c^2 + ncv_2}{nc + v_2} \tag{4.33}$$

A Taylor expansion for $v_2$ is leading to

$$v_T = \frac{c}{n} + v_2\left(1 - \frac{1}{n^2}\right) - \frac{v_2^2}{nc}\left(1 - \frac{1}{n^2}\right) + \frac{v_2^3}{n^2c^2}\left(1 - \frac{1}{n^2}\right) - +\ldots \tag{4.34}$$

This equation is, concerning terms of first order, equal to the relation given in equation Eq (4.31).

A calculation using the Lorentz-Transformation for the situation according to Fig 4.2 show the results presented in Tab. 4.1. In Fig 4.3 the results are presented in a diagram. As expected after the end of the experiment all values are located on the $ct'$- line. Furthermore, it is evident that the transformation equations confirm the expected relations and that no contradictions can be observed.

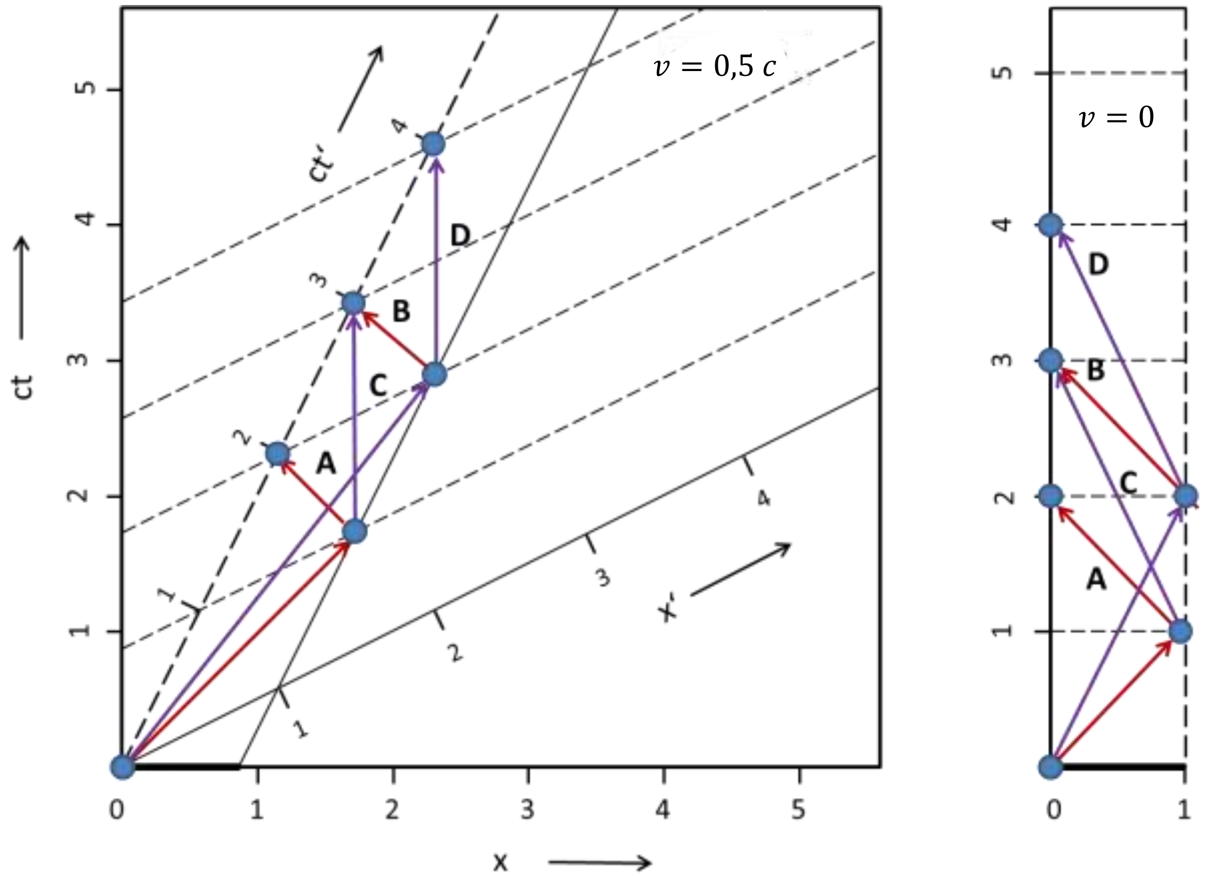



Fig. 4.3: Minkowski-diagram for cases A, B, C and D according to Fig. 4.2. Left: moved ($v = 0{,}5\,c$), right: at rest ($v = 0$)

| Case | $E_0$ | $E_0'$ | $E_1$ | $E_1'$ | $E_2$ | $E_2'$ |
|---|---|---|---|---|---|---|
| A | [0; 0] | [0; 0] | [1,73; 1,73] | [1; 1] | [1,15; 2,31] | [0; 2] |
| B | [0; 0] | [0; 0] | [2,31; 2,89] | [1; 2] | [1,73; 3,46] | [0; 3] |
| C | [0; 0] | [0; 0] | [1,73; 1,73] | [1; 1] | [1,73; 3,46] | [0; 3] |
| D | [0; 0] | [0; 0] | [2,31; 2,89] | [1; 2] | [2,31; 4,62] | [0; 4] |

Tab. 4.1: Calculated values for the situations presented in Fig. 4.2

The validity of this equation was verified in a multitude of experiments, first by H. Fizeau using flowing water and later e.g. by R. V. Jones using rotating transparent discs [37,38]. It is therefore an important part of physics and belongs both to the foundations of optics and relativistic considerations.

## 4.3 Triggering of engines after synchronization

It was already discussed in detail and demonstrated based on several examples that after mere kinematic considerations during the exchange of signals in laboratory systems after an “Einstein-Synchronization” no discrepancies will occur. A similar situation exists, when signals are used not only for synchronization of clocks but to trigger engines which influence the movement of the laboratory. The following situation shall be discussed:

1. From the middle of a laboratory signals are sent at the same time in different directions A and B.
2. When a signal is detected at A or B an engine will be started instantly in transverse direction compared to the direction of the incoming signal. The acceleration at A and B follow the same orientation.
3. Tests are executed in a situation at rest and in motion.

First, it is clear that A and B will start their engines at the same time when the laboratory is in a situation at rest (Fig. 4.4, right-hand side). This is not the case for a moved system, however. While the observer in motion after a previous synchronization realizes that the engines will start at the same time, an observer at rest will monitor that, because of the longer running time of the signal from the middle to A´ compared to B´, the engine at B´ will start first. Because of the acceleration transverse to the moving direction according to this consideration a momentum is generated, and the laboratory should start to turn.

In the literature cases like this are discussed quite often. A similar approach was examined by M. Born and during considerations of electrodynamics the assumption was made that an observer (here: the laboratory) with an unlimited rigidity would create discrepancies [39]. An unlimited rigidity (sometimes also called “Born’s rigidity”) cannot be valid, however, because all real material objects show a limited and not an infinite speed of sound which would be necessary for unlimited rigidity. The situation was discussed at length by A. Sommerfeld [15d].

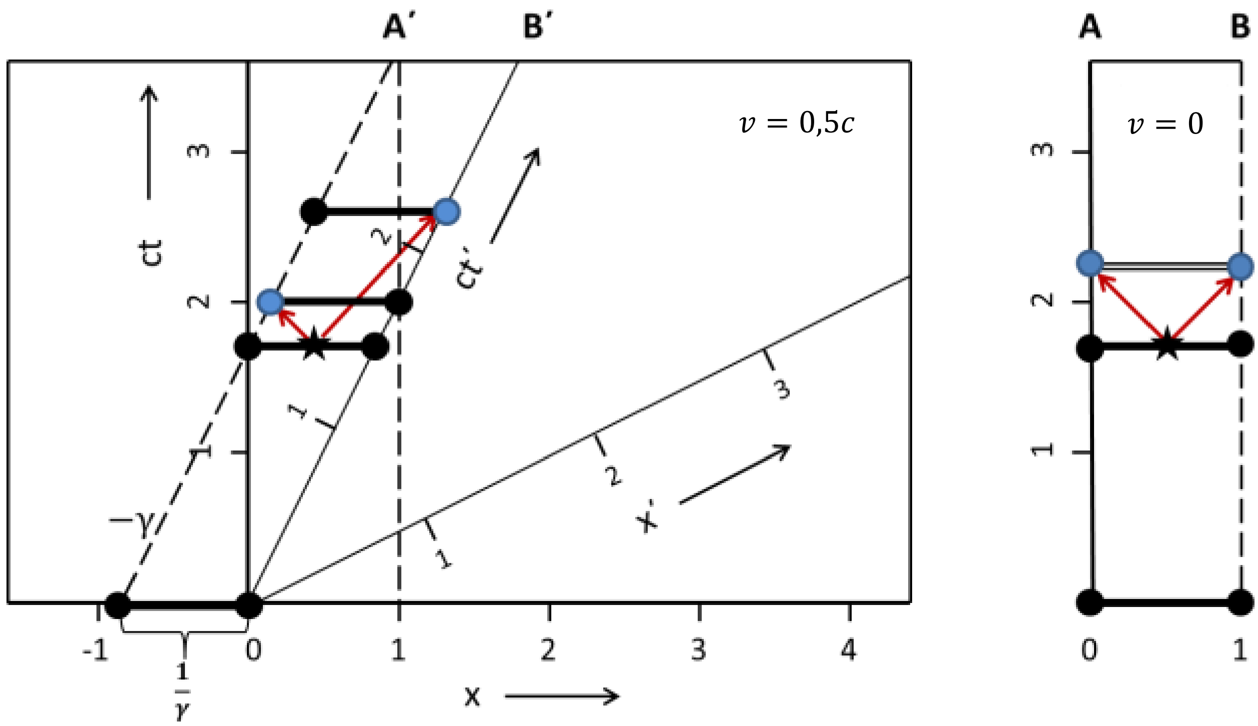



Fig. 4.4: Laboratory with signals to trigger an engine in transverse direction ($v = 0{,}5\ c$). Left: System in motion; Right: System at rest

If the situation is considered in a way that the propagation of a signal is using the speed of sound (or any other limited velocity up to the speed of light), the relativistic addition of velocities lead to the same case that was discussed in chapter 4.2. The propagation of the movements in transverse direction caused by the different engines will arrive at the same time in the middle of the laboratory and thus no momentum will be generated.

## 4.4 Exchange of signals between observers with spatial geometry

Up to now the exchange of signals between observers with an elongation in only one direction was discussed. To extend this for objects with spatial geometry, an experimental setup like in chapter 2.2.2 is chosen with the difference, however, that for the laboratories objects with equilateral triangles were selected.

The signals are therefore not emitted longitudinal, but with an angle of 60° to this direction (see Fig. 4.5). When the observers in both systems pass each other at A, B, resp. A′ and B′ a signal is sent to C resp. C′. Both C and C′ are reflecting the signals back to the sender and the measured times are monitored. For an observer at rest the situation of a system in motion is defined as presented in Fig. 4.6. First, the base of the equilateral triangle with length $a$ is shortened by the factor $\gamma$ in moving direction, which is resulting in the effect that 4 cases for contacts between the corners of the triangles will occur. These situations are shown in the left-hand side of Fig. 4.6. Whereas inside the moving system the distance from A′ to C′ (cases 1 and 3) and B′ to C′ (cases 2 and 4) is subjectively viewed as shown (in the diagram presented with dotted lines), for the system at rest the way of the signal is following $d$ as defined in the right-hand side of the diagram.

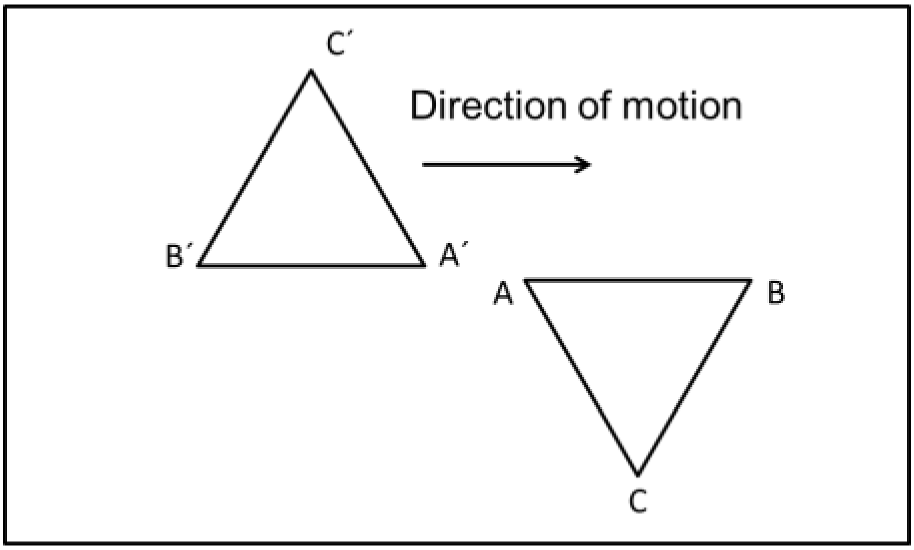



Fig. 4.5: Experimental set-up of experiments for observers with spatial geometry

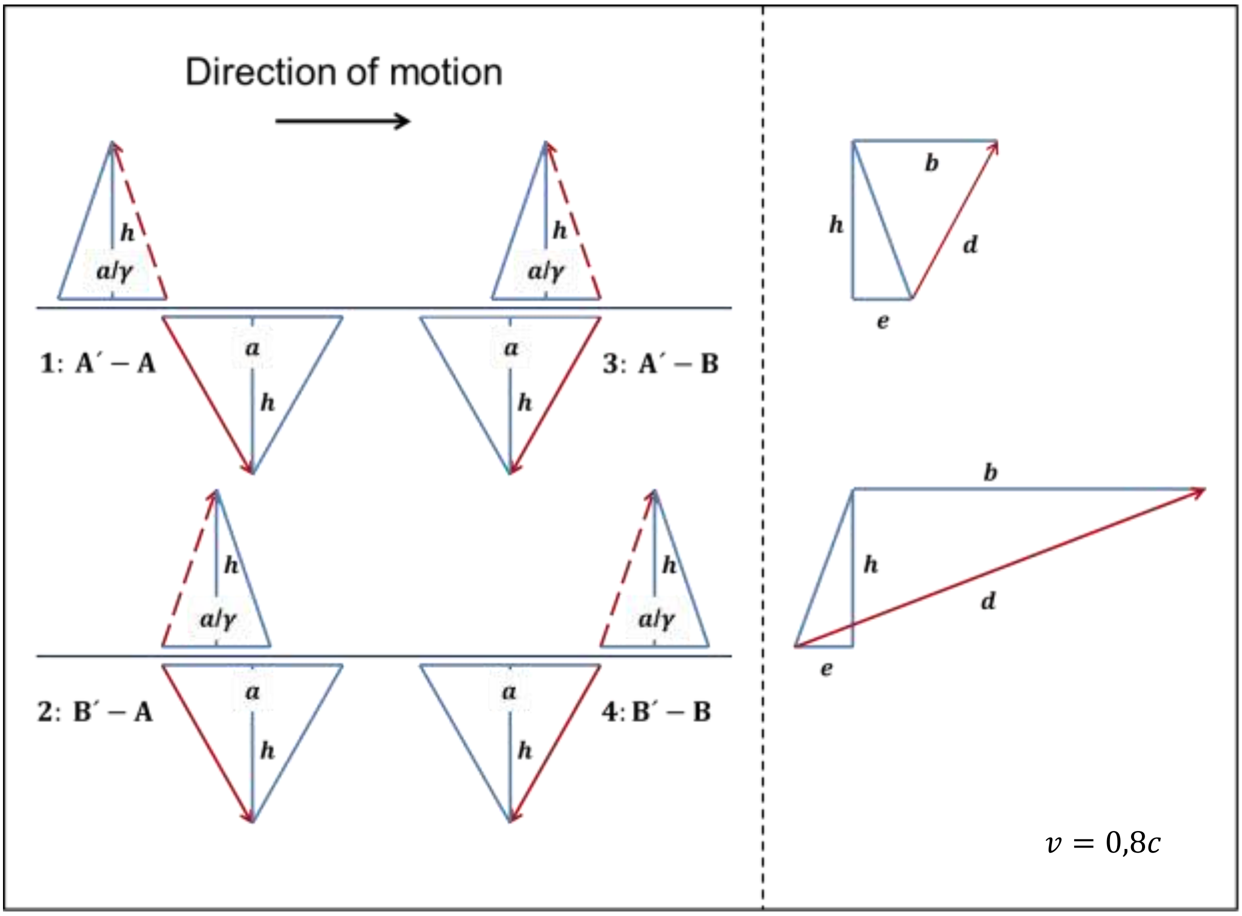



Fig. 4.6: Situation for contact and geometrical dependencies.

The geometrical dependency for distance $d$ for cases 1 and 3 is defined by the Pythagorean theorem

$$(b-e)^2 + h^2 = d^2 \tag{4.40}$$

and with the relation

$$\frac{b}{d} = \frac{v}{c} \tag{4.41}$$

This leads to

$$\left(d\frac{v}{c} - \frac{a}{2\gamma}\right)^2 + \frac{3}{4}a^2 = d^2 \tag{4.42}$$

resulting in

$$d_{1/2} = -a\gamma\left(\frac{v}{2c} \pm 1\right) \tag{4.43}$$

If a signal is sent from B′ to C′ (cases 2 and 4) a slightly different approach is valid with

$$(b+e)^2 + h^2 = d^2 \tag{4.44}$$

and

$$d_{1/2} = a\gamma\left(\frac{v}{2c} \pm 1\right) \tag{4.45}$$

Only results with positive algebraic sign are permitted, so

$$\mathrm{A}' \to \mathrm{C}': \qquad \frac{d}{a} = \gamma\left(1 - \frac{v}{2c}\right) \tag{4.46}$$

$$\mathrm{B}' \to \mathrm{C}': \qquad \frac{d}{a} = \gamma\left(1 + \frac{v}{2c}\right) \tag{4.47}$$

If the value for time is standardized to 1 then

$$t_{\mathrm{A}'\to\mathrm{C}'} = \gamma\left(1 - \frac{v}{2c}\right) \tag{4.48}$$

$$t_{\mathrm{B}'\to\mathrm{C}'} = \gamma\left(1 + \frac{v}{2c}\right) \tag{4.49}$$

When the values for the returning signals are evaluated, it is instantly clear because of symmetry reasons

$$t_{C'\to B'} = t_{\mathrm{A}'\to\mathrm{C}'} = \gamma\left(1 - \frac{v}{2c}\right) \tag{4.50}$$

$$t_{\mathrm{C}'\to A'} = t_{\mathrm{B}'\to\mathrm{C}'} = \gamma\left(1 + \frac{v}{2c}\right) \tag{4.51}$$

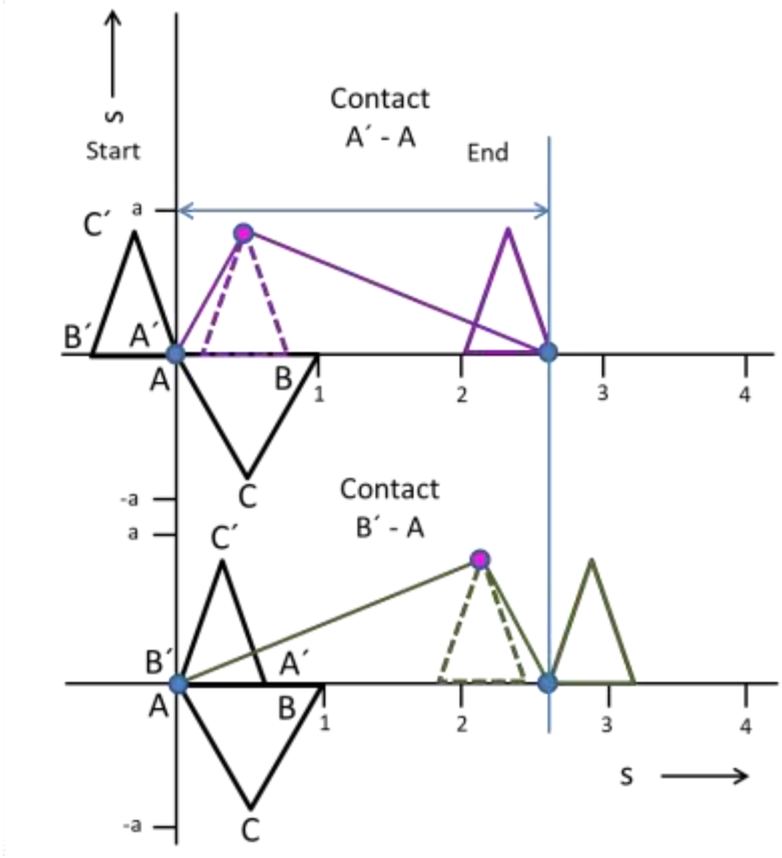
For a full calculation, the elapsing time between the contacts must be determined. When the time for contact A − A′ (case 1) is set to zero, then the following periods can be calculated using

$$\text{case1}\to\text{case2:} \qquad t_{1\to2} = \frac{c}{\gamma v} \tag{4.52}$$

$$\text{case1}\to\text{case3:} \qquad t_{1\to3} = \frac{c}{v} \tag{4.53}$$

$$\text{case1}\to\text{case4:} \qquad t_{1\to4} = \frac{c}{\gamma v} + \frac{c}{v} \tag{4.54}$$

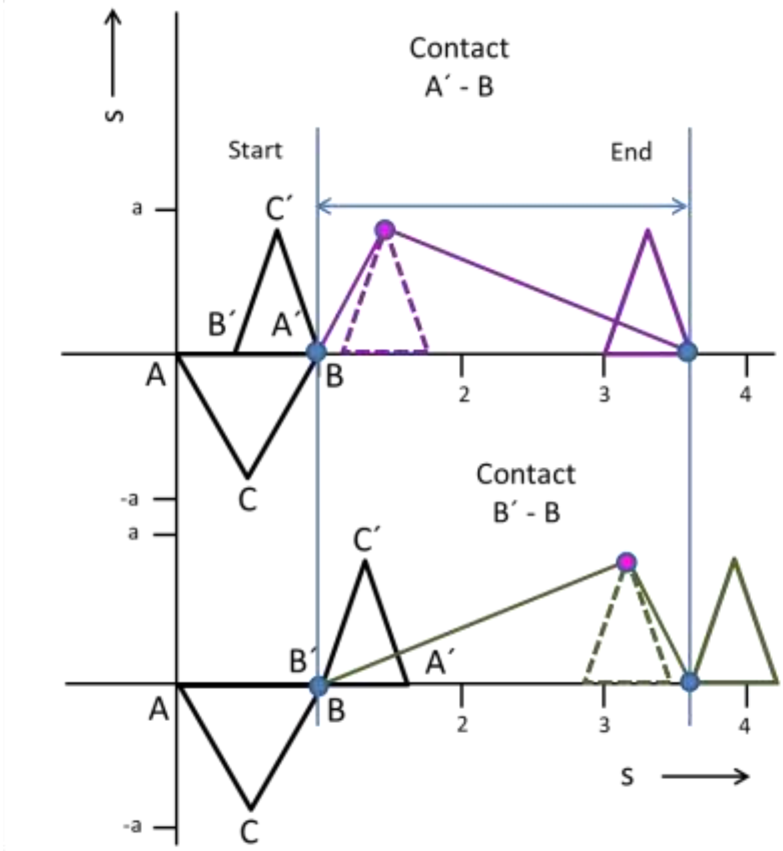
With a suitable combination of these equations, it is possible to discuss the results of all situations of the experiment.




$v = 0,8c$

| A´ – A | System | System´ |
|---|---|---|
| $t_0$ | 0 | 0 |
| $t_1$ | 1 | 1 |
| $t_2$ | 2 | 3,33 |

| B´ – A | System | System´ |
|---|---|---|
| $t_0$ | 0,75 | 0,75 |
| $t_1$ | 1,75 | 3,08 |
| $t_2$ | 2,75 | 4,08 |




| A´ – B | System | System´ |
|---|---|---|
| $t_0$ | 1,25 | 1,25 |
| $t_1$ | 2,25 | 2,25 |
| $t_2$ | 3,25 | 4,58 |

| B´ – B | System | System´ |
|---|---|---|
| $t_0$ | 2 | 2 |
| $t_1$ | 3 | 4,33 |
| $t_2$ | 4 | 5,33 |

Fig. 4.7: Sequence of signals for the 4 possible contacts in the system.

In Fig. 4.7 the diagram for the experiment with a velocity of $v = 0{,}8\ c$ is presented. This high speed was chosen to provide an acceptable visual effect in the diagram, but this does not mean, however, that there are any restrictions in the universality of this relation.

For the 4 different contact situations the values for the total travelling time of signals sent from A′ resp. B′ to C′ and after reflection to their emitting points are added in the diagram. Furthermore, the equivalent measurements for the system presented by A, B, C are presented. To keep the evaluation simple the travelling time of a signal is standardized in a way that the distance $a$ is set to 1. To make sure that the measurements can be compared with each other, the travelling times are adjoined by the times which elapsed since the sending of the first signal according to relations Eq. (4.52) to (4.54). The contact of A′ and A is representing the initial zero-value followed by B′/A, then A′/B and at last by B′ and B with $t = 2$.

According to the Theory of Special Relativity the "principle of identity" and after using the Lorentz Transformation the "principle of equivalence" must be valid. First it can be stated that the time for travelling the distance A→ C→ A and B→ C→ B is taking the total time $t = 2$, whereas for the distances A′→ C′→ A′ and B′→ C′→ B′ the time $t = 2.333 = 2\gamma$ is needed. This is exactly according to the anticipation valid for the situation of a moving observer.

When the time periods are considered, which are measured by C and C′ between the signals, then the same effect can be monitored, which was already discussed in chapter 2.2.2. This means, that the values of C and C′ for the contacts of A/B′ and B′/A are changing. It is obvious, that the principle of relativity requires, that C resp. C′ must receive the signal of the observer in their system A resp. A′ first. This is important to realize a proper sequence of contacts.

Generally, it was shown that all combinations sending signals in any arbitrary spatial direction are respecting the principle of relativity.

## 5.1 Signal exchange during rotation (Sagnac-effect)

In contrast to translational movements, there are measurable effects between outgoing and returning light beams in rotating systems. This does not contradict the principle of relativity, as by definition these are not inertial systems. The first successful experiments on this were carried out by Georges Sagnac (1860-1926) in 1913 [100].

The schematic experimental setup is shown in Fig. 4.8. Part a) shows that monochromatic light is emitted from a light source, which is partially reflected by a semi-transparent mirror and split into 2 opposing directions. After complete circulation and recombining, an interferometer is used to detect small transit time differences between the light beams. The apparatus is first calibrated at rest and then measurements are taken while the system is rotating. All elements of the experimental setup, i.e. light source, mirrors, and detector are also rotated. As shown in Fig. 4.8 b), the light beams emitted in the direction of rotation travel a longer distance than those moving in the opposite direction and this difference can be measured.

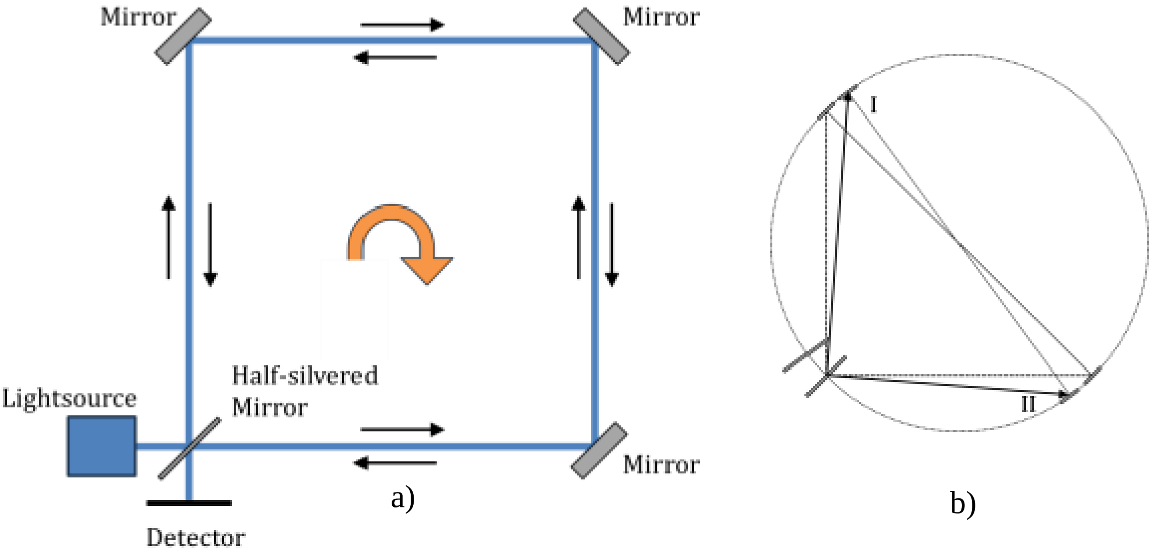



Fig. 4.8: Setup of a Sagnac interferometer. a) Rotatable test arrangement
b) Changing the measuring length of the first segment by rotation
Type I (in direction of rotation): Lengthening; type II (counter-rotating): Shortening

The designations shown in Fig. 4.9 can be used to calculate the values. The following relationship applies to the length of the arc segment s from A to B

$$s = r \cdot \omega \cdot (t_0 + \Delta t_0) \tag{4.60}$$

where r is the radius and ω is the angular frequency. In addition, $t_0$ is the time required by the light beam in the stationary system between 2 mirrors and $\Delta t_0$ is the additional time required for a rotational movement. The following also applies in general

$$a = ct_0 \qquad e = c\Delta t_0 \qquad b = a + e \qquad a = r\sqrt{2}$$

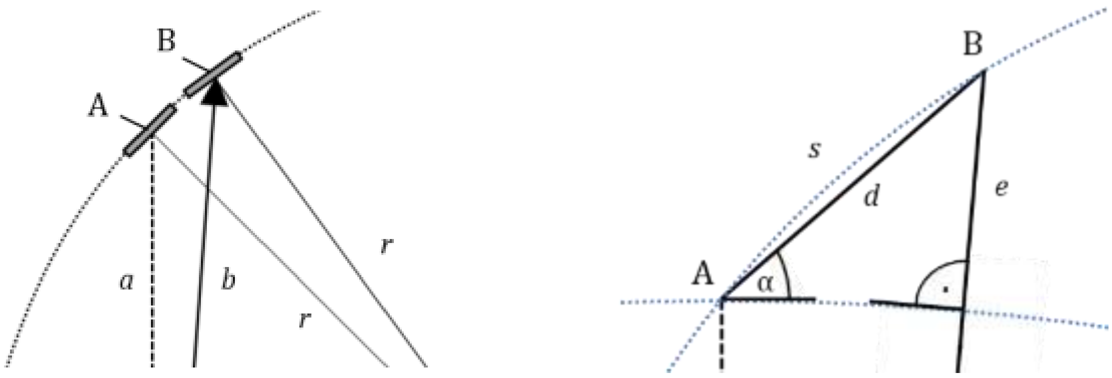



Fig. 4.9: Formula symbols used for the calculations

If $\Delta t \ll t_0$ is assumed, the following relationships apply as a good approximation

$$s = d = r \cdot \omega \cdot t_0 \tag{4.61}$$

$$\sin\alpha = \sin(45°) = \frac{1}{\sqrt{2}} = \frac{e}{d} = \frac{c \cdot \Delta t_0}{r \cdot \omega \cdot t_0} \tag{4.62}$$

and thus

$$\Delta t_0 = \frac{r\omega t_0}{\sqrt{2} \cdot c} = \frac{a^2\omega}{2c^2} \tag{4.63}$$

There are 4 segments, so the time delay for one cycle is

$$\Delta t_+ = 2\frac{a^2\omega}{c^2} \tag{4.64}$$

The shortening of the time for the light beam on the opposite path has the same value, so the final result is

$$\Delta t_t = \Delta t_+ + \Delta t_- = 4\frac{a^2\omega}{c^2} \tag{4.65}$$

With a length a of 1m and 10 revolutions per second, this results in $\Delta t_t = 4{,}4 \cdot 10^{-16}$s corresponding to a wavelength in visible light that allows interference measurements.

G. Sagnac was convinced that he had measured an ether effect with his (similarly constructed) apparatus; however, Max v. Laue had already demonstrated in 1911 that such an experiment was compatible with the principle of relativity [101].

In 1925, A. A. Michelson and H. G. Gale carried out an experiment with dimensions of 613 m in length and 339 m in width [102,103]. This made it possible to measure the rotation of the earth with a relative accuracy of 2%.

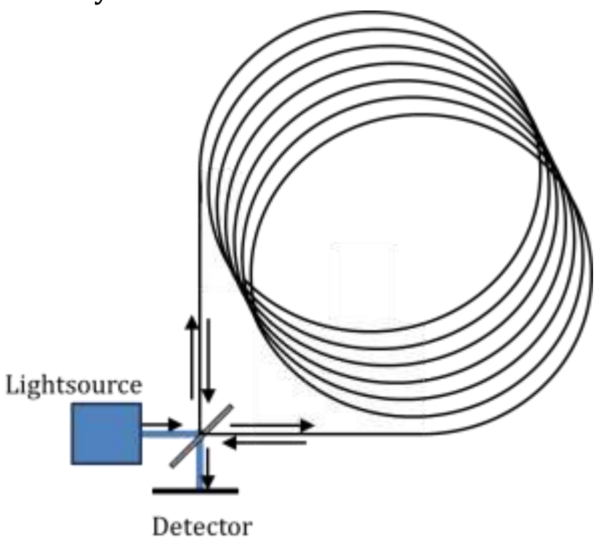



Fig. 4.10: Construction of a Sagnac interferometer with an optical fiber

In addition to the structure with beam reflection by mirrors, coiled fiber optic cables can also be used as shown in Fig. 4.10. These are widely used today in areas such as aerospace, navigation, ships, and robotics. They are less susceptible to mechanical wear than mechanical gyrocompasses as they contain no moving parts. Another trend in their development is the miniaturization of optical gyroscopes. With the advent of micro-electro-mechanical systems (MEMS), it has become possible to produce smaller and more cost-efficient gyroscopes that can be used for a variety of applications, from smartphones to drones.

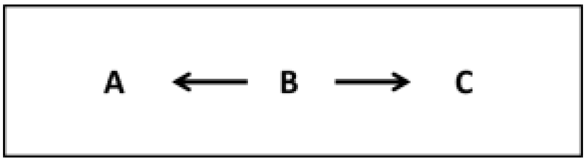
# 5. Clock transport

It is well known, that according to Special Relativity during an exchange of signals between two observers only a mutual consideration of the time needed in both directions is possible. Nevertheless, in the past effort was made to measure the one-way speed of light inside a system in motion. One of these attempts to perform a separate measurement was the examination of the effect that occurs, when clocks are moved at slow speed inside a moving observer. In this case a system in motion is defined, where two clocks after an Einstein Synchronization are lined up and one is following the other. To execute the experiment the clocks are moved in this system in a way, that after the end of the trial they have changed their positions. When the experiment is carried out at low speed the synchronization should maintain its original values and after a further synchronization process a difference should appear.

Since some time it is clear, however, that the effects measured by both clocks is changing exactly corresponding to their position inside the system and therefore leading to a null result (see i.e. [19,40]). This important verification and the necessary calculations are presented here, first simply by means of an example and afterwards in a general way. Further in this chapter the well-known twin paradox will be discussed, and it will appear as a special case of the clock transport.

## 5.1 Clock transport in direction of motion

To define an appropriate experimental set-up it is assumed, that in a laboratory 3 observers A, B and C are lined up equidistant.




First the case is considered that the observers are at rest. To start the experiment observer B is sending out synchronized clocks with the same speed to A and C. After the arriving of the clocks at A and C it is found that these - depending on the speed they were moved - are running slow compared to the clocks at rest because of time dilatation. Further A and C after exchanging of experiment data conclude that the moved clocks arrived at the same time at their positions.

It is now considered that the laboratory is accelerated and afterwards moving with a constant speed. The existing clocks shall then be synchronized. If an effect that could be measured inside the system would occur, it must be possible to find it out in one (or both) of the ways presented in the following:

1. Observers A and C find differences in the arriving time of the clocks sent out by observer B in comparison to the results of the experiments in a system at rest.
2. The moved clocks show differences when they arrive at A and C compared to the situation of a system at rest.

It shall be presented in the following, that inside a system at rest compared to a system in motion the same results will be achieved. This simplified statement can be extended to the proposition that it is valid also for any arbitrary inertial system, which means it is a system not accelerated and without rotation. The statement is therefore valid universally.

### 5.1.1 Qualitative Considerations

Fig. 5.1 shows the situation, that in a laboratory at rest (left) and in motion (right) at the time zero a light signal is emitted from position B in direction to the back end (A) and the front end (C). These signals are reaching A and C at the positions $c_1$ and $a_1$ as shown in the diagram. In this presentation further the situation with moving clocks starting from point B is added.

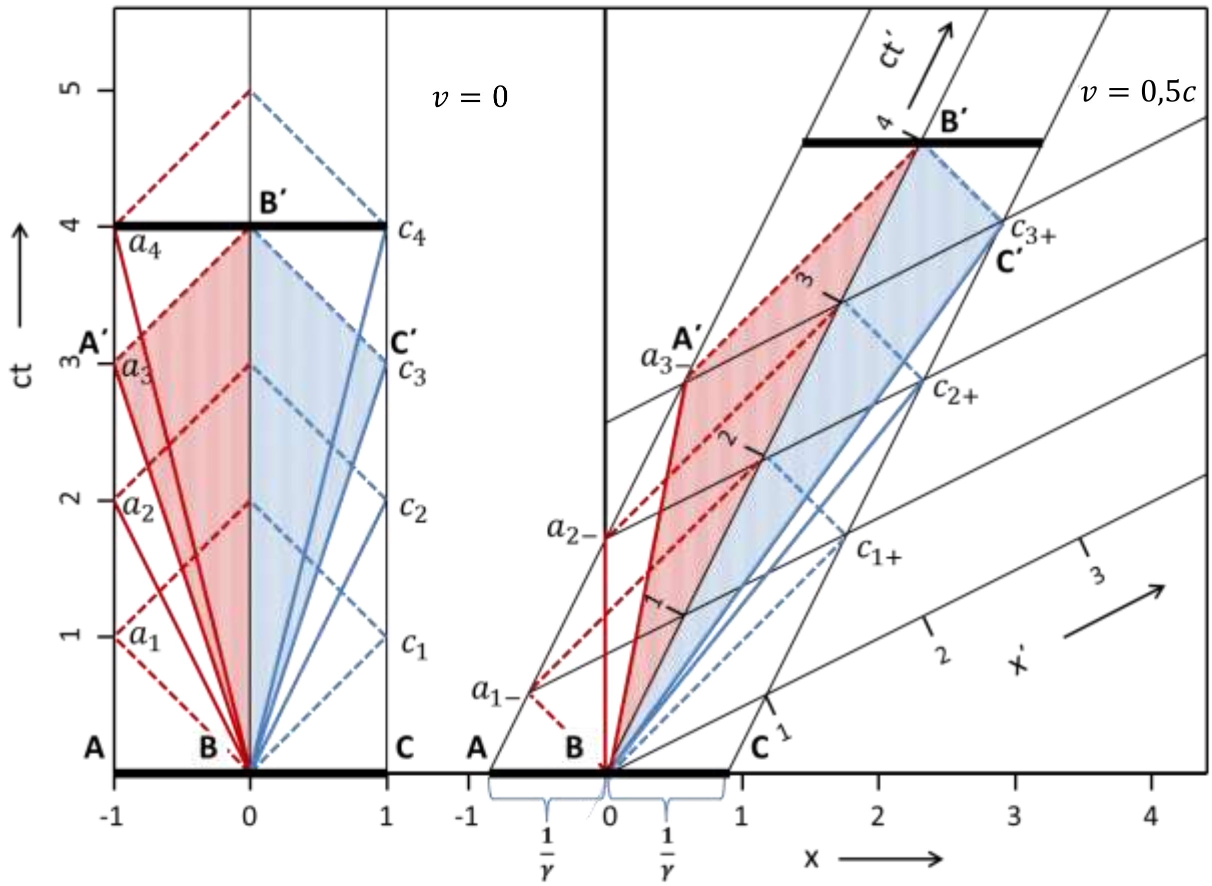



Fig. 5.1: Space-time-diagram for clock transport
Dotted lines: Signal exchange

First the laboratory at rest shall be looked at (left-hand side of the diagram). When the clocks are starting at time zero with a velocity of $1/2c$ they are reaching the positions $c_2$ as well as $a_2$ after 2 time-units, when the speed is $1/4c$ then 4 time-units are necessary for the positions $c_4$ and $a_4$ etc. All possible times for receiving the signals can be realized depending on the velocities of the moved clocks.

When a moving system is considered, however, for an observer at rest some differences in the situation would occur (right-hand side of the diagram), i.e. differences in the times to reach $c_n$ and $a_n$, further the distance 1 is changing to $1/\gamma$ etc. These changes are described by the Lorentz-Transformation.

In the following the situation for an observer in motion shall be discussed. This is presented in Fig. 5.1 by means of marked zones (blue: in moving direction, red: opposite direction). The following relation applies for the system at rest

$$v = \left(0{,}5\ \pm \frac{1}{3}\right) \cdot c \tag{5.01}$$

and for the observers in motion

$$v_{c3+} = \frac{0{,}5 + 0{,}\overline{3}}{1 + 0{,}5 \cdot 0{,}\overline{3}} c\ =\ 0{,}714c \tag{5.02}$$

$$v_{a3-} = \frac{0{,}5 - 0{,}\overline{3}}{1 - 0{,}5 \cdot 0{,}\overline{3}} c\ =\ 0{,}2c \tag{5.03}$$

To simplify the calculations the following definitions shall be introduced: The values for time, space and speed of light $c$ are scaled to 1, the results of the velocities are therefore defined as fractions of $c$.

The arrival time and the factor $\gamma$ is

$$\begin{aligned} t_{c3+} &= 4{,}041 \qquad \gamma_{c3+} = 1{,}429 \\ t_{a3-} &= 2{,}887 \qquad \gamma_{a3-} = 1{,}021 \end{aligned} \tag{5.04}$$

The passed (subjective) time for the observers is

$$system\ in\ motion\text{:}\quad \frac{t_{c3+}}{\gamma_{c3+}} = \frac{t_{a3-}}{\gamma_{a3-}} = 2{,}828 \tag{5.05}$$

This result is consistent with the values of the system at rest, because

$$t_{c3} = t_{a3} = 3 \qquad \gamma_{c3} = \gamma_{a3} = 1{,}061 \tag{5.06}$$

is valid and so the same result is obtained.

$$system\ at\ rest\text{:}\quad \frac{t_{c3}}{\gamma_{c3}} = \frac{t_{a3}}{\gamma_{a3}} = 2{,}828 \tag{5.07}$$

The presented deductions show that the subjectively measured time period for the transition to A and C of the moved observers is identical. Further the presentation makes clear, that the time measured for the arrival of the simultaneously moved clocks by the observers

A and C in their synchronized system is also the same. This makes it impossible inside a uniformly proceeding system, which is moving without acceleration or rotation, to take measurements with clocks or any other devices and find conclusions out of the received results about the velocity compared to other systems or to find deviations in the synchronization.

### 5.1.2 General derivation

The presented issue will now be verified in a general form. First it is necessary to define the following parameters:

| System at rest | System in motion | |
|---|---|---|
| - | $v_0$ | Velocity of the system in motion |
| $\Delta v$ | $v_+, v_-$ | Travelling speed of the moved observers |
| - | $\Delta t_A, \Delta t_A$ | Synchronization difference to system at rest |
| $t_0$ | $t_+, t_-$ | Arrival time of moved observers |
| $t'_0$ | $t'_+, t'_-$ | Subjective travelling time of moved observers |
| $\gamma_\Delta$ | $\gamma_+, \gamma_-$ | Lorentz-factor of moved observers |

These parameters are presented in a modified Minkowski-diagram (see Fig. 5.2). The experimental set-up is the following:

From position B in the middle of a laboratory at rest, signals are sent to the positions at both ends A and C and arrive here at the time $t'$ (left side of the diagram, positions marked with A′ and C′). At the same time 2 synchronized clocks start moving from the position B with an arbitrary velocity $\Delta v$ which is the same for both. They arrive at their positions at time $t''$ (marked with A′′ and C′′); directly afterwards signals are sent back to position B. In the right part of the diagram the situation is presented for an observer in motion. The differences in moving direction and opposite to it are in conformance with the Lorentz equations.

In the following it is demonstrated that the observers taking part in this experiment are not able to detect differences in the measurements of the elapsing time. In detail these are the considerations:

1. The observers in motion cannot decide on basis of their measurements whether the system is moving or not.
2. The observers at rest find during their measurements - independent of the velocity of the moving system - the same time periods for the arriving of the moving observers.

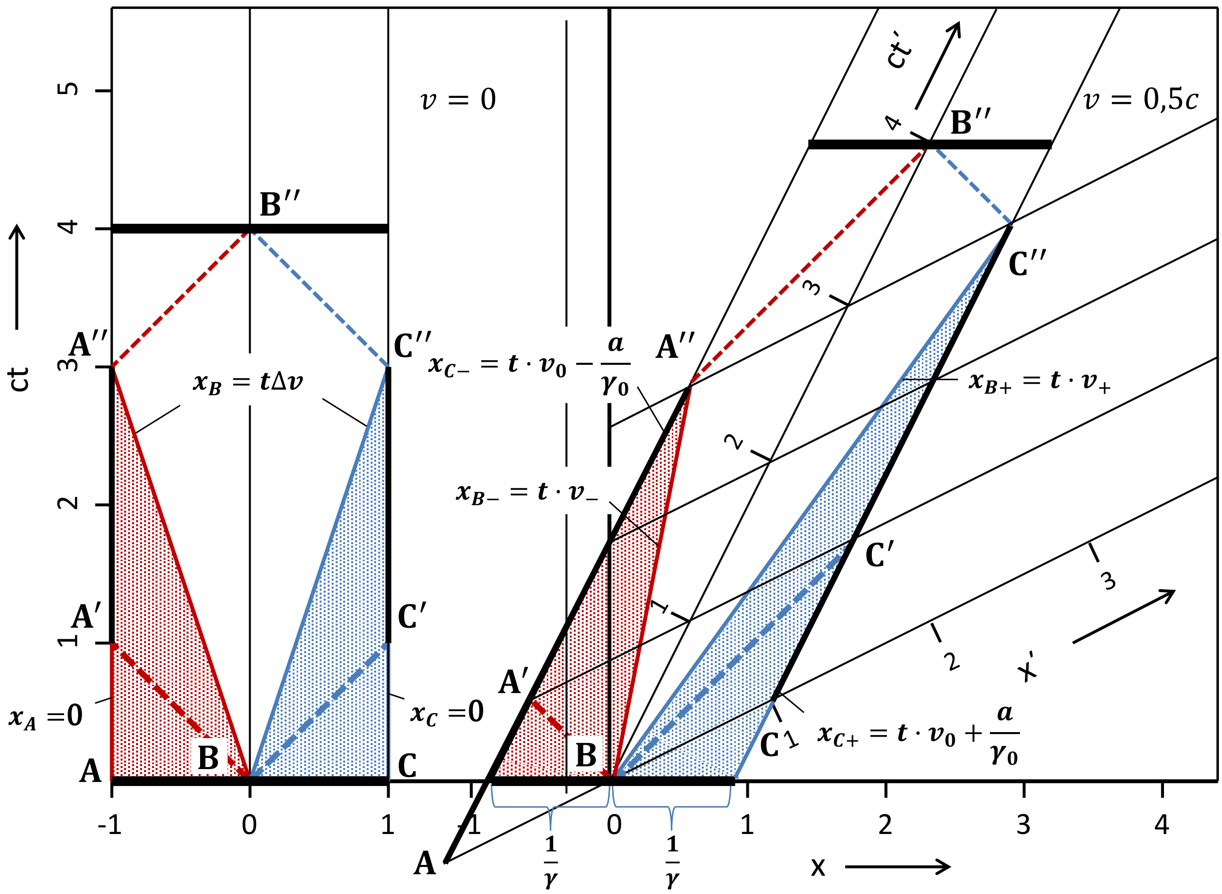



Fig. 5.2: Space-time-diagram for clock transport with defined parameters
Dotted lines: Signal exchange

The different issues are now dealt with separately.

### 5.1.3 Identical time schedules for the arriving of moved observers

The following issues shall be reviewed:

a) The synchronization differences in a moving system $\Delta t_A$ and $\Delta t_C$ for the observers A and C relating to B

b) The time periods $t_-$ and $t_+$ the observers in motion need to reach the positions A and C

c) The difference between both values. When the result (multiplied by $\gamma_0$) is corresponding to the values of the system at rest, then the measuring results are not distinguishable from each other.

*a) Synchronization differences*

To determine the synchronization differences, it is first necessary to identify the travelling time a light signal needs starting from B to the positions A´ resp. C´. This is

$$\Delta t_{B\to A'} = \frac{a}{c\gamma_0(1+\frac{v_0}{c})} \tag{5.08}$$

$$\Delta t_{B\to C'} = \frac{a}{c\gamma_0(1-\frac{v_0}{c})} \tag{5.09}$$

The value which is necessary to reach the starting point is subtracted

$$\Delta t_{A'\to A} = \Delta t_{C'\to C} = \frac{a}{c}\gamma_0 \tag{5.10}$$

Thus, the synchronization leads to values

$$\Delta t_A = \frac{a}{c\gamma_0(1+\frac{v_0}{c})} - \frac{a}{c}\gamma_0 = -\frac{\gamma_0 a v}{c^2} \tag{5.11}$$

and

$$\Delta t_C = \frac{a}{c\gamma_0(1-\frac{v_0}{c})} - \frac{a}{c}\gamma_0 = \frac{\gamma_0 a v}{c^2} \tag{5.12}$$

*b) Time for observers in motion*

The time a moved observer needs to reach the positions A′ resp. C′ in a system at rest is

$$t_0 = \frac{a}{\Delta v} \tag{5.13}$$

To determine this in a system in motion the values of $x_{B+}$ and $x_{C+}$ (with $t \to t_+$) resp. $x_{B-}$ and $x_{C-}$ (with $t \to t_-$) are set equal and this results in (see Fig. 5.2)

$$t_+ = \frac{a}{\gamma_0(v_+ - v_0)} \tag{5.14}$$

$$t_- = \frac{a}{\gamma_0(v_0 - v_-)} \tag{5.15}$$

*c) Consideration of differences*

In the following the differences between $\Delta t_A$ and $t_-$ resp. $\Delta t_C$ and $t_+$ are considered. In a system at rest this is

$$\Delta t_{A\to A''} = \Delta t_{C\to C''} = \frac{a}{\Delta v} \tag{5.16}$$

In a system in motion this changes to

$$\Delta t_{A\to A''} = \Delta t_{C\to C''} = \gamma_0 \frac{a}{\Delta v} \tag{5.17}$$

If

$$t_- = \Delta t_A + \Delta t_{A\to A''} \tag{5.18}$$

with

$$\frac{a}{\gamma_0(v_0 - v_-)} = \frac{a}{c\gamma_0(1+\frac{v_0}{c})} - \frac{a}{c}\gamma_0 + \gamma_0\frac{a}{\Delta v} \tag{5.19}$$

$$t_+ = \Delta t_C + \Delta t_{C\to C''} \tag{5.20}$$

$$\frac{a}{\gamma_0(v_+ - v_0)} = \frac{a}{c\gamma_0(1-\frac{v_0}{c})} - \frac{a}{c}\gamma_0 + \gamma_0\frac{a}{\Delta v} \tag{5.21}$$

is valid, no differences can be detected inside a system.

To simplify the calculation the equations shall be multiplied with $c/a$ and the values of the velocities are replaced by their quotient to the speed of light $c$

$$v'_+ = \frac{v_+}{c} \quad v'_- = \frac{v_-}{c} \quad v'_0 = \frac{v_0}{c} \quad \Delta v' = \frac{\Delta v}{c} \tag{5.22}$$

Eq. 5.19 is developing to

$$\frac{1}{\gamma_0(v'_0 - v'_-)} = \frac{1}{\gamma_0(1 + v'_0)} - \gamma_0 + \frac{\gamma_0}{\Delta v'} \tag{5.23}$$

and Eq. 5.21 changes to

$$\frac{1}{\gamma_0(v'_+ - v'_0)} = \frac{1}{\gamma_0(1 - v'_0)} - \gamma_0 + \frac{\gamma_0}{\Delta v'} \tag{5.24}$$

Inserting the values

$${\gamma_0}^2 = \frac{1}{1 - {v'_0}^2} \tag{5.25}$$

then after simple transformation of Eq. 5.23

$$(1 + v'_-)(1 - v'_0) = -v'_0 + \frac{v'_0}{\Delta v'} + v'_- - \frac{v'_-}{\Delta v'} \tag{5.26}$$

can be derived. Further

$$v'_- = \frac{v'_0 - \Delta v'}{1 - v'_0 \cdot \Delta v'} \tag{5.27}$$

and from Eq. 5.24

$$(1 - v'_+)(1 + v'_0) = -v'_+ + \frac{v'_+}{\Delta v'} + v'_0 - \frac{v'_0}{\Delta v'} \tag{5.28}$$

$$v'_+ = \frac{v'_0 + \Delta v'}{1 + v'_0 \cdot \Delta v'} \tag{5.29}$$

is valid. These results correspond exactly to the definitions of $v'_-$. and $v'_+$. It is thus shown that inside a system the observers A and C are not able to find differences in the arriving time of a moved observer. The subjective time periods are completely independent whether the system is moving or not.

### 5.1.4 Identical time periods at arrival for moving observers

The time period a moving observer needs to reach the positions A or C in a system at rest is

$$t_0 = \frac{a}{\Delta v} \tag{5.30}$$

and in the moving system

$$t_+ = \frac{a}{\gamma_0(v_+ - v_0)} \tag{5.31}$$

$$t_- = \frac{a}{\gamma_0(v_0 - v_-)} \tag{5.32}$$

The time subjectively measured by the moving observer is here

$$t'_0 = \frac{a}{\gamma_\Delta \Delta v} \tag{5.33}$$

$$t'_+ = \frac{a}{\gamma_+ \gamma_0 (v_+ - v_0)} \tag{5.34}$$

$$t'_- = \frac{a}{\gamma_- \gamma_0 (v_0 - v_-)} \tag{5.35}$$

If the subjectively measured time is identical then the relation applies

$$t'_0 = t'_+ = t'_- \tag{5.36}$$

First this is discussed for the case $t'_0 = t'_+$. Thus

$$\frac{a}{\gamma_+ \gamma_0 (v_+ - v_0)} = \frac{a}{\gamma_\Delta \Delta v} \tag{5.37}$$

must be valid. This leads to

$$\frac{\gamma_\Delta}{\gamma_+ \gamma_0} = \frac{(v_+ - v_0)}{\Delta v} \tag{5.38}$$

To simplify the calculation again the values of the velocities are replaced by their quotient to the speed of light $c$

$$v'_+ = \frac{v_+}{c} \quad v'_- = \frac{v_-}{c} \quad v'_0 = \frac{v_0}{c} \quad \Delta v' = \frac{\Delta v}{c} \tag{5.39}$$

When in equation (5.38) the values of $\gamma$ are inserted, then

$$\frac{(1 - v_+'^2)(1 - v_0'^2)}{1 - \Delta v'^2} = \frac{(v'_+ - v'_0)^2}{\Delta v'^2} \tag{5.40}$$

and

$$(1 - v'_+ v'_0)^2 \Delta v'^2 = (v'_+ - v'_0)^2 \tag{5.41}$$

When $v_+$ is replaced by

$$v'_+ = \frac{v'_0 + \Delta v'}{1 + v'_0 \cdot \Delta v'} \tag{5.42}$$

then

$$\left(1 - \frac{v'_0 + \Delta v'}{1 + v'_0 \cdot \Delta v'} v'_0\right)^2 \Delta v'^2 = \left(\frac{v'_0 + \Delta v'}{1 + v'_0 \cdot \Delta v'} - v'_0\right)^2 \tag{5.43}$$

If this equation is expanded completely, then 20 terms will occur which will add up to zero. The same procedure can be applied to $t'_0 = t'_-$. With

$$\frac{\gamma_\Delta}{\gamma_- \gamma_0} = \frac{(v'_0 - v'_-)}{\Delta v'} \tag{5.44}$$

and

$$v_- = \frac{v'_0 - \Delta v'}{1 - v'_0 \cdot \Delta v'} \tag{5.45}$$

the same result will be realized. Thus, it is shown that the subjective measurements of the moving observers do not differ from the results achieved at rest.

It is now generally verified that inside a moving system no possibility exists to find deviations caused by “slow clock transport” when using synchronized clocks in comparison to a reference system at rest.

## 5.2 Twin paradox

One of the best-known examples connected with the theory of Special Relativity is the twin paradox. This issue covers a long history in literature (see i.e. a comprehensive summary in [41]). In general, a pair of twins is looked at, where one is at rest (remaining at earth) while the other is leaving with a fast spaceship and comes back later. This twin will be aged less compared to the one who remained on earth. The paradox occurs because according to Special Relativity both twins should be considered as equal and therefore the travelling twin after his return should find the remaining twin also in a condition aged less.

The solution to overcome the contradictions is possible because the twin in the spaceship is changing the inertial system during his trip.

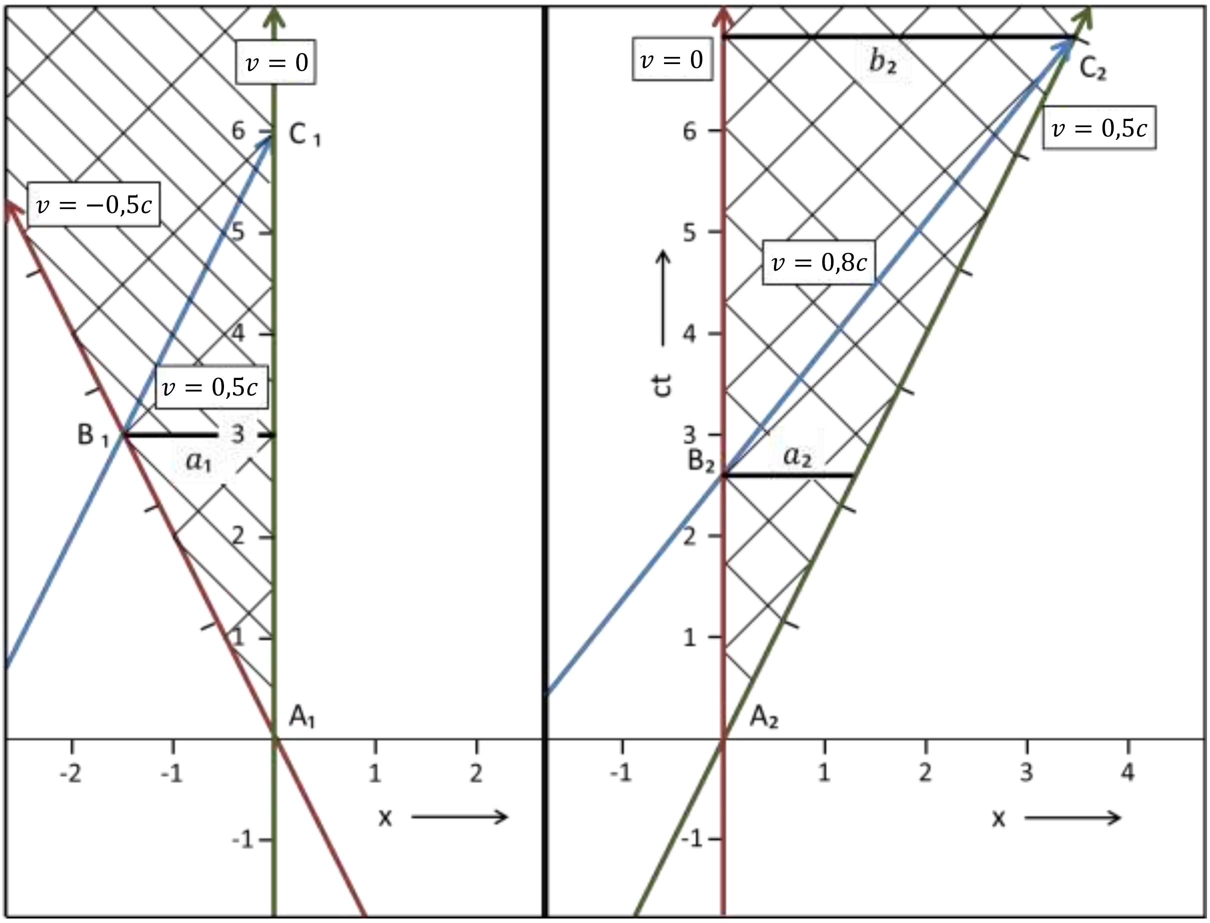



Fig. 5.3: Presentation of the twin paradox
Left: Observer A at rest, B in motion
Right : Observer A in motion, B at rest (at the beginning)

In Fig. 5.3 this case is presented on the left side of the diagram. On the right side the situation is presented, that the observers change their perspective and the one who was first considered as at rest is moving and vice versa. To avoid influences during changing of the direction, the experimental set-up is modified in a way that 3 observers take part (in Fig. 5.3 marked with the colors green, red, and blue) and each of the observers is in possession of a precise clock [41]. At the positions $A_1$ and $B_1$ resp. $A_2$ and $B_2$ the clocks are synchronized and at the end of the trial the results are evaluated. In this presentation the problem finally has the same status as the issue of a slow clock transport.

If the situations are comparable, then the subjective measuring results must be the same for all observers taking part in the trial. This shall be demonstrated in the following. The important issues are the total travelling time from the start to the end of the journey, and the subjective time periods for the moving observers, which must be identical from the start to the returning point and from that to the end. The total time for the observer at rest is defined as $t_0$ as shown in the left part of the diagram. The other parameters are presented in the following table.

| System at rest | System in motion | |
|---|---|---|
| $t_T$ | $t'_T$ | Total time from start (A) to the end of journey (C) |
| $t_1$ | $t'_1$ | Time for the first part of the journey (A→ B) |
| $t_2$ | $t'_2$ | Time for the second part of the journey (B→ C) |
| - | $v'_1$ | Velocity for $A_1 \rightarrow B_1$, $B_1 \rightarrow C_1$, $A_2 \rightarrow C_2$ |
| - | $v'_2$ | Velocity for $B_2 \rightarrow C_2$ |
| - | $\gamma_1$ | Lorentz factor for $v'_1$ |
| - | $\gamma_2$ | Lorentz factor for $v'_2$ |

Remark:

The velocities are always taken as ratio to the speed of light, i.e.

$$v'_1 = \frac{v_1}{c} \qquad v'_2 = \frac{v_2}{c} \tag{5.50}$$

*a) Total time*

Left: The total time $t_T$ is defined as

$$t_T = t_0 \tag{5.51}$$

and for $t'_T$ is valid

$$t'_T = t_1 + t_2 = \frac{t_0}{\gamma_1} \tag{5.52}$$

where in this case because of symmetry reasons applies

$$t'_1 = t'_2 = \frac{t'_T}{2} = \frac{t_0}{2\gamma_1} \tag{5.53}$$

Right: Because the subjective time periods $t_1$ shall be the same in both cases it must be valid

$$t_1 = \frac{t_0}{2\gamma_1} \tag{5.54}$$

The time $t_2$ can be derived using relations concerning $b_2$ (see Fig. 5.3, right), because for $v'_1$ and $v'_2$ applies

$$v'_1(t_1 + t_2) = v'_2 t_2 \tag{5.55}$$

$$t_2 = \frac{v'_1 t_1}{v'_2 - v'_1} \tag{5.56}$$

Further for $v'_2$ because of the same velocities during the round trip for the relativistic addition of velocities according to Eq. (4.21) applies

$$v'_2 = \frac{2v'_1}{1 + {v'_1}^2} \tag{5.57}$$

This leads to

$$t_T = t_1 + t_2 = \frac{t_0}{2\gamma_1} + \frac{v'_1 t_0}{2\gamma_1(v'_2 - v'_1)} = \frac{t_0}{2\gamma_1}\left(1 + \frac{v'_1}{v'_2 - v'_1}\right) \tag{5.58}$$

After insertion of Eq. (5.57) in Eq. (5.58) follows with

$$\gamma_1 = \sqrt{\frac{1}{1 - {v'_1}^2}} \tag{5.59}$$

$$t_T = \frac{t_0}{2}\gamma_1\left(1 - {v'_1}^2\right)\left(1 + \frac{v'_1}{\frac{2v'_1}{1 + {v'_1}^2} - v'_1}\right) \tag{5.60}$$

$$t_T = \frac{t_0}{2}\gamma_1\left(1 - {v'_1}^2\right)\left(1 + \frac{v'_1\left(1 + {v'_1}^2\right)}{v'_1 - {v'_1}^3}\right) \tag{5.61}$$

$$t_T = \gamma_1 t_0 \tag{5.62}$$

Because of

$$t'_T = \frac{t_T}{\gamma_1} \tag{5.63}$$

it applies

$$t'_T = t_0 \tag{5.64}$$

The measurements of subjective times are thus the same.

*b) Single times*

First it is necessary to calculate time $t_2$, which is subjectively elapsing for the observer in motion between $B_2$ and $C_2$.

According to Eq. (5.56) and (5.54) for the observer at rest applies

$$t_2 = \frac{v_1' t_0}{2\gamma_1(v_2' - v_1')} \tag{5.65}$$

This leads to

$$t'_2 = \frac{v_1' t_0}{2\gamma_2\gamma_1(v_2' - v_1')} \tag{5.66}$$

When the subjective time periods for the left- and right-hand side of the diagram shall be the same then

$$\frac{t_0}{2\gamma_1} = \frac{v_1' t_0}{2\gamma_2\gamma_1(v_2' - v_1')} \tag{5.67}$$

This can be derived easily. First

$$\gamma_2 = \frac{v_1'}{v_2' - v_1'} \tag{5.68}$$

and using Eq. (5.57)

$$\gamma_2 = \frac{1 + {v_1'}^2}{1 - {v_1'}^2} \tag{5.69}$$

applies. Because of

$$\gamma_2 = \sqrt{\frac{1}{1 - {v_2'}^2}} \tag{5.70}$$

it applies

$$\frac{1 - {v_1'}^2}{1 + {v_1'}^2} = \sqrt{1 - \frac{4{v_1'}^2}{\left(1 + {v_1'}^2\right)^2}} \tag{5.71}$$

$$1 - {v_1'}^2 = \sqrt{\left(1 + {v_1'}^2\right)^2 - 4{v_1'}^2} \tag{5.72}$$

$$1 - {v_1'}^2 = \sqrt{1 - 2{v_1'}^2 + {v_1'}^4} \tag{5.73}$$

which is obviously the same. It is thus shown that the subjective measured times for the total distance and for the single parts of the trip are identical. The “paradox” is therefore not showing discrepancies.

## 5.3 Clock transport in arbitrary directions

When the clock transport in arbitrary spatial directions is considered the relation Eq. (4.20) must be used for relativistic addition of velocities.

$$v_T = \frac{\sqrt{v_1^2 + v_2^2 + 2v_1v_2cos\alpha - \left(\frac{v_1v_2sin\alpha}{c}\right)^2}}{1 + \frac{v_1v_2cos\alpha}{c^2}} \tag{4.20}$$

A simple example with $\alpha = 90°$ shows

$$v'_T = \sqrt{v_1'^2 + v_2'^2 - v_1'^2v_2'^2} \tag{5.80}$$

This equation can be interpreted as a variant of the relation presented in Fig. 5.3 with the difference that all observers are moving with an additional speed of $v_2$. In this case the time dilatation during the trip from $A_1 \longrightarrow B_1$ is increasing in view of an observer at rest from $\gamma_1$ to $\gamma_1 \cdot \gamma_2$. This means that the following relation

$$\gamma_T = \gamma'_1\gamma'_2 \tag{5.81}$$

must apply. This yield

$$\gamma_T = \frac{1}{\sqrt{1 - v_{ges}'^2}} \tag{5.82}$$

$$= \frac{1}{\sqrt{1 - (v_1'^2 + v_2'^2 - v_1'^2v_2'^2)}} = \frac{1}{\sqrt{(1 - v_1'^2)(1 - v_2'^2)}} \tag{5.83}$$

which is obviously identical with Eq. (5.81). So, it is verified for this case also, that a linear combination of different motions will not lead to a possibility to measure differences of the elapsing time.

Summarizing the calculations, it was verified here, that no possibility exists to carry out measurements inside a system moving with constant speed and decide about its state of motion. All the discussed variants of the exchange of signals and the “slow clock transport” lead to a null result. Of course, this cannot be a surprise, because according to Special Relativity this is predicted.

# 6. Relations for mass, momentum, force, and energy

In this chapter results connected with the relativistic mass increase will be presented. First the well-known effect on the kinetic energy will be discussed, followed by some new investigations. These are the "spring paradox", the relativistic consideration of the elastic collision (important for the examination of collisions of elementary particles), the exchange of signals during and after acceleration and the concept of a relativistic rocket equation. Because some of the delineations show no approach to an analytical solution, numerical evaluation concepts combined with examples for calculations are added in separate files for these cases.

None of the examinations show any contradictions to the Lorentz Transformation or the basic principles of relativity.

## 6.1 Relativistic mass increase and energy

During the historical development of the investigations concerning relativistic mass, it was first realized that there are differences between a "longitudinal" and "transversal" mass increase for high velocities. These terms were introduced by H. A. Lorentz [13,42], because during the acceleration of electrons differences were measured depending on their movement. According to experiments the transversal mass $m_t$ and the longitudinal mass $m_l$ showed the following values:

$$m_t = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{6.01}$$

$$m_l = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \tag{6.02}$$

During these experiments the mass was measured in a way, that the acting force was divided by the acceleration using Newton´s law

$$m = \frac{F}{a} \tag{6.03}$$

The transverse acceleration is leading to a constant circular motion, while a longitudinal acceleration is increasing the velocity of the object and therefore both the longitudinal and transverse mass of the body is raised.

According today´s standard of knowledge the equation (6.01) is presenting the correct increase of mass during acceleration, whereas Eq. (6.02) is derived, when instead of Eq. (6.03) the complete notation of Newton´s formula for the force is used

$$F = \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\mathrm{d}(mv)}{\mathrm{d}t} = \frac{\mathrm{d}m}{\mathrm{d}t} v + m\frac{\mathrm{d}v}{\mathrm{d}t} \tag{6.04}$$

If Eq. (6.01) is combined with Eq. (6.04) then

$$F = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\right) v + \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\frac{\mathrm{d}v}{\mathrm{d}t} \tag{6.05}$$

With

$$\frac{\mathrm{d}m}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}m}{\mathrm{d}v}\cdot\frac{\mathrm{d}v}{\mathrm{d}t} \tag{6.06}$$

the equation develops to

$$F = \left(-\frac{1}{2}\right)\left(\frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\right)\left(-2\frac{v}{c^2}\right)\frac{\mathrm{d}v}{\mathrm{d}t} v + \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$F = \left(\frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\right)\left(\frac{v^2}{c^2}\right)\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{m_0\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{\mathrm{d}v}{\mathrm{d}t} \tag{6.07}$$

and thus, the value in Eq. (6.02) for the longitudinal mass is the result. So, the equations for the different masses are identical and therefore since the mid of the 20th century the separation was cancelled and today normally the general term “relativistic mass” according to Eq. (6.01) is used.

It is apparent that equation Eq. (6.07) can be directly transformed to

$$F = \frac{m_0}{\gamma^3} a \tag{6.08}$$

This means that for a constant force acting from the *system at rest*, the acceleration occurring in the moving system (also measured from the system at rest) differs by a factor $\gamma^3$. This law was derived by H. A. Lorentz for an electric field acting on an electron from the outside. When considering accelerations caused by effects within a moving system (such as valid for a rocket engine), the same laws apply. As shown in chapter 6.4, the factor $\gamma^3$ results also if the relativistic velocity addition is chosen as the only criterion for derivation.

In the following the kinetic energy of a body in motion shall be discussed. To realize this, the relativistic (longitudinal) mass according to (6.07) is considered, because this is the complete equation that describes an increase of the velocity. The force which is necessary to accelerate a mass is therefore defined as

$$F = \frac{m_0 \cdot a}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \tag{6.09}$$

The necessary acceleration energy is

$$W_{1,2} = \int_{v_1}^{v_2} F \cdot \mathrm{d}s = \int_{v_1}^{v_2} \frac{m_0 \cdot a}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \cdot \mathrm{d}s = \int_{v_1}^{v_2} \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \cdot \frac{\mathrm{d}v}{\mathrm{d}t} \mathrm{d}s \tag{6.10}$$

Because of

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \tag{6.11}$$

it applies

$$W_{1,2} = \int_{v_1}^{v_2} \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} v \, \mathrm{d}v \tag{6.12}$$

and finally

$$W_{1,2} = \left. \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \right|_{v_1}^{v_2} \tag{6.13}$$

For $v_1 = 0$ and $v_2 = v$ follows

$$W = E_{kin} = m_0 c^2 \left( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) = m_0 c^2 (\gamma - 1) \tag{6.14}$$

The Taylor expansion of the square root leads to

$$\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2} \frac{v^2}{c^2} + \frac{1 \cdot 3}{2 \cdot 4} \frac{v^4}{c^4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{v^6}{c^6} + + \cdots \tag{6.15}$$

and for $v \ll c$ the classical formula for the kinetic energy is derived

$$E_{kin} \cong \frac{m_0}{2} v^2 \tag{6.16}$$

The equation (6.14) was developed by A. Einstein already in the year 1905 [22]. It contains implicit the first consideration of the equivalence of mass and energy and leads generally to

$$E = mc^2 \tag{6.17}$$

This is most probably the best-known formula in modern physics.

## 6.2 Spring paradox

In the following the situation shall be discussed, in which way a simple spiral spring and a mass attached to it will behave, when different experiments in a system at rest and in motion will be performed. To realize this at first 3 different experimental arrangements will be examined and in a second step the correlations for the energy are investigated and finally assessed.

### 6.2.1 Simple elongation of a spring

The simplest way to realize a static displacement of a spiral spring (this means without oscillation) is straining using a weight. This procedure is not suitable for a discussion using Special Relativity, however, because the value of the displacement is defined by the gravitational constant and thus by the mass of the earth. It is therefore not possible to carry out an undisputed examination. In this case a concept using General Relativity would be necessary.

Because of this reason a different technique for the generation of a displacement is necessary. For realization, the straining with a repulsive force is chosen, when caused by steadily flowing gas a constant force will be applied to the spring. Thus, the spring constant $k$ can be derived by

$$F = k \cdot s \tag{6.20}$$

In this case $F$ is the norm of the generated force and $s$ of the displacement. When this experimental set-up is transferred into motion and the elongation of the spring is in a position transverse to the system at rest, the observers at rest and in motion must detect the same displacement of the spring because the "principle of identity" is valid. For the observer in motion the spring constant must be the same as in the case discussed before. The observer at rest will, however, because of time dilatation and increasing of the relativistic mass, realize the following differences:

1. The number of gas-molecules per time unit generating the repulsion force is reduced by the factor $\gamma$.
2. The mass of any single molecule of the gas is increased by the factor $\gamma$.
3. The velocity of the gas molecules moving in transverse direction (in relation to the observed direction of motion) is reduced by the factor $\gamma$.

It must be added to point 3 that the total speed of a flowing gas molecule is exactly the same compared to the situation for an experiment at rest. The reason for this is that the way is increasing by the factor $\gamma$ but the angle of the gas flow is different by the factor $\alpha = arctan\, v/c$ to the transverse direction. This is the same situation why a light beam is travelling a longer way to a target in transverse direction in view of an observer at rest. The transverse component of the velocity is not affected by this, however, and is therefore reduced by the factor $\gamma$. These relations must be valid to make sure, that the moved observer is realizing the same situation compared to an observer at rest. In summary the considerations lead to the equation

$$k = \gamma \cdot k' \tag{6.21}$$

This means, that the spring constant in the system in motion is lower by the factor $\gamma$ when it is monitored by the observer in a system at rest. This fact, which is surprising at first sight, is necessary to make sure that no discrepancies with other experimental configurations appear. This will be shown in the following.

### 6.2.2 Rotation

Instead of using a repulsion force the displacement of a spring can also be generated by its existing torsion characteristics. First in a system at rest the value for the peripheral velocity depending on the dislocation of the spring and so the existing force is determined. When this set-up is accelerated to a higher velocity and the experiment is repeated (using again the orientation transverse to motion) the following value for the centrifugal force is calculated

$$F_z' = m' \cdot \frac{v'^2}{r} = \frac{F_z}{\gamma} \tag{6.22}$$

Reason for the difference to the system at rest is the fact that the peripheral velocity $v$ is occurring in a quadratic form in this equation. The relation is valid because the speed is slower in view of the observer at rest and the mass $m$ is increasing in the discussed manner.

### 6.2.3 Harmonic oscillation

A similar situation is observed when the spring is performing an oscillation. In this case the following differential equation is valid

$$\ddot{x} + \omega_0^2 \cdot x = 0 \tag{6.23}$$

with

$$\omega_0^2 = \frac{k}{m} \tag{6.24}$$

and

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}} \tag{6.25}$$

where $\omega_0$ is the angular frequency and $T_0$ the oscillation time. When this experimental set-up is accelerated to a higher speed (again transverse to the direction of motion) the amplitude will be reduced by the factor $\gamma$. This leads to the following relation

$$T_0' = 2\pi\sqrt{\frac{m'}{k'}} = 2\pi\sqrt{\frac{\gamma^2 \cdot m}{k}} = \gamma \cdot T_0 \tag{6.26}$$

In this case also a reduction of the spring constant is necessary to avoid discrepancies with the principle of relativity.

### 6.2.4 Literature survey

In the literature no variants of these experiments are discussed (at least not known by the author). There is, however, an additional interpretation of the experiment with a “broken lever” (first discussed by G. N. Lewis and R. C. Tolman), which is a variant of the Trouton-

Noble Experiment, were a similar situation is discussed by P. S. Epstein [43]. Based on the general approach by A. Sommerfeld [44] the following relations were developed

$$f_x = f'_x \qquad f_y = \frac{f'_y}{\gamma} \tag{6.27}$$

where $f_x$ and $f_y$ are the components of the "Newtonian force". This description explains the relations developed for springs like the decrease of the force in transverse direction by an observer at rest.

### 6.2.5 Considerations of energy

Due to these relations a further effect appears, however, which is leading to an apparent contradiction. Considering the internal energy of the spring

$$E_{pot} = \int_0^s F(s)ds = \int_0^s k \cdot s\, ds \tag{6.28}$$

it is obviously clear, that during straining the energy is depending on the force resp. on the spring constant in a linear relationship. Assessing the examples discussed before this would mean, that the mechanical energy of a spring is decreasing with higher velocities. This is clearly a violation of the universal principle of conservation of energy. If a strained spring is accelerated and then released an observer at rest would measure a lower energy compared to the value which was necessary when loading the spring. Looking the other way round the spring would have a higher internal energy after a deceleration.

To dissolve the apparent paradox first an additional examination of the total energy shall be carried out. For this purpose, the total energy of a mass is observed which is moving with a velocity $v_1$. This situation is according to the equation established in chapter. 6.1

$$E_1 = \gamma_1 m_0 c^2 \tag{6.29}$$

Now the case is investigated, that the mass is moving in a direction transverse to this (relative to the observer at rest), with a speed of $v_2$ measured by the observer in motion. The observer at rest will find a reduced value of

$$v'_2 = \frac{v_2}{\gamma_1} \tag{6.30}$$

because of time dilatation. According to the relativistic addition of velocities (see chapter 4.1, Eq. (4.20) with $\alpha = 90°$) this will lead to

$$v_T = \sqrt{\left(\frac{v_1}{c}\right)^2 + \left(\frac{v_2}{\gamma_1 c}\right)^2 - \left(\frac{v_1 v_2}{\gamma_1 c^2}\right)^2} \tag{6.31}$$

The energy of this mass is

$$E_T = \gamma_T m_0 c^2 \tag{6.32}$$

The differences of these energies are

$$\Delta E = \gamma_T m_0 c^2 - \gamma_1 m_0 c^2 \tag{6.33}$$

with

$$\Delta E = \frac{m_0 c^2}{\sqrt{1-\left(\frac{v_T}{c}\right)^2}} - \frac{m_0 c^2}{\sqrt{1-\left(\frac{v_1}{c}\right)^2}} \tag{6.34}$$

A Taylor expansion using $v_1, v_2 \ll c$ for this equation and the insertion of $v_T$ according to Eq (6.31) leads to the value

$$\Delta E \cong \left[1+\frac{1}{2}\left(\left(\frac{v_1}{c}\right)^2+\left(\frac{v_2}{\gamma_1 c}\right)^2-\left(\frac{v_1 v_2}{\gamma_1 c^2}\right)^2\right)-\left(1+\frac{1}{2}\left(\frac{v_1}{c}\right)^2\right)\right] m_0 c^2$$

$$= \frac{1}{2}\left[\left(\frac{v_2}{\gamma_1 c}\right)^2-\left(\frac{v_1 v_2}{\gamma_1 c^2}\right)^2\right] m_0 c^2$$

$$= \frac{1}{2}\left[\left(\frac{v_2}{\gamma_1 c}\right)^2\left(1-\left(\frac{v_1}{c}\right)^2\right)\right] m_0 c^2 = \frac{1}{2} m_0 v_2^2 \tag{6.35}$$

This is exactly the relation for the kinetic energy of a body in motion for nonrelativistic condition and shows that the balance of energy is obeyed in this case. The discrepancies concerning the energy of a spring are generated by the fact, that the force is a physical value with a direction. In this case the strange situation occurs that force and acceleration having different orientations. This issue was already discovered by P. S. Epstein in the year 1911 [43]. Although in this paper - according to the knowledge at that time - the mass was assigned the character of a tensor and the relationships discussed in chapter 6.1 for the force in moving direction and transverse to it where unknown, this is the solution to solve the discrepancies of the paradox.

## 6.3 Relativistic elastic collision

A further non-linear examination is possible for relativistic elastic collision. This will not be of importance when macroscopic observers are considered, because velocities to create a noticeable effect would certainly destroy the participating bodies on impact. However, when the effect on the behavior of elementary particles is examined, e.g. in particle colliders, it is an interesting question, how the tracking of the reaction changes when it is viewed by observers with different velocities relative to the experimental set-up.

The foundation for the calculation is – like for the non-relativistic examination – the laws of conservation for energy and momentum. The relevant relations for momentum and energy are

Rel. momentum: $$\vec{p} = \gamma m \vec{v} \tag{6.40}$$

Rel. kinetic energy: $$E = (\gamma - 1) m c^2 \tag{6.41}$$

When in a simple example it is assumed that 2 masses are colliding centrally without deviation, then for the momentum the presentation as vector can be skipped and the conservation laws are

$$m_1\gamma_1 v_1 + m_2\gamma_2 v_2 = m_1\gamma_3 v_3 + m_2\gamma_4 v_4 \tag{6.42}$$

$$(\gamma_1 - 1)m_1c^2 + (\gamma_2 - 1)m_2c^2 = (\gamma_3 - 1)m_1c^2 + (\gamma_4 - 1)m_1c^2 \tag{6.43}$$

where $v_1$ and $v_2$ are the velocities before and $v_3$ and $v_4$ after collision. This leads to

$$p = m_1\gamma_1 v_1 + m_2\gamma_2 v_2 = m_1\gamma_3 v_3 + m_2\gamma_4 v_4 \tag{6.44}$$

and

$$\frac{E_0}{c^2} = (\gamma_1 - 1)m_1 + (\gamma_2 - 1)m_2 = (\gamma_3 - 1)m_1 + (\gamma_4 - 1)m_2 \tag{6.45}$$

The determination of the results for $v_3$ and $v_4$ is not possible in closed analytical form and so for the solution a numerical approach is necessary. For the required calculation the principle of bisection is used. An example for the required computation is presented in annex A in the attachment.

For the examination of the non-relativistic case the equation for the momentum in Eq. (6.44) is modified

$$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4 \tag{6.46}$$

where simply the values for $\gamma$ are skipped, and further the use of the approximation formula

$$\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots \tag{6.47}$$

for $v \ll c$ and insertion into Eq. (6.45) leads to

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_3^2 + \frac{1}{2}m_2 v_4^2 \tag{6.48}$$

When Eq. (6.46) and Eq. (6.48) are suitably transformed it applies

$$m_1(v_1 - v_3) = m_2(v_4 - v_2) \tag{6.49}$$

and

$$m_1(v_1 - v_3)(v_1 + v_3) = m_2(v_4 - v_2)(v_4 + v_2) \tag{6.50}$$

Hence, after division of both equations

$$v_1 + v_3 = v_4 + v_2 \tag{6.51}$$

and after insertion in Eq. (6.49) the classical equations for the central collision can be derived in a simple way

$$v_3 = 2\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} - v_1 \tag{6.52}$$

and

$$v_4 = 2\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} - v_2 \tag{6.53}$$

It is obvious that the result represents a simple analytical solution and that for this case no numerical calculations are necessary.

Still open is the question, how the results will be tracked by observers with different velocities relative to the collision. To examine this, the circumstances for the situation before and after collision must be considered in detail. In annex A the calculation of the values of $v_3$ and $v_4$ is presented first, furthermore the equations for the relativistic addition of velocities according to the following relations are calculated, which is then subject to further comparison:

$$v_T(\, v_1,\ v_2) = \frac{v_1 - \ v_2}{1 - \frac{v_1\, v_2}{c^2}} \tag{6.54}$$

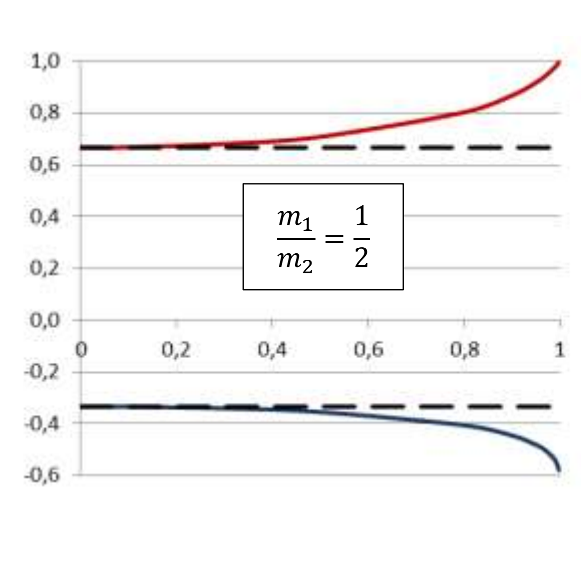
$$v_T(\, v_4,\ v_3) = \frac{v_4 - \ v_3}{1 - \frac{v_4\, v_3}{c^2}} \tag{6.55}$$

For a meaningful comparison between both results the quotient will be calculated first and then, because of the small deviation, the appearing value will be subtracted by1 resulting the error range

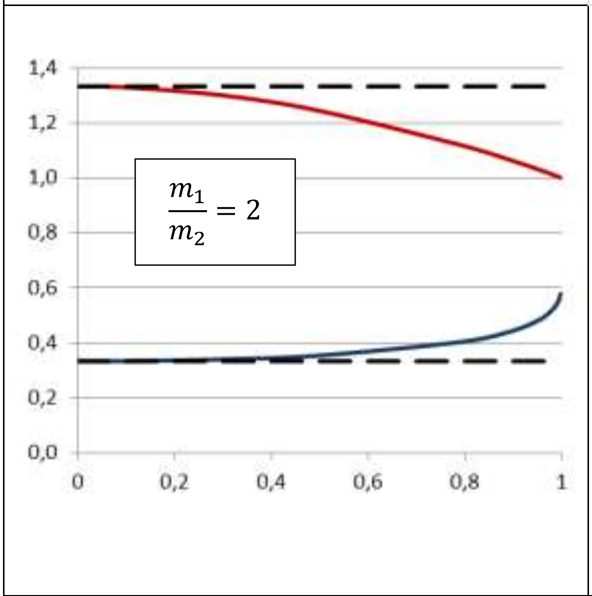
$$\delta_v = \frac{v_T(\, v_1,\ v_2)}{v_T(\, v_4,\ v_3)} - 1 \tag{6.56}$$

In Fig. 6.1 the values of the velocities $v_1/c$ from 0.0001 to 0.999 are presented for the mass-ratio $m_1 : m_2$ of 1: 2 and 2: 1 corresponding to the starting conditions $v_2 = 0$ and $v_1 = v_2$. To ensure comparability between the examined different velocities, for any value of $v_1/c$ the results of $v_3/v_1$ and $v_4/v_2$ were calculated and shown in a table, furthermore the findings are presented in graphical form. The graphs of the relations between the velocities show an asymptotic approach to the values of the non-relativistic cases calculated using Eq. (6.52) and Eq. (6.53), which were also inserted in the diagrams. The calculation of $\delta_v$ shows clearly, that all observers come to the same result irrespective of their velocities. This is corresponding to the examination of the non-relativistic case (see Eq. (6.52) and Eq. (6.53)).

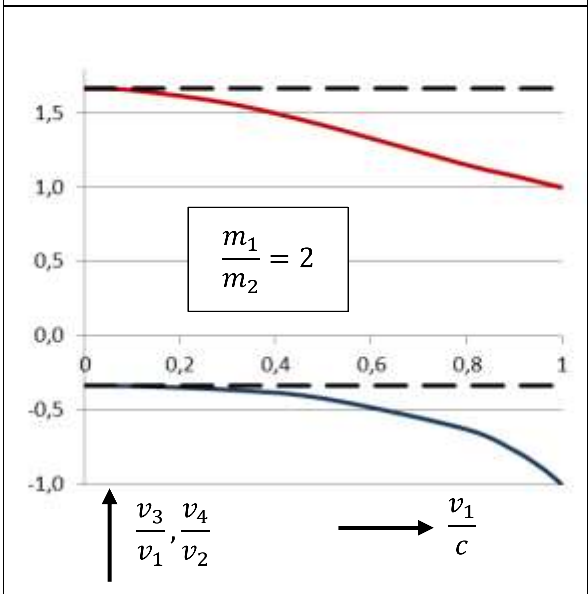
In a further examination the error range $\delta_v$ for different velocities is presented. Whereas high velocities show almost no noteworthy deviations this is changing considerably for lower values. This is caused by the decreasing accuracy during the calculation of small values because of round-off errors. Using standard spreadsheet calculation programs on a PC (such as Microsoft Excel©) the possible calculation limit is reached at values for $\delta_v$ of approximately $10^{-15}$. It is not possible to calculate with higher precision, smaller values are classified as 0. The question of accuracy is also of great importance for numerical solutions; this topic is dealt with in a comprehensive way in annex D, where 3 different approaches (recursion, Newton's calculus, bisection) are described and compared.




| $\frac{v_1}{c}$ | $\frac{v_2}{c}$ | $\frac{v_3}{v_1}$ | $\frac{v_4}{v_2}$ | $\delta_v$ |
|---|---|---|---|---|
| 0,0001 | -0,0001 | -0,3333 | 0,6666 | $-4{,}7\cdot10^{-8}$ |
| 0,001 | 0 | -0,3333 | 0,6666 | $-2{,}6\cdot10^{-10}$ |
| 0,1 | 0 | -0,3340 | 0,6682 | $2{,}4\cdot10^{-15}$ |
| 0,3 | 0 | -0,3403 | 0,6805 | $-3{,}0\cdot10^{-15}$ |
| 0,5 | 0 | -0,3544 | 0,7083 | 0 |
| 0,8 | 0 | -0,4054 | 0,8030 | 0 |
| 0,9 | 0 | -0,4449 | 0,8679 | 0 |
| 0,95 | 0 | -0,4807 | 0,9174 | 0 |
| 0,97 | 0 | -0,5023 | 0,9437 | 0 |
| 0,99 | 0 | -0,5392 | 0,9773 | 0 |
| 0,999 | 0 | -0,5793 | 0,9973 | 0 |




| $\frac{v_1}{c}$ | $\frac{v_2}{c}$ | $\frac{v_3}{v_1}$ | $\frac{v_4}{v_2}$ | $\delta_v$ |
|---|---|---|---|---|
| 0,0001 | 0 | 0,3333 | 1,3333 | $-2{,}6\cdot10^{-8}$ |
| 0,001 | 0 | 0,3330 | 1,3330 | $1{,}3\cdot10^{-10}$ |
| 0,1 | 0 | 0,3340 | 1,3296 | $-1{,}0\cdot10^{-14}$ |
| 0,3 | 0 | 0,3403 | 1,3050 | $3{,}6\cdot10^{-15}$ |
| 0,5 | 0 | 0,3544 | 1,2442 | $3{,}3\cdot10^{-15}$ |
| 0,8 | 0 | 0,4054 | 1,1159 | $3{,}1\cdot10^{-15}$ |
| 0,9 | 0 | 0,4449 | 1,0621 | 0 |
| 0,95 | 0 | 0,4807 | 1,0317 | 0 |
| 0,97 | 0 | 0,5023 | 1,0202 | 0 |
| 0,99 | 0 | 0,5392 | 1,0070 | 0 |
| 0,999 | 0 | 0,5793 | 1,0007 | $6{,}9\cdot10^{-15}$ |




| $\frac{v_1}{c}$ | $\frac{v_2}{c}$ | $\frac{v_3}{v_1}$ | $\frac{v_4}{v_2}$ | $\delta_v$ |
|---|---|---|---|---|
| 0,0001 | -0,0001 | -0,3333 | 1,6660 | $-1{,}8\cdot10^{-8}$ |
| 0,001 | -0,001 | -0,3330 | 1,6666 | $1{,}7\cdot10^{-10}$ |
| 0,1 | -0,1 | -0,3363 | 1,6550 | $-5{,}6\cdot10^{-15}$ |
| 0,3 | -0,3 | -0,3614 | 1,5670 | 0 |
| 0,5 | -0,5 | -0,4194 | 1,4186 | 0 |
| 0,8 | -0,8 | -0,6276 | 1,1523 | 0 |
| 0,9 | -0,9 | -0,7697 | 1,0777 | 0 |
| 0,95 | -0,95 | -0,8697 | 1,0387 | 0 |
| 0,97 | -0,97 | -0,9174 | 1,0228 | 0 |
| 0,99 | -0,99 | -0,9708 | 1,0075 | 0 |
| 0,999 | -0,999 | -0,9970 | 1,0007 | 0 |

Fig. 6.1: Relativistic elastic collision for $0{,}0001 < v_1/c < 0.999$. Relations for velocities $v_3/v_1$ (blue), $v_4/v_2$ (red). Error range $\delta_v$ (For definition: see text). Non-relativistic case: dotted line.

Finally, it can be stated that during relativistic elastic collision no effects appear which would make it possible to identify the existence of a system of absolute rest in the universe. However, new attempts are made (year 2017) to identify results of this kind using precision measurements of particle mass (in this case: electrons) [45]. According to the considerations presented here it is not possible that experiments of this type can be successful at all.

## 6.4 Exchange of signals during and after acceleration

In this chapter it is investigated how accelerated systems behave in relativistic situations and which measurement results are obtained for other, non-accelerated observers with constant velocity. The acceleration is not generated from outside sources - e.g. by an electromagnetic field acting on a charged object - as it was investigated by H. A. Lorentz (cf. chapter 6.1), but shall be caused by thrust like it is the case for a rocket.

First, a simple situation is considered in which the system under investigation is subjected to constant acceleration, with changes in mass due to the emission of propellant gases initially being disregarded. Important results can be determined by analytical and numerical methods. Then, in a more advanced approach, consideration of the decrease in rocket mass with acceleration is added. If for the propulsion a proportional change of the ejection mass compared to the remaining rocket mass is assumed, the acceleration remains constant during a trial and the behavior is the same as in the previously investigated case.

In contrast, a constant mass decrease per time unit (as required when the classical rocket formula is used) leads to increasing acceleration values. These calculations in full scale (including acceleration and covered distance) can only be carried out numerically; a corresponding program and the results obtained with it are shown in the appendix. Further, the final velocity of a rocket, which can be calculated using the classical and relativistic rocket formula, is determined and the agreement of the results is shown.

### 6.4.1 Exchange of signals in systems with constant acceleration

In the following the case shall be discussed that a rocket accelerates uniformly and is observed from other inertial systems. During the acceleration process, signals are emitted by observer S inside the rocket at regular intervals of $\Delta t_S$. Further observer A also participates in the experiment and moves at the beginning of the acceleration with the same speed as S. Out of an additional inertial system, a second observer B is moving with an arbitrary velocity relative to A. Both observers A and B are recording the signals of S.

First, the acceleration of the rocket monitored by observer A is investigated. An analytical calculation is complicated by the fact that the relation for the relativistic velocity addition is not linear. During the acceleration, for the current velocity $v_A$ the velocity change d$v_A$ (from the point of view of A) is described by

$$v_A + dv_A = \frac{v_A + dv_S}{1 + \frac{v_A \cdot dv_S}{c^2}} \tag{6.60}$$

where $dv_S$ represents the change of the velocity observed in the moving system S. The use of a Taylor expansion results in

$$v_A + dv_A = v_A + dv_S\left(1 - \frac{v_A^2}{c^2}\right) + \ (dv_S)^2\left(\frac{v_A^3 - v_A \cdot c^2}{c^4}\right) + + \dots \tag{6.61}$$

With a differential consideration for $dv_S \rightarrow 0$, values of $(dv_S)^2$ and higher order can be neglected. Equation (6.61) thus obtains the form

$$dv_A = dv_S\left(1 - \frac{v_A^2}{c^2}\right) \tag{6.62}$$

The applicable accelerations are now defined for both systems

$$a_S = \frac{dv_S}{dt_S} \qquad\qquad a_A = \frac{dv_A}{dt_A} \tag{6.63}$$

Furthermore

$$dt_S = \ dt_A \cdot \gamma = \frac{dt_A}{\sqrt{1 - \left(\frac{v_A}{c}\right)^2}} \tag{6.64}$$

and finally

$$a_A = \frac{dv_A}{dt_A} = \frac{dv_S}{dt_S}\left(1 - \frac{v_A^2}{c^2}\right)^{3/2} = a_S\left(1 - \frac{v_A^2}{c^2}\right)^{3/2} = \frac{a_S}{\gamma^3} \tag{6.65}$$

Thus, between $a_A$ and $a_S$ the same factor $\gamma^3$ appears as it was derived when determining the correlations for the occurring forces in case of relativistic mass increase (cf. chapter 6.1).

In the following, the relations between the subjectively observed times, velocities, and distances for stationary and moving observers shall be determined. For this purpose, first the velocity is considered. From eq. (6.65) follows immediately

$$dt_A = \frac{1}{a_S}\left(1 - \frac{v_A^2}{c^2}\right)^{-3/2} dv_A \tag{6.66}$$

Assuming, that values for $a_S$ are constant and integrating Eq. (6.66), we obtain

$$t_A = \frac{v_A}{a_S}\left(1 - \frac{v_A^2}{c^2}\right)^{-1/2} + C = \frac{v_A \cdot \gamma(v_A)}{a_S} + C \tag{6.67}$$

If concrete values are used (e.g. time runs from 0 to $t_A$), the integration constant $C$ equals zero. This equation describes - with subjectively constant acceleration of the rocket - the dependency between time and velocity from the point of view of A. With a given velocity, time can be determined directly, in the opposite case, a numerical procedure must be applied to determine $v_A$ when using the equation. To avoid this, however, equation Eq. (6.67) can be extended and transformed via

$$\left(\frac{a_S \cdot t_A}{c}\right)^2 = \left(\frac{v_A \cdot \gamma(v_A)}{c}\right)^2 = \frac{\frac{v_A^2}{c^2} + 1 - 1}{1 - \frac{v_A^2}{c^2}} = \frac{1}{1 - \frac{v_A^2}{c^2}} - 1 \tag{6.68}$$

Transformed to $v_A$ the result is

$$v_A = \frac{a_S \cdot t_A}{\sqrt{1 + \left(\frac{a_S \cdot t_A}{c}\right)^2}} \tag{6.69}$$

This representation is also found in the literature, using approaches similar to the one chosen here [32] as well as using rapidity [91]. [Note: rapidity θ describes a concept in which velocities are added up according to Galileo's principle; the relationship with relativistic velocity is $\theta = arctanh(v/c)$]. Equations (6.67) and (6.69) are equivalent and can be used depending on the computational requirements.

To calculate the time subjectively elapsing in the rocket, equations (6.64) and (6.66) are combined, yielding the relation

$$dt_S = \frac{1}{a_S}\left(1 - \frac{v_A^2}{c^2}\right)^{-1} dv_A \tag{6.70}$$

Integration leads to

$$t_S = \frac{c}{a_S}\operatorname{arctanh}\left(\frac{v_A}{c}\right) + C \tag{6.71}$$

For direct calculation of the dependency on $t_A$ instead of $v_A$, Eq. (6.69) can be substituted into (6.71).

The distance travelled $x_A$ can be calculated using Eq. (6.66) with

$$dx_A = v_A dt_A = \frac{1}{a_S}\left(1 - \frac{v_A^2}{c^2}\right)^{-3/2} dv_A \tag{6.72}$$

Integration yields

$$x_A = \frac{c^2}{a_S}\left(1 - \frac{v_A^2}{c^2}\right)^{-1/2} + C \tag{6.73}$$

In contrast to the previous cases, the integration constant must be determined here. This is done by using the boundary condition $x_A = 0$ for the velocity $v_A = 0$. Substituting in Eq. (6.73) this leads to

$$0 = \frac{c^2}{a_S}(1 - 0)^{-1/2} + C \quad \Rightarrow \quad C = -\frac{c^2}{a_S}$$

and inserted into Eq. (6.73), the final form is given by

$$x_A = \frac{c^2}{a_S}\left\{\left(1 - \frac{v_A^2}{c^2}\right)^{-1/2} - 1\right\} = \frac{c^2}{a_S}(\gamma - 1) \tag{6.74}$$

Again, the relationship between $v_A$ and $t_A$ from equation (6.69) can be used alternatively to obtain a direct dependence on $t_A$.

Equation (6.74) has the peculiarity that for small values of $v_A$ the end results can become very inaccurate. The value of $\gamma$ approaches 1 in this case; but since the value 1 is subtracted in the formula, larger errors can occur with usual calculation accuracy. It is recommended here to use a Taylor expansion where these problems do not appear. Appendix B contains a derivation in chapter B.3 and it is shown under which boundary conditions Eq. (6.74) or the Taylor method is more accurate.

Furthermore, a numerical method is also presented in this annex B, where the use of additions of relativistic velocities with sufficiently small steps leads to the same results. An analytical method is easier to use but would lead to problems in case of modifications, such as changing the acceleration during the experiment. With numerical methods, on the other hand, such a situation can be implemented easily. This becomes clear in the situation described in the next chapter, in which the real behavior of creating thrust realized by ejection of a propellant gas from a rocket and the resulting influences on the system are considered in detail.

In the following it shall be demonstrated that based on these simple correlations no contradictions will occur concerning the experimental findings of observers travelling with different velocities compared to the system, which is at rest at the start of acceleration of the rocket. The only precondition necessary is, that from the rocket signals to observers A and B are transmitted, and that these signals have a constant subjective frequency concerning the system inside the rocket. The situation of all participants is presented in the following diagram.

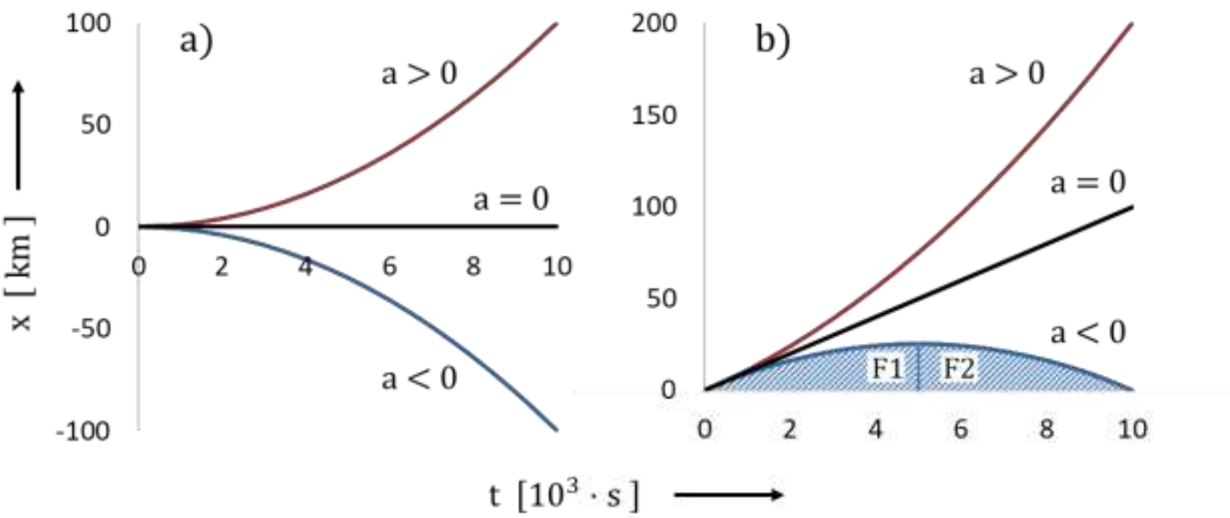



Fig. 6.2: Comparison of different acceleration conditions calculated for $a = 10\ \mathrm{m/s^2}$, $a = 0$ and $a = -10\ \mathrm{m/s^2}$
a) $v_0 = 0$, b) $v_0 = 50\ \mathrm{m/s}$

Observer B is at rest in all cases relative to the presentation of the diagram (i.e. from the point of view of A and S, he is moving relative to them at the start of the experiment with velocity $v_0$), while A is moving on the line $\mathrm{a} = 0$. Thus, in subplot a) with $v_0 = 0$, the results for A and B coincide, while in b) participant A is increasing the distance in relation to B with constant velocity $v_0$. The aim of the following calculations is to show that the values of A in part a) and also b) are identical from the point of view of B using the Lorentz equations. The principle of relativity is valid because the subjectively measured times are independent of the speed of the observers.

To prove this, Fig. 6.3 shows a situation in which subplot a) shows the rocket passing observer B (blue line in the x/t diagram), decelerates and then approaches again. In subplot b) the rocket starts from a position at rest and is accelerated uniformly. In this case, the course of an additional test participant A moving uniformly at velocity $v_0$ is also shown (blue line). To make the results easier to distinguish, the reference points in subplot a) have been marked with P and in b) with Q and R.

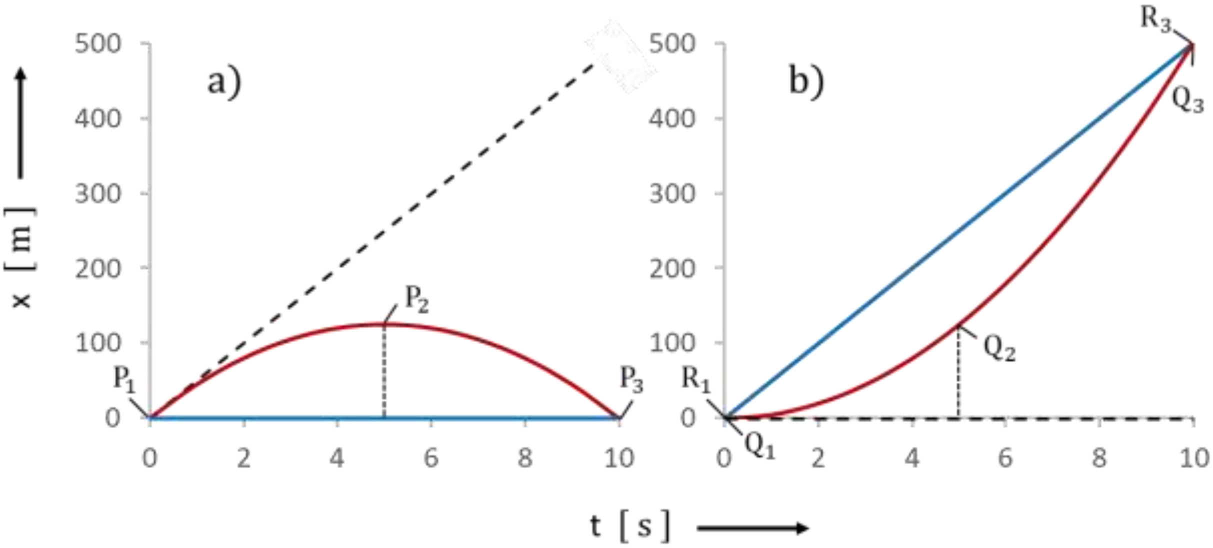



Fig. 6.3: Identical accelerations observed by different participants
a) $v_0 = 50 \text{ m/s}, \ a_S = -10 \text{ m/s}^2$ b) $v_0 = 0, \ a_S = 10 \text{ m/s}^2$

With the very small values for $v_0$ chosen here for the presentation in the diagram, in principle no significant deviations between relativistic and non-relativistic consideration can be provided. Therefore, calculations were carried out which are based on a system velocity of 369 km/s. As already pointed out in several other cases, this is the velocity with which our solar system is moving relative to the uniform cosmic background radiation and thus is of great interest for possible experiments to be performed. It remains to be clarified how large the difference is in the present case between relativistic and non-relativistic consideration. In order to show this, values for the non-relativistic case (Galileo) were also added to the table. As it is well known, these relations are given by

$$v = a \cdot t \tag{6.75}$$

$$x = \frac{1}{2} a \cdot t^2 \tag{6.76}$$

If it is assumed that a spaceship passes earth with 369 km/s and decelerates with $10 \text{m/s}^2$, the maximum distance would be reached at about $6{,}8 \cdot 10^6$ km (subplot a, point $P_2$) in non-relativistic consideration. The total time until the earth is reached again at $P_3$ is about 20.5 hours. The exact values and also the results calculated for a relativistic consideration are summarized in a table (Tab. 6.1).

The information included in this representation will be broken down in the following. For this purpose, it is necessary to note the sequence of the calculations. First, the subplot a) is considered:

1. $P_1 \rightarrow P_2$
   The values of $t_S(P_2)$ are calculated using Eq. (6.67), $t_A(P_2)$ is derived from Eq. (6.71) and $x_A(P_2)$ from Eq. (6.74) for the velocity $v_A = 369$ km/s. The use of Eq. (6.74) is permitted, although it was initially derived considering the case $v_A = 0$; because of symmetrical reasons first case $P_2 \rightarrow P_1$ is calculated and the result is then transferred to $P_1 \rightarrow P_2$.
2. $P_2 \rightarrow P_3$
   Because of symmetry reasons the values of $t_S(P_3)$ und $t_N(P_3)$ must be twice as large as for $(P_2)$. The value of $x_A(P_3) = 0$ by definition.

For subplot b) the values are accordingly:

1. $Q_1 \rightarrow Q_2$
   Symmetry reasons result in $t_S(P_2) = t_S(Q_2)$, $t_A(P_2) = t_A(Q_2)$ and $x_A(P_2) = x_A(Q_2)$.
2. $Q_2 \rightarrow Q_3$
   In this case the assumption is used that subjectively within differently moved inertial systems no differences may arise at the same changes of state; this means $t_S(P_3) = t_S(Q_3)$ is set (the two fields are green and marked with arrow). If this assumption is correct, no differences may show up in a later comparison of results. First, the value for $v_A(Q_3)$ is calculated from Eq. (6.71), then $t_A(Q_3)$ from Eq. (6.67) and $x_A(Q_3)$ from Eq. (6.74).

| Pos. | Kriterium | $v_A$ | $t_S$ | $t_A$ | $x_A$ |
|---|---|---|---|---|---|
| $P_1$ | | 369 | 0 | 0 | 0 |
| $P_2$ | relativ. | 0 | 36900,0186344619 | 36900,0279516977 | 6808057,73532358 |
| $P_2$ | Galilei | 0 | 36900 | 36900 | 6808050 |
| $P_3$ | relativ. | -369 | 73800,0372689239 | 73800,0559033954 | 0 |
| $P_3$ | Galilei | -369 | 73800 | 73800 | 0 |
| | | | | | |
| $Q_1$ | | 0 | 0 | 0 | 0 |
| $Q_2$ | relativ. | 369 | 36900,0186344619 | 36900,0279516977 | 6808057,73532358 |
| $Q_2$ | Galilei | 369 | 36900 | 36900 | 6808050 |
| $Q_3$ | relativ. | 737,998881935078 | 73800,0372689239 | 73800,1118068352 | 27232241,2567440 |
| $Q_3$ | Galilei | 738 | 73800 | 73800 | 27232200 |
| | | | | | |
| $R_3$ | relativ. | 369 | 73800,0559034565 | 73800,1118068942 | 27232241,2567440 |
| $R_3$ | Galilei | | 73800 | 73800 | 27232200 |
| | | | | | |
| $\delta_K$ | | | $-8{,}28559 \cdot 10^{-13}$ | $-8{,}001 \cdot 10^{-13}$ | |

Tab. 6.1: Results of calculations for $v_0 = 369$ km/s using $a_S = -10$ m/s$^2$ (values P) and $a_S = 10$ m/s$^2$ (values Q) for a non-relativistic (Galileo) and relativistic approach. Points are defined according to Fig. 6.3.

For a further evaluation, the case must be calculated, how the situation arises in subplot b) for a linearly (unaccelerated) moving observer (blue line). To realize this, the boundary condition is used that accelerated and non-accelerated observers meet at the point $Q_3$, i.e. the values $x_N$ for $Q_3$ and $R_3$ must be the same in this case (these fields are also green and marked with an arrow). From

$$t_A = \frac{x_A}{v_0} \tag{6.77}$$

and

$$t_S = \frac{t_A}{\gamma} \tag{6.78}$$

the values of $t_A$ and $t_S$ can be calculated.

With the data determined here, a comparison between individual values can be carried out. First, the values for $t_A$ for the accelerated and non-accelerated case are compared at point $Q_3 = R_3$, which by definition must be the same, since both start and end from the same point ($Q_1 \rightarrow Q_3$ and $R_1 \rightarrow R_3$). The values are marked in blue. Despite different calculations, they lead to approximately the same result, with the deviation according to the calculation for

$$\delta_{K1} = \frac{t_A(Q_3)}{t_A(R_3)} - 1 \tag{6.79}$$

to be determined. The same behavior occurs when the values for $t_A(P_3)$ and $t_S(Q_3(L)$ are compared (marked in yellow)

$$\delta_{K2} = \frac{t_A(P_3)}{t_S(R_3)} - 1 \tag{6.80}$$

These must be equal for the following reason: The stationary observer in subplot a) determines that the passing rocket arrives at his position again after uniform negative acceleration at the time $t_A$. The uniformly moving observer in subplot b) must subjectively observe the same behavior. For the situation of an observer at rest in subplot b), represented by the course of the dashed line, the value for $t_A$ is higher in this case, but can be traced back to the subjective measured value of the moving system by simple division by $\gamma$. No relevant calculation differences can be determined here.

With the boundary conditions selected here using $v_A = 369$ km/s, deviations of approx. $8 \cdot 10^{-13}$ occur for $\delta_K$. If, on the other hand, higher values for $v_A$ are selected, as e.g. in Tab. 6.2 with $v_A = 0{,}5c$, no deviations are detectable within the scope of the calculation accuracy, but with smaller values for $v_A$ they increase. This is due to the occurrence of very small values of $\gamma$, especially in Eq. (6.74). At small velocities, the value for $\gamma$ is only slightly larger than 1; if the value of 1 is subtracted from this, large deviations can result depending on the accuracy of the calculation. This effect is shown in more detail in annex B, chapter B.3 and for this purpose a significant improvement of the accuracy is demonstrated by using a Taylor expansion.

Instead of the analytical approach chosen here, the regularities can also be determined numerically. A procedure for this is compiled in Annex B. If the occurring deviations are considered, an advantage for the numerical procedure is shown with low values of $v_A$, with higher velocities it is the other way round; the accuracy depends beyond that substantially on the number of the selected iteration steps. After performing the numerical calculations, it is shown here that the subjectively existing acceleration between motionless and moving observer differs by a factor $\gamma^3$; in contrast to the analytical method, where this was determined by basic considerations, this is a result of the calculations performed. In the Annex B the results are presented in detail. Also added is a comparison with results of the numerical method from Annex C, in which the amount of propellant gas ejected was kept constant in relation to the residual mass of the rocket, thus achieving uniform acceleration.

| Pos. | Kriterium | $v_A$ | $t_S$ | $t_A$ | $x_A$ |
|---|---|---|---|---|---|
| $P_1$ | | 149896,229 | 0 | 0 | 0 |
| $P_2$ | relativ. | 0 | 16467783,9204409 | 17308525,6327320 | 1390379100217,26 |
| $P_2$ | Galilei | 0 | 14989622,9 | 14989622,9 | 6808050 |
| $P_3$ | relativ. | -149896,229 | 32935567,8408818 | 34617051,2654639 | 0 |
| $P_3$ | Galilei | -149896,229 | 29979245,8 | 29979245,8 | 0 |
| | | | | | |
| $Q_1$ | | 0 | 0 | 0 | 0 |
| $Q_2$ | relativ. | 149896,229 | 16467783,9204409 | 17308525,6327320 | 1390379100217,26 |
| $Q_2$ | Galilei | 149896,229 | 14989622,9 | 14989622,9 | 6808050 |
| $Q_3$ | relativ. | 239833,96640 | 32935567,8408818 | 39972327,7333333 | 5991701191578,78 |
| $Q_3$ | Galilei | 299792,458 | 29979245,8 | 29979245,8 | 4493775893684,09 |
| | | | | | |
| $R_3$ | relativ. | 149896,229 | 34617051,2654639 | 39972327,7333333 | 5991701191578,78 |
| $R_3$ | Galilei | 299792,458 | 29979245,8 | 29979245,8 | 4493775893684,09 |
| | | | | | |
| $\delta_K$ | | | 0 | 0 | |

Tab. 6.2: Results of calculations for $v_0 = 0{,}5c$ using $a_S = -10\ \mathrm{m/s^2}$ (values P) and $10\ \mathrm{m/s^2}$ (values Q) for a non-relativistic (Galileo) and relativistic approach. Points are defined according to Fig. 6.3.

An evaluation of the chosen general conditions reveals at first sight that a rocket technology generating the required thrust long enough is not existing today; with such a system it would be possible to reach Mars in a few days. This becomes even clearer if a long journey is considered under the conditions chosen here. If it is assumed that a body of 100 tons with constant acceleration of 1g crosses the galaxy (100,000 light years, subjective time on board: approx. 12 years), the rocket with a propellant density of 70 kg/m$^3$ would have to have a size of 14 x 14 x 14 km$^3$ at departure, even if an optimal conversion of mass into kinetic energy is assumed [91]. This does not include any statements on the deceleration of the rocket after the journey or on the influence of micrometeorites and gas causing a speed reduction, or the protection of the passengers by additionally required masses due to necessary shielding devices.

Despite the obvious impossibility of implementation on an industrial scale, however, the results calculated here are unambiguous and show that - although the influence is small - they must be taken into account when even small acceleration phases are considered.

Finally, questions of the influence of acceleration on the measurements shall be examined in general. According to the Theory of General Relativity it is not possible for an observer to decide with measurements in a closed system, whether he is exposed to an acceleration effect caused by increasing velocity or by a gravitation field. Although it is not without controversy that additional (gravitational) time dilatation will appear in accelerated systems, the potential effect shall be estimated to complete a general consideration.

For the conditions chosen here with an acceleration value of 10 $m/s^2$, which corresponds approximately to the effect of the earth's acceleration due to gravity of 9.81 $m/s^2$, a time dilation of about $7 \cdot 10^{-10}$ results, which has been confirmed by many measurements [80]. If this value is multiplied by the total time from Tab. 6.2, an effect of $5{,}17 \cdot 10^{-5}$ s results. This would mean that the calculated time difference between relativistic and non-relativistic consideration is extended by a value of 0.28%. Thus, because of the small deviation, this potential effect can be neglected here.

### 6.4.2 Relativistic rocket propulsion

Now the question arises, how a rocket behaves in reality, which is accelerated by outflowing gas and accordingly loses mass. An observer B, who monitors this process from another inertial frame and measures the velocity $v_0$ for S and A at the beginning of the experiment, will find differences to the measurements of S due to the time dilation and the relativistic mass increase, namely

1. The quantity of the gas-molecules generating the repulsion force is reduced by the factor $\gamma(v_0)$ per time unit.
2. The mass of any single molecule of the gas is increased by the factor $\gamma(v_0)$.
3. The remaining mass of the rocket is increased by the factor $\gamma(v_0)$.
4. The speed of the outflowing gas corresponds to the theorem of relativistic addition of velocities.
5. The elapsing time between outgoing signals is increased by the factor $\gamma(v_0)$.
6. The total time for acceleration during an experiment is increased by the factor $\gamma(v_0)$.

For the exact determination of the situation, all influences related to these criteria must be calculated with respect to the reduction of the rocket mass due to the gas ejection for propulsion. These conditions are considered for cases with constant gas ejection (which leads to a steady increase in acceleration) and with constantly reduced gas ejection (to ensure constant acceleration).

The relativistic momentum is used to establish the equations relevant to solve this problem. It is determined in general that all functions referring to the outflowing gas are marked with $f'$; relations connected with the moving rocket, on the other hand, are represented without this marking.

Following this general definition, the relativistic momentum of a rocket before starting acceleration is

$$p_0 = m_0 v_0 \gamma_0 \tag{6.81}$$

where $v_0$ is the velocity of the rocket relative to a reference frame at the start of the trial. After the first step the relation changes to

$$p_1 = m_1 v_1 \gamma_1 \tag{6.82}$$

and the values for step 1 are calculated as follows:

1. It is assumed that during the first step of acceleration the rocket is losing mass $\Delta m_0$ with the jet velocity $v_0'$; the gas used to form the high-speed jet to generate the repulsion force is generally called “propellant mass”.

2. The momentum of the rocket $p_1$ (related to the remaining mass $m_1 = m_0 - \Delta m_0$) and $p_1'$ of the propellant mass $\Delta m_0$ are added and set equal to the momentum $p_0$ of the rocket (using of the law of conservation of momentum). From this, the changing velocity of the rocket is calculated. This results in

$$p_1 + p_1' = (m_0 - \Delta m_0) v_1 \gamma_1 + \Delta m_0 v_1' \gamma_1' = m_0 v_0 \gamma_0 \tag{6.83}$$

and generally

$$p_K + p_K' = (m_{K-1} - \Delta m_{K-1}) v_K \gamma_k + \Delta m_{K-1} v_K' \gamma_K' = m_{K-1} v_{K-1} \gamma_{K-1} \tag{6.84}$$

The values for $v$ and $v'$ show in different directions (this is explaining the “+” in the formula). Relative to the rocket, the gas flow maintains at a constant speed of $v_0'$. The relativistic addition of velocities is leading to

$$v_{K+1}' = \frac{v_{K+1} + v_0'}{1 + \frac{v_{K+1} v_0'}{c^2}} \tag{6.85}$$

Using the equations (6.84) and (6.85) for every step $K$ the velocity of the rocket can be calculated; this means the complete numerical evaluation is following a nested loop with a subroutine for any $v_K$.

To perform such a calculation, programming was done in Visual Basic (VBA). The VBA program code is compiled in Annex C with the corresponding formulas and a flow chart. The main purpose of these calculations is the comparison of systems which are at rest at the time of the start of the trial to those which are relatively moved. For this purpose, two exemplary calculation variants were programmed, whereby firstly the acceleration and in the second case the outflow velocity of the propellant mass were kept constant. The differences associated with both concepts are presented in the following.

*a) Propellant mass proportional to the remaining mass of the rocket*

The precondition of propellant mass proportional to the remaining mass of the rocket results in constant acceleration values for the rocket over the entire observation period. This situation corresponds to the case already described in chapter 6.4.1.

Table 6.3 shows the results of two calculations with $v_0 = 0$ and $v_0 = 0{,}5c$ as initial velocities. The selected values are quite different and this also the case for the results. In order to enable a comparison of the values with each other, the final velocity of the rocket from the view of an observer at rest was defined as the difference $v_T = v_N - v_0$. The value $t_T$ is the total time, which results subjectively from the view of the unmoved system when applying the Lorentz equations for an observer moving with system velocity $v_0$ until the arrival of a signal from the rocket.

In addition, the distance $x_N$ covered by the rocket from the view of the stationary observer up to the emission of the impulse is listed. Furthermore, the result for the remaining mass $m_N$ of the rocket after completion of the experiment is shown (related to the initial value $m_0 = 1$). In addition, the values for the accelerations $a_N$ and also the calculations for $\gamma^3 a_N$ are presented.

| K | N | $v_T$ | $t_T$ | $m_N$ | $x_N$ | $a_N$ | $\gamma^3 a_N$ |
|---|---|---|---|---|---|---|---|
| 1 | 10 | 3,9999999999413 | 400,00266852469 | 0,3486784401000 | 800,0000000579 | 9,9999999975878 | 10,000000000258 |
| 2 | $10^2$ | 3,9999999999407 | 400,00266852463 | 0,3660323412732 | 800,0000000592 | 9,9999999973564 | 10,000000000027 |
| 3 | $10^3$ | 3,9999999999408 | 400,00266852463 | 0,3676954247710 | 800,0000000594 | 9,9999999973343 | 10,000000000005 |
| 4 | $10^4$ | 3,9999999999424 | 400,00266852463 | 0,3678610464329 | 800,0000000596 | 9,9999999973292 | 10,000000000000 |
| 5 | $10^5$ | 3,9999999999581 | 400,00266852464 | 0,3678776017666 | 800,0000000616 | 9,9999999975070 | 10,000000000177 |
| 6 | $10^6$ | 4,0000000001169 | 400,00266852155 | 0,3678792572316 | 800,0000000717 | 9,9999999975070 | 10,000000000177 |
| 7 | $10^7$ | 4,0000000016930 | 400,00266859493 | 0,3678794227775 | 800,0000004935 | 10,000000005125 | 10,000000007795 |

| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ | $\delta a_N$ | $\delta \gamma^3 a_N$ |
|---|---|---|---|---|---|---|
| 1/2 | $-5{,}7021 \cdot 10^{-13}$ | $-5{,}7980 \cdot 10^{-11}$ | $1{,}7354 \cdot 10^{-2}$ | $1{,}2930 \cdot 10^{-9}$ | $-2{,}3141 \cdot 10^{-10}$ | $-2{,}3141 \cdot 10^{-10}$ |
| 2/3 | $1{,}3989 \cdot 10^{-13}$ | $-1{,}0232 \cdot 10^{-12}$ | $1{,}6631 \cdot 10^{-3}$ | $1{,}3006 \cdot 10^{-10}$ | $-2{,}2089 \cdot 10^{-11}$ | $-2{,}2089 \cdot 10^{-11}$ |
| 3/4 | $1{,}5601 \cdot 10^{-12}$ | 0 | $1{,}6562 \cdot 10^{-4}$ | $2{,}1396 \cdot 10^{-10}$ | $-5{,}0804 \cdot 10^{-12}$ | $-5{,}0804 \cdot 10^{-12}$ |
| 4/5 | $1{,}5710 \cdot 10^{-11}$ | $8{,}98 \cdot 10^{-12}$ | $1{,}6555 \cdot 10^{-5}$ | $2{,}0430 \cdot 10^{-9}$ | $1{,}7776 \cdot 10^{-10}$ | $1{,}7776 \cdot 10^{-10}$ |
| 5/6 | $1{,}5883 \cdot 10^{-10}$ | $-3{,}09 \cdot 10^{-9}$ | $1{,}6555 \cdot 10^{-6}$ | $1{,}0087 \cdot 10^{-8}$ | 0 | 0 |
| 6/7 | $1{,}5760 \cdot 10^{-9}$ | $7{,}3485 \cdot 10^{-8}$ | $1{,}6555 \cdot 10^{-7}$ | $4{,}2175 \cdot 10^{-7}$ | $7{,}6180 \cdot 10^{-9}$ | $7{,}6180 \cdot 10^{-9}$ |

| K | N | $v_T$ | $t_T$ | $m_N$ | $x_N$ | $a_N$ | $\gamma^3 a_N$ |
|---|---|---|---|---|---|---|---|
| 1 | 10 | 2,9999709803087 | 400,00266851663 | 0,3486784401000 | 69235026,29063 | 6,4951038669616 | 10,000066715204 |
| 2 | $10^2$ | 2,9999700801272 | 400,00266851582 | 0,3660323412732 | 69235026,29036 | 6,4950395173348 | 9,9999676404299 |
| 3 | $10^3$ | 2,9999699985783 | 400,00266851575 | 0,3676954247710 | 69235026,29034 | 6,4950389552623 | 9,9999667750399 |
| 4 | $10^4$ | 2,9999700695917 | 400,00266851581 | 0,3678610464329 | 69235026,29036 | 6,4950385228968 | 9,9999661093612 |
| 5 | $10^5$ | 2,9999708701507 | 400,00266851649 | 0,3678776017666 | 69235026,29059 | 6,4950356404560 | 9,9999616715177 |
| 6 | $10^6$ | 2,9999825792620 | 400,00266852517 | 0,3678792572316 | 69235026,29360 | 6,4949779917004 | 9,9998729144571 |
| 7 | $10^7$ | 3,0001320510055 | 400,00266865907 | 0,3678794227775 | 69235026,33995 | 6,4955544754057 | 10,000760496920 |

| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ | $\delta a_N$ | $\delta \gamma^3 a_N$ |
|---|---|---|---|---|---|---|
| 1/2 | $-9{,}0018 \cdot 10^{-7}$ | $-8{,}1502 \cdot 10^{-10}$ | $1{,}7354 \cdot 10^{-2}$ | $-2{,}6439 \cdot 10^{-4}$ | $-6{,}4350 \cdot 10^{-5}$ | $-9{,}9075 \cdot 10^{-5}$ |
| 2/3 | $-8{,}1549 \cdot 10^{-8}$ | $-7{,}2987 \cdot 10^{-11}$ | $1{,}6631 \cdot 10^{-3}$ | $-2{,}5108 \cdot 10^{-5}$ | $-5{,}6207 \cdot 10^{-7}$ | $-8{,}6539 \cdot 10^{-7}$ |
| 3/4 | $7{,}1013 \cdot 10^{-8}$ | $6{,}4006 \cdot 10^{-11}$ | $1{,}6562 \cdot 10^{-4}$ | $2{,}2203 \cdot 10^{-5}$ | $-4{,}3237 \cdot 10^{-7}$ | $-6{,}6568 \cdot 10^{-7}$ |
| 4/5 | $8{,}0056 \cdot 10^{-7}$ | $6{,}7701 \cdot 10^{-10}$ | $1{,}6556 \cdot 10^{-5}$ | $2{,}3431 \cdot 10^{-4}$ | $-2{,}8824 \cdot 10^{-6}$ | $-4{,}4378 \cdot 10^{-6}$ |
| 5/6 | $1{,}1709 \cdot 10^{-5}$ | $8{,}6780 \cdot 10^{-9}$ | $1{,}6555 \cdot 10^{-6}$ | $3{,}0045 \cdot 10^{-3}$ | $-5{,}7649 \cdot 10^{-5}$ | $-8{,}8757 \cdot 10^{-5}$ |
| 6/7 | $1{,}4947 \cdot 10^{-4}$ | $1{,}3391 \cdot 10^{-7}$ | $1{,}6555 \cdot 10^{-7}$ | $4{,}6355 \cdot 10^{-2}$ | $5{,}7648 \cdot 10^{-4}$ | $8{,}8758 \cdot 10^{-4}$ |

Tab. 6.3: Values of $v_T$, $t_T$, $m_N$, $x_N$, $a_N$, $\gamma^3 a_N$ for proportional reduction of propellant mass.
Top: $v_0 = 0$, bottom: $v_0 = 0{,}5\,c$ (149.896,458 km/s).
$\varDelta m_0 = 0{,}25\%/\mathrm{s}$, $t_S = 400\mathrm{s}$. The values for $m_N$ are normalized to 1.
Values for $v_T$ in $\mathrm{km/s}$, $t_T$ in s, $x_N$ in km, $a_N$ and $\gamma^3 a_N$ in $\mathrm{m/s^2}$.

For the calculations a loss of propellant mass per time unit of $\Delta m_0 = 0{,}25\%/\mathrm{s}$ was specified. This leads to an acceleration of $10\mathrm{m/s}^2$ and thus a comparability with the other already performed calculations is given. The experimental time chosen was $t_S = 400\mathrm{s}$, and this leaves the realistic magnitude of a residual mass of almost 37% of the initial value after the completion of the experiment. For better evaluation, the deviations between the values $\delta v_T = v_T(K)$ and $v_T(K-1)$ are shown according to the relationships also used elsewhere (e.g., as defined in Eq. (6.79)), and in the same way for $\delta t_T$, $\delta m_N$, $\delta x_N$, $a_N$ and $\gamma^3 a_N$, where $K$ corresponds here in each case to a potency of ten in the number of calculation steps between 10 and $10^7$ (cf. Tab. 6.3). First, it should be noted in principle that the values for $\delta v_T$, $\delta t_T$ and $\delta x_N$ show unsystematic fluctuations and exhibit the smallest deviations from each other considering the number of iteration steps between $\mathrm{N} = 10^2$ and $10^4$. Hereby it is clear that the visible differences are not caused by a physically explainable effect, but only by the use of the numerical method.

Furthermore, it can be seen that the value of the remaining mass $m_N$ becomes more accurate with each increase by a factor of 10 in the number of iteration steps (Iteration $10^3 \rightarrow 10^4 = 1.6562 \cdot 10^{-4}$; $10^4 \rightarrow 10^5 = 1.6566 \cdot 10^{-5}$ and so on, see Tab. 6.3). This is not of further importance here and therefore an evaluation is not carried out at this point; however, this changes in the following considerations for the case of constant propellant mass and will be further investigated there.

The results of the calculations for $\gamma^3 a_N$ show again that the ratio for the accelerations between differently moving observers reveals the factor $\gamma^3$.

The determination made here with a proportional loss of propellant mass with respect to the residual mass of the rocket allows a direct comparison with the analytical and numerical results from Section 6.4.1. and the conformity proves to be very good. A detailed evaluation is presented in Annex B.4.

### *b) Propellant mass constant*

This case proves to be significantly more complex with regard to the evaluation compared to the situation discussed before. This is due to the fact that the values of $v_T$, $t_T$ and $x_N$, which are important for the observation, show the same behavior as $m_N$ before and become more precise with increasing number of iteration steps. Therefore, they must be analyzed in particular (in contrast to the case before, $m_N$ does not show this behavior here!).

This becomes clear when considering the case shown in Tab. 6.4. In the upper part of the table, as before, the results of the calculations of the relevant values are given, below – marked with section I – the compilation of the deviations $\delta v_T$, $\delta t_T$, $\delta m_N$ und $\delta x_N$ follows. The first and the last calculation deviate in values from the systematics of the other results and were not considered further. Therefore, only the blue colored fields were used for final calculations and the values reproduced in section II were extrapolated from them. The results presented in the lower part of the table show the outcome of these calculations. The mass reduction was set to $\Delta m_0 = 0{,}5\%/\mathrm{s}$, which leads to a test duration of $t_0 = 100\mathrm{s}$ for the final mass value of 50% desired here.

In the Annex C, besides the derivation of the program structure, further results of the calculations for different boundary conditions were presented in the tables C.2, C.3 and C.4.

In addition to the figures for the system velocity of $v_0 = 0$ discussed here, calculated values for 369 km/s plus 2,000 km/s and 10,000 km/s were also added to provide a better overview. In these cases, a lower remaining mass after the test was also determined with a rest of 10%.

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ | $\Delta t_o$ |
|---|---|---|---|---|---|
| 10 | 2,67508561278727 | 100,000397329364 | 0,500000000000000 | 119,116010675216 | 10 |
| $10^2$ | 2,76261372200990 | 100,000408141269 | 0,500000000000000 | 122,357320955608 | 1 |
| $10^3$ | 2,77158897232187 | 100,000409292747 | 0,500000000000055 | 122,702523336750 | $10^{-1}$ |
| $10^4$ | 2,77248872482278 | 100,000409408634 | 0,500000000000055 | 122,737265091767 | $10^{-2}$ |
| $10^5$ | 2,77257872237194 | 100,000409420246 | 0,499999999996724 | 122,740741494222 | $10^{-3}$ |
| $10^6$ | 2,77258772224753 | 100,000409422400 | 0,500000000041133 | 122,741089155569 | $10^{-4}$ |
| $10^7$ | 2,77258862465211 | 100,000409440862 | 0,499999999708066 | 122,741124020357 | $10^{-5}$ |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ | |
| $\bar{x}$ | 8,9931 | $1,4064 \cdot 10^{-3}$ | | $3,4698 \cdot 10^{2}$ | |
| $10^2$ | $8,7528 \cdot 10^{-2}$ | $1,0812 \cdot 10^{-5}$ | 0 | 3,2413 | I |
| $10^3$ | $8,9753 \cdot 10^{-3}$ | $1,1515 \cdot 10^{-6}$ | $5,4956 \cdot 10^{-14}$ | $3,4520 \cdot 10^{-1}$ | |
| $10^4$ | $8,9975 \cdot 10^{-4}$ | $1,1589 \cdot 10^{-7}$ | 0 | $3,4742 \cdot 10^{-2}$ | |
| $10^5$ | $8,9998 \cdot 10^{-5}$ | $1,1612 \cdot 10^{-8}$ | $-3,3309 \cdot 10^{-12}$ | $3,4764 \cdot 10^{-3}$ | |
| $10^6$ | $8,9998 \cdot 10^{-6}$ | $2,1540 \cdot 10^{-9}$ | $4,4409 \cdot 10^{-11}$ | $3,4766 \cdot 10^{-4}$ | |
| $10^7$ | $9,0240 \cdot 10^{-7}$ | $1,8462 \cdot 10^{-8}$ | $-3,3307 \cdot 10^{-10}$ | $3,4865 \cdot 10^{-5}$ | |
| $10^8$ | $8,9931 \cdot 10^{-8}$ | $1,4069 \cdot 10^{-11}$ | | $3,4698 \cdot 10^{-6}$ | |
| $10^9$ | $8,9931 \cdot 10^{-9}$ | $1,4069 \cdot 10^{-12}$ | | $3,4698 \cdot 10^{-7}$ | |
| $10^{10}$ | $8,9931 \cdot 10^{-10}$ | $1,4211 \cdot 10^{-13}$ | | $3,4698 \cdot 10^{-8}$ | |
| $10^{11}$ | $8,9931 \cdot 10^{-11}$ | 0 | | $3,4698 \cdot 10^{-9}$ | |
| $10^{12}$ | $8,9933 \cdot 10^{-12}$ | 0 | | $3,4699 \cdot 10^{-10}$ | |
| $10^{13}$ | $8,9928 \cdot 10^{-13}$ | 0 | | $3,4703 \cdot 10^{-11}$ | |
| $10^{14}$ | $9,0150 \cdot 10^{-14}$ | 0 | | $3,4674 \cdot 10^{-12}$ | II |
| $10^{15}$ | $8,8818 \cdot 10^{-15}$ | 0 | | $3,4106 \cdot 10^{-13}$ | |
| $10^{16}$ | 0 | 0 | | 0 | |
| | $v_T$ | $t_T$ | | $x_N$ | |
| $10^7$ | 2,77258862155768 | 100,000409422541 | | 122,741123853607 | |
| $10^8$ | 2,77258871148869 | 100,000409422555 | | 122,741127323411 | |
| $10^9$ | 2,77258872048179 | 100,000409422556 | | 122,741127670391 | |
| $10^{10}$ | 2,77258872138110 | 100,000409422556 | | 122,741127705089 | |
| $10^{11}$ | 2,77258872147103 | 100,000409422556 | | 122,741127708559 | |
| $10^{12}$ | 2,77258872148003 | 100,000409422556 | | 122,741127708906 | |
| $10^{13}$ | 2,77258872148093 | 100,000409422556 | | 122,741127708941 | |
| $10^{14}$ | 2,77258872148102 | 100,000409422556 | | 122,741127708944 | |
| $10^{15}$ | 2,77258872148102 | 100,000409422556 | | 122,741127708945 | |
| $10^{16}$ | 2,77258872148102 | 100,000409422556 | | 122,741127708945 | |

Tab. 6.4: Values of $v_T$, $t_T$, $m_N$ and $x_N$ for linear reduction of propellant mass. Section I: Iterations, Section II: Extrapolated. All values in km and s. Calc.-Type: "A1", $v_0' = -4$ km/s, $\Delta m_0 = 0{,}5\%/s$, $t_0 = 100$s, $v_0 = 0$

Again, the most important statement results from the comparison of the calculated values for $t_T$, which represent the signal propagation times until reaching an observer moving with $v_0$, calculated in view of the system at rest. For a better comparison of the times, here as in other cases, the comparative formula

$$\delta t_T = \frac{t_T(v_K)}{t_T(v_{K-1})} - 1 \tag{6.86}$$

was chosen. Table 6.5 shows the results of values for $t_T$ and $\delta t_T$, where the calculation was based on $t_T$ using iteration steps of $\mathrm{N} = 10^{16}$. No systematic deviations can be found when results for different system velocities are compared.

| | 1 | | 2 | | 3 | |
|---|---|---|---|---|---|---|
| $v_0$ | $t_T$ | $\delta t_T$ | $t_T$ | $\delta t_T$ | $t_T$ | $\delta t_T$ |
| 0 | 100,000409422556 | | 1.000,00992905474 | | 10.002,4827416511 | |
| 369 | 100,000409421505 | $1{,}05 \cdot 10^{-11}$ | 1.000,00992904501 | $9{,}73 \cdot 10^{-12}$ | 10.002,4827411418 | $5{,}09 \cdot 10^{-11}$ |
| 2000 | 100,000409421509 | $-3{,}40 \cdot 10^{-14}$ | 1.000,00992904483 | $1{,}76 \cdot 10^{-13}$ | 10.002,4827388902 | $2{,}25 \cdot 10^{-10}$ |
| 10000 | 100,000409421471 | $3{,}74 \cdot 10^{-13}$ | 1.000,00992904385 | $9{,}82 \cdot 10^{-13}$ | 10.002,4827278462 | $1{,}10 \cdot 10^{-9}$ |

Tab. 6.5: $t_T$ and $\delta t_T$ with constant propellant mass per time unit for different $v_0$.
$\Delta m_0$ is normalized to 1.
1: $v_0' = -4\ \mathrm{km/s}$, $\Delta m_0 = 0{,}5\%/s$, $t_0 = 100s$
2: $v_0' = -4\ \mathrm{km/s}$, $\Delta m_0 = 0{,}09\%/s$, $t_0 = 1.000s$
3: $v_0' = -100\ \mathrm{km/s}$, $\Delta m_0 = 0{,}009\%/s$, $t_0 = 10.000s$

For the consideration of the final velocity $v_T$ the possibility of a comparison with the values determined according to the classical rocket formula arises. The formula derived by K. E. Tsiolkovsky in 1903 is based on the non-relativistic momentum equation and aims to calculate the terminal velocity of a rocket as a function of the exit velocity of the gas for a constant propellant mass. For non-relativistic consideration with $v \ll c$, first Eq. (6.85) is reduced to

$$v_K' = v_K + v_0' \tag{6.87}$$

To solve the equation Eq. (6.84), the stipulation that $\gamma = 1$ (not relativistic) applies. Since the mass of the rocket decreases with increasing index $K$, but the velocity rises, the following relations apply additionally

$$m_K = m_{K-1} - \Delta m_{K-1} \qquad\qquad v_K = v_{K-1} + \Delta v_{K-1}$$

In addition, for differential consideration the following definitions are introduced:

$$m_K \to m \qquad\qquad \Delta m \to \mathrm{d}m$$

$$v_K \to v \qquad\qquad \Delta v \to \mathrm{d}v$$

This results in the following approach for Eq. (6.84):

$$(m + \mathrm{d}m - \mathrm{d}m)v + \mathrm{d}m(v + v_0') = (m + \mathrm{d}m)(v - \mathrm{d}v) \tag{6.88}$$

$$mv + v\mathrm{d}m + v_0'\mathrm{d}m = mv - m\mathrm{d}v + v\mathrm{d}m - \mathrm{d}m\mathrm{d}v \tag{6.89}$$

and because of $\mathrm{d}m\mathrm{d}v \to 0$

$$m\mathrm{d}v + v_0'\mathrm{d}m = 0 \tag{6.90}$$

If mass and velocity of the outflowing gas (and thus the momentum) are kept constant, the integration of eq. (6.90) leads to the classical rocket formula

$$\int_0^v \mathrm{d}v = -\, v_0' \int_{m_0}^{m} \frac{\mathrm{d}m}{m} \tag{6.91}$$

$$v = v_0' \, ln\left(\frac{m_0}{m}\right) \tag{6.92}$$

where $m_0$ is the mass at the start from an unmoved platform. If the starting point is moving, the velocities are simply added. This becomes necessary e.g. at the drop of a rocket stage, when the mass decreases and also the momentum changes.

Besides the classical rocket formula according to Tsiolkovsky, also a relativistic rocket formula exists. This was derived in 1946 by J. Akeret [90]. The derivation is clearly more complex and requires additionally the use of the energy conservation theorem; the derivation is shown in the appendix C under point C.4. The result of this relativistic rocket equation according to Eq. (C.33) is

$$\frac{v}{c} = \frac{1 - \left(\frac{m}{m_0}\right)^{2v_0'/c}}{1 + \left(\frac{m}{m_0}\right)^{2v_0'/c}} \tag{6.93}$$

If the classical and/or the relativistic rocket equations $v_R$ are taken as a limiting case to the presented solution of the numerically derived relativistic rocket formulas, and the results from the values for $v_T$ calculated in appendix C, tables C2, C3 and C4 are related to each of them, the following values for a comparison can be obtained

$$\delta_R = \frac{v_R}{v_T} - 1 \tag{6.93}$$

The results of these calculations are shown in Fig. 6.4. First, it becomes clear that for low system velocities, especially in the case $v_0 = 0$, no sufficient accuracy is achieved for iteration steps from $\mathrm{N} = 10$ to $\mathrm{N} = 10^7$ and they are therefore to be considered only with restrictions. On the other hand, if the extrapolated values calculated up to $\mathrm{N} = 10^{16}$ are added, a significantly improved result is obtained. When the values for classical and relativistic rocket formulas are compared, no differences can be found for $v_0' = -\,4$ km/s, while for $v_0' = -100$ km/s, discrepancies can be seen for small system velocities ($v_0 = 0$ und 369 km/s). To show the differences, the results for the classical rocket formula (Tsiolkovsky) and relativistic (Akeret) were presented separately in subplots c) and d).

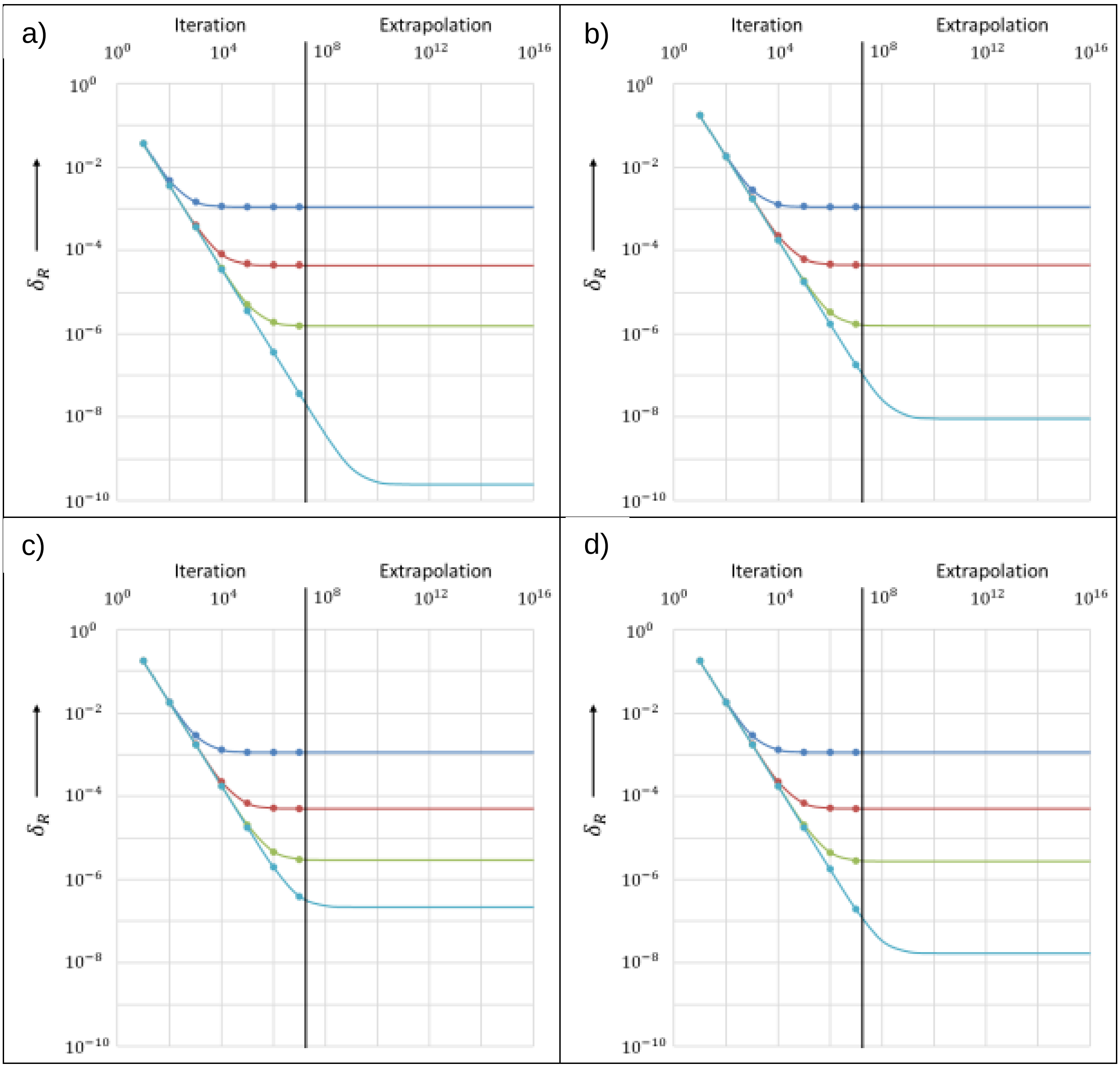



Fig. 6.4: Dependency $\delta_R$ between relativistic und classical rocket formula related to the number of iteration steps acc. to Tab. C.2, C3 and C4.
a) $v'_0 = -4\ \text{km/s},\ \Delta m_0 = 0{,}5\%/\text{s},\ t_0 = 100\text{s}$
b) $v'_0 = -4\ \text{km/s},\ \Delta m_0 = 0{,}09\%/\text{s},\ t_0 = 1.000\text{s}$
c) and d) $v'_0 = -100\ \text{km/s},\ \Delta m_0 = 0{,}009\%/\text{s},\ t_0 = 10.000\text{s}$
c) classic (acc. to K. E. Tsiolkowski), d) relativistic (acc. to J. Akeret).
a) to d) at the bottom $v_0 = 0$ then ascending $v_0 = 369,\ 2000, 10000\ \text{km/s}$
$\Delta m_0$ normalized to 1.

To evaluate the behavior at higher velocities, results from the numerical rocket equations are compared with corresponding values from the classical and relativistic rocket formulas. In Tab. 6.6, the calculated values of the final velocity are entered for the parameters $v'_0/c$ (gas velocity of a rocket in relation to the speed of light) and for the ratio of the masses at the final stage compared to the start.

An evaluation shows that up to a velocity of the propellant gas of 0.01c, there are no major differences between the calculations. At 0.1c the differences between the classical rocket formula and the other two variants already become clear and at 0.5c the speed of light is exceeded according to the classical nonrelativistic method at a mass release of approx. 90%. The values according to J. Akeret and those of the own numerical calculation, which of course remain below the speed of light, hardly differ.

| $m/m_0$ \ $v_0'/c$ | A | B | C | A | B | C | A | B | C |
|---|---|---|---|---|---|---|---|---|---|
| | 0,01 | 0,01 | 0,01 | 0,1 | 0,1 | 0,1 | 0,5 | 0,5 | 0,5 |
| 0,5 | 0,006931 | 0,006931 | 0,006932 | 0,069315 | 0,069204 | 0,069432 | 0,346574 | 0,333333 | 0,362675 |
| 0,2 | 0,016094 | 0,016093 | 0,016093 | 0,160944 | 0,159568 | 0,159727 | 0,804719 | 0,666667 | 0,681958 |
| 0,1 | 0,023026 | 0,023022 | 0,023022 | 0,230259 | 0,226274 | 0,226122 | 1,151293 | 0,818182 | 0,818378 |
| 0,01 | 0,046052 | 0,046019 | 0,046021 | 0,460517 | 0,430506 | 0,428238 | 2,302585 | 0,980198 | 0,973447 |
| 0,001 | 0,069078 | 0,068968 | 0,069009 | 0,690776 | 0,598480 | 0,593888 | 3,453878 | 0,998002 | 0,996217 |

Tab. 6.6: End velocity of a rocket (values in relation to the speed of light) depending on the calculation method
Parameter top: Values for propellant gas (values in relation to the speed of light)
Parameter left: Ratio of final mass to the mass at the start
A: Classical, acc. to K. E. Tsiolkowski
B: Relativistic, acc. to J. Akeret
C. Numerical, calculation acc. to annex C ( $\Delta m_0 = 10^{-5}$ %/s, $\Delta t_S = 100$s)

The essential difference between analytical and numerical calculation is that for the analytical method no output quantity of the gas *per time unit* must be given and that therefore the result is independent of the acceleration occurring during a rocket launch. Therefore, there is also no information about which distance the rocket has covered in which time. This means, only the data determined according to the described numerical method can be used for the previously performed calculations; the analytical rocket formula does not provide the necessary information.

To illustrate this, results for gas ejection velocities of $v_0' = -0{,}5$c and $v_0' = -100$ km/s are presented below. In Tab. 6.7, gas ejection rates of $\Delta m_0 = 10^{-7}$ to $10^{-4}$/s (corresponding to $10^{-5}$ and $10^{-2}$ %/s) were selected for the numerical determination and the values of $v_T, t_T, x_K$ and $a_K$ were calculated on these. First, it should be noted that in all cases the final velocity $v_T$ remains constant for the respective gas exit velocity. When the gas ejection rate (per time unit) is increased by a factor of ten, the values for the total duration of the experiment $t_T$ as well as the distance traveled $x_K$ increase by the same factor. The acceleration $a_K$, on the other hand, decreases by the same amount.

Finally, an essential difference between the numerical method and the relativistic rocket formula must be pointed out. While the latter was derived using the law of conservation of energy, the numerical method (as well as the classical rocket formula according to Tsiolkovsky) is based exclusively on the law of conservation of momentum. For the calculation, this means that the momentum of the propulsion gas could in theory be increased unlimited by approaching the speed of light more and more, and thus extremely high rocket velocities could be achieved connected with a low mass output. However, in reality this is not possible, because for the acceleration of the propellant gas considerable amounts of energy (and thus because of $E = mc^2$ additional mass losses) would be needed, which are not considered in the calculation. For these extreme values, therefore, the numerical method presented cannot be used.

| | $v_0' = -0{,}1c$ | | | | $v_0' = -100\text{km/s}$ | | | |
|---|---|---|---|---|---|---|---|---|
| $\Delta m_0$ | $v_T$ | $t_T$ | $x_K$ | $a_K$ | $v_T$ | $t_T$ | $x_K$ | $a_K$ |
| $10^{-7}$ | 67789,6421 | 9713871,60 | 2017951968 | 27,5336472 | 230,263962 | 900233,452 | 669757541 | 0,0999999043 |
| $10^{-6}$ | 67789,6421 | 971387,160 | 2017951968 | 275,336472 | 230,263962 | 900233,452 | 66975754,1 | 0,9999990426 |
| $10^{-5}$ | 67789,6421 | 97138,7160 | 2017951968 | 2753,36472 | 230,263962 | 90023,3452 | 6697575,41 | 9,9999904265 |
| $10^{-4}$ | 67789,6421 | 9713,87160 | 201795196,8 | 27533,6472 | 230,263962 | 9002,33452 | 669757,541 | 99,999904265 |

Tab. 6.7: End velocity $v_T$, total time $t_T$, covered distance $x_K$ and acceleration $a_K$ as a function of the gas ejection velocity and the gas quantity $\Delta m_0$ (per time unit). $v_T$ in $\text{km/s}$ , $t_T$ in s, $x_K$ in km, $a_K$ in $\text{m/s}^2$, $\Delta m_0$ in $1/s$ (normalized to 1)

The problem of determining the energy requirement for rocket propulsion systems has been discussed for a long time and can be solved by defining various loss factors. As an example, the representation used by U. Walter [91] is given in Fig. 6.5.

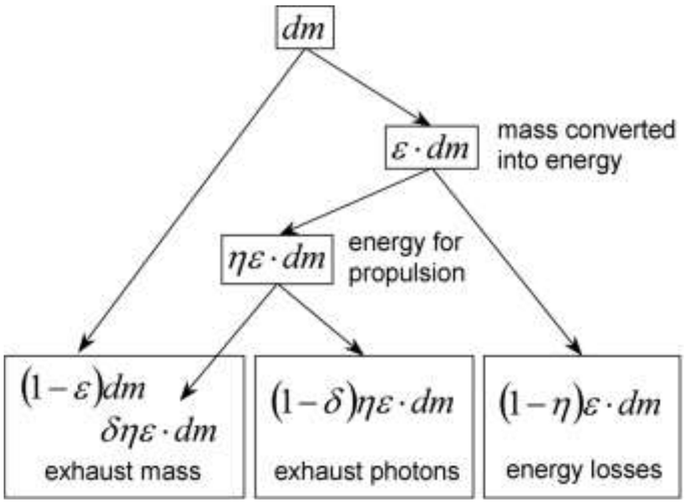



Fig. 6.5: Energy scheme for a relativistic rocket with energy losses and expelled propulsion mass and photons (extracted from [91])

Further information on this topic can be found in the following literature [91,92].

# 7. Non-elastic processes

The situation concerning the elastic behavior during collisions was already discussed at length in chapter 6. The analysis of non-elastic processes is also of great importance for further considerations and shall now be examined in detail. At first the non-elastic collision will be scrutinized, where during the experimental situation two or more bodies are combined and an energy-absorption takes place. The reversing effect is observed during particle disintegration; in this case kinetic energy is set free because of conversion of mass into energy and carried away by the decay products. Non-elastic collision and particle disintegration can thus be interpreted as complementary processes.

## 7.1 Relativistic non-elastic collision

For the relativistic consideration of non-elastic collisions, the situation of observers with different velocities will be examined. For that purpose, a simple example shall be looked at and, after exact evaluation, the consequences derived will be discussed. The experimental conditions are as follows:

Two bodies are approaching each other and combine after axial contact, which means ideal plastic behavior is assumed. The collision shall be completely central and so no rotation will appear. In this case it is not necessary to use a vectorial calculation and the following calculation for the momentum can be used

$$p_3 = p_1 + p_2 = m_1\gamma_1 v_1 + m_2\gamma_2 v_2 = m_3\gamma_3 v_3 \qquad (7.01)$$

where $v_1$ and $v_2$ are the velocities before and $v_3$ after the collision, the same definition is valid for the masses $m_1$, $m_2$ and $m_3$. If it is assumed that mass $m_3$ is at rest after the collision, then the values of $p_1$ and $p_2$ will neutralize each other because the conservation-principle of momentum must be respected. This means, that the absolute values of $p_1$ and $p_2$ are equal but the algebraic sign is different and so the total momentum after collision $p_3$ is zero.

However, the kinetic energy before and after the collision is not equal. This becomes clear when the equation of kinetic energy before the collision is considered (see also explanations in chapter 6.1)

$$E_{kin} = (\gamma_1 - 1)m_1c^2 + (\gamma_2 - 1)m_2c^2 \qquad (7.02)$$

When again the situation is considered that mass $m_3$ is at rest after collision, then the kinetic energy is zero ether. Because kinetic energy is a scalar and not a vector like it is the case for momentum, it is compulsory that it must be transformed into another form. Otherwise, the conservation principle of energy would be violated. When it is assumed in this case that kinetic energy is transformed completely into mass the following equation is valid

$$\Delta m_3 = (\gamma_1 - 1)m_1 + (\gamma_2 - 1)m_2 \tag{7.03}$$

where $\Delta m_3$ is the increase of mass according to the transformation of kinetic energy.

To examine the situation an experiment with two different cases will be looked at, where in one instance an observer will be at rest and in another case moving. For simplification of the calculations, it is assumed that the masses of the bodies involved are equal, i.e. $m_1 = m_2 = m$. The cases will be marked with A and B; this identification will be continuously used for the relevant situations as index for the parameters depending on the velocities. It will be presumed in the first instance that the simple relation $m_3 = m_1 + m_2$ is valid. However, during the following considerations it will become clear that this assumption is leading to discrepancies, and it will be proven that Eq. (7.03) is valid in any case without restrictions.

A: Referring to an observer A at absolute rest the velocity is $v_{3A} = 0$

Because $m_1 = m_2$ was presumed this stand for the fact, that before the collision the two bodies are moving with equal speed but different directions, this means that beside $v_{3A} = 0$ also $v_{1A} = -v_{2A}$ is valid.

B: Referring to an observer B at absolute rest the velocity is $v_{1B} = 0$

All calculations refer to $v_{1B} = 0$.

The following relations apply:

| | Observer A<br>$v_{3A} = 0 \quad v_{1A} = -v_{2A}$ | Observer B<br>$v_{1B} = 0$ |
|---|---|---|
| Momentum before collision | $p_{1A} = m\gamma_{1A}v_{1A}$<br>$p_{2A} = -m\gamma_{1A}v_{1A}$ | $p_{1B} = 0$<br>$p_{2B} = m\gamma_{2B}v_{2B}$ |
| Momentum after collision | $p_{3A} = 0$ | $p_{3B} = 2m\gamma_{3B}v_{3B}$ |
| Kinetic energy before collision | $\frac{E_{1A}}{c^2} = (\gamma_{1A} - 1)m$<br>$\frac{E_{2A}}{c^2} = (\gamma_{1A} - 1)m$ | $\frac{E_{1B}}{c^2} = 0$<br>$\frac{E_{2B}}{c^2} = (\gamma_{2B} - 1)m$ |
| Kinetic energy after collision | $\frac{E_{3A}}{c^2} = 0$ | $\frac{E_{3B}}{c^2} = 2(\gamma_{3B} - 1)m$ |

In the presented table the results for momentum and kinetic energy are presented which apply for *identical experimental conditions* in view of the observers A and B. These will be discussed further in the next chapters using the relativistic addition of velocities for comparison.

### 7.1.1 Results based on relativistic addition of velocities

For observer A the simple case $v_{1\mathrm{A}} = -v_{2\mathrm{A}}$ is valid. The calculation of the velocity for observer B makes is necessary to use the relativistic addition of velocities, which was already described in chapter 4.1. Because of symmetry reasons the relation $v_{3\mathrm{B}} = v_{1\mathrm{A}}$ applies and this is leading to

$$v_{2\mathrm{B}} = \frac{2v_{1\mathrm{A}}}{1 + \left(\frac{v_{1\mathrm{A}}}{c}\right)^2} \tag{7.04}$$

Example:

| | | | |
|---|---|---|---|
| **Observer A** | $v_{1\mathrm{A}} = 0{,}5c$ | $v_{2\mathrm{A}} = -0{,}5c$ | $v_{3\mathrm{A}} = 0$ |
| **Observer B** | $v_{1\mathrm{B}} = 0$ | $v_{2\mathrm{B}} = 0{,}8c$ | $v_{3\mathrm{B}} = 0{,}5c$ |

### 7.1.2 Results based on relations for momentum

Observer A is considering the total value of the momentum before and after the collision as zero because of the relation $v_{1\mathrm{A}} = -v_{2\mathrm{A}}$ and thus

$$p_{3\mathrm{A}} = p_{1\mathrm{A}} + p_{2\mathrm{A}} = m\gamma_{1\mathrm{A}}v_{1\mathrm{A}} - m\gamma_{1\mathrm{A}}v_{1\mathrm{A}} = 0 \tag{7.05}$$

Observer B finds the following relations:

$$p_{1\mathrm{B}} = 0 \tag{7.06}$$

$$p_{2\mathrm{B}} = m\gamma_{2\mathrm{B}}v_{2\mathrm{B}} \tag{7.07}$$

$$p_{3\mathrm{B}} = 2m\gamma_{3\mathrm{B}}v_{3\mathrm{B}} \tag{7.08}$$

Because of the conservation principle of momentum, the values for $p_{2B}$ and $p_{3B}$ according to (7.01) must be equal, so

$$\gamma_{2\mathrm{B}}v_{2\mathrm{B}} = 2\gamma_{3\mathrm{B}}v_{3\mathrm{B}} \tag{7.09}$$

This equation allows the calculation of $v_{3\mathrm{B}}$ depending on $v_{2\mathrm{B}}$.

Because of the structure of the equation an analytical solution is not possible and so a numerical solution must be used. In annex D different approaches are presented; here the use of simple recursion, a procedure according to Newton and the bisection method were chosen to effectuate a solution. In all cases the results for $v_{3\mathrm{B}}$ were calculated using different values for $v_{2\mathrm{B}}$.

As expected, all iteration methods lead to the same values; the procedures using simple recursion and according to Newton share the advantage, that they converge very quickly for small values of $v/c$. However, as a drawback the convergence is reducing for increasing

$v/c$ and the use is no longer possible when extremely high values are taken. Increasing to values higher than $v/c > 0.9c$ the bisection method is the only procedure which is still working.

Example:

| | | | |
|---|---|---|---|
| Observer A | $v_{1A} = 0{,}5c$ | $v_{2A} = -0{,}5c$ | $v_{3A} = 0$ |
| Observer B | $v_{1B} = 0$ | $v_{2B} = 0{,}8c$ | $v_{3B} = 0{,}5547c$ |

### 7.1.3 Results based on relations for energy

Observer A will consider the case that the kinetic energy of the colliding masses will be transformed completely into another form of energy (e.g. heat). This loss of energy has the value of

$$\frac{E_{1A} + E_{2A}}{c^2} = 2m(\gamma_{1A} - 1) \tag{7.10}$$

For observer B this is implicating that the difference between the kinetic energy before and after the collision is balanced and thus

$$2m(\gamma_{3B} - 1) = m(\gamma_{2B} - 1) - 2m(\gamma_{1A} - 1) \tag{7.11}$$

$$\gamma_{3B} = \frac{\gamma_{2B} - 2\gamma_{1A} + 3}{2} \tag{7.12}$$

This equation shows a simple analytical solution using

$$\frac{v_{3B}}{c} = \pm\sqrt{1 - \frac{1}{\gamma_{3B}^2}} = \pm\sqrt{1 - \frac{4}{(\gamma_{2B} - 2\gamma_{1A} + 3)^2}} \tag{7.13}$$

Example:

| | | | |
|---|---|---|---|
| Observer A | $v_{1A} = 0{,}5c$ | $v_{2A} = -0{,}5c$ | $v_{3A} = 0$ |
| Observer B | $v_{1B} = 0$ | $v_{2B} = 0{,}8c$ | $v_{3B} = 0{,}5293c$ |

(Negative results of the square root are not relevant because of plausibility reasons.)

### 7.1.4 Evaluation of the results

In Fig. 7.1 the deviations between the velocities according to the different calculations are presented.

Here the following definitions apply:

$$\delta = \frac{v_{3B} - v_{1A}}{v_{1A}} \tag{7.14}$$

where $\delta_p$ is the percental difference for the momentum (chapter 7.1.2) and $\delta_E$ for the energy (chapter. 7.1.3). It is clear at first sight that the height and also the position of the maxima are not sharing any similarities.

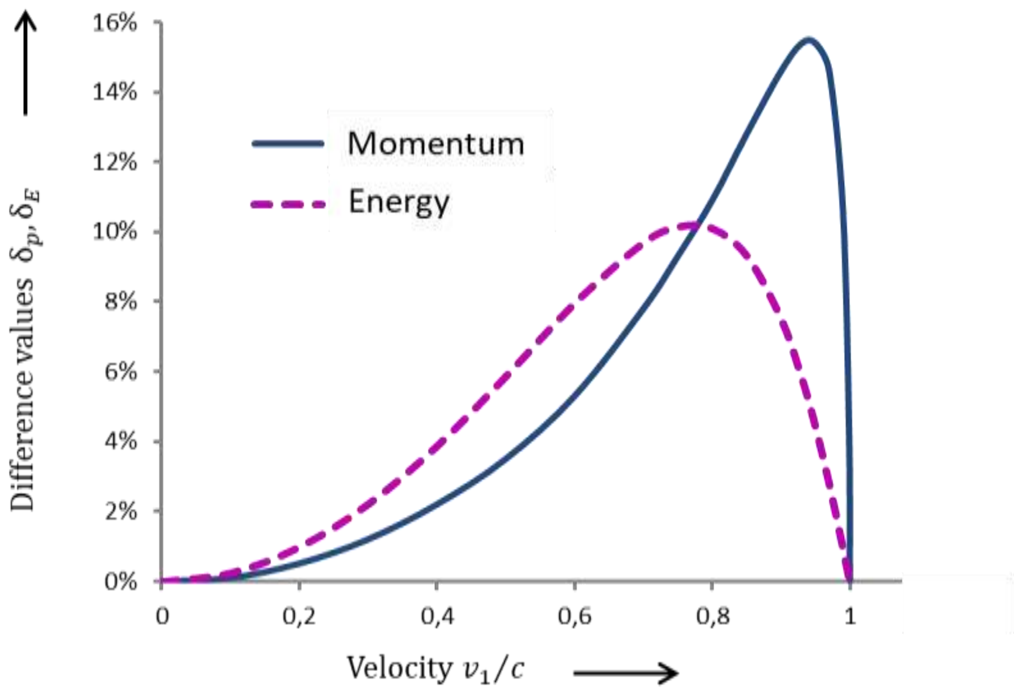



Fig. 7.1: Difference values $\delta_p$ and $\delta_E$ depending on $v_1$

It is obvious that for the non-elastic collision the consideration of the relations for relativistic addition of velocities and the conservation laws for momentum and energy using these calculations are leading to completely different results. This means that in these cases severe discrepancies would occur between the relativistic principles of identity and equivalence (for definition of the principles see chapter 1.6).

Up to now the velocity $v_{3B}$ of the two combined masses was calculated based on the validity of the laws of momentum and energy without any further correction. To find a solution for the observed problems, in the following the attempt is made to examine the effect on momentum and energy which occurs, when the relativistic addition of velocities is supposed to be valid without further discussion. To realize this, the correction values $C_p$ for the momentum and $C_E$ for the energy are defined and used in the relevant relations.

<u>a) Momentum</u>

Equation Eq. (7.09) is modified to

$$C_p \cdot 2\gamma_{3B} v_{3B} = \gamma_{2B} v_{2B} \qquad (7.15)$$

using the relation $v_{3B} = v_{1A}$ (see chapter 7.1.1)

$$C_p = \frac{\sqrt{1 - \left(\frac{v_{1A}}{c}\right)^2}}{2v_{1A}} \frac{v_{2B}}{\sqrt{1 - \left(\frac{v_{2B}}{c}\right)^2}} \qquad (7.16)$$

Because of Eq. (7.04) is

$$C_p = \frac{\sqrt{1-\left(\frac{v_{1A}}{c}\right)^2}}{2v_{1A}} \frac{\frac{2v_{1A}}{1+\left(\frac{v_{1A}}{c}\right)^2}}{\sqrt{1-\left(\frac{\frac{2v_{1A}}{c}}{1+\left(\frac{v_{1A}}{c}\right)^2}\right)^2}}$$

$$= \sqrt{\frac{1-\left(\frac{v_{1A}}{c}\right)^2}{\left[1-\left(\frac{v_{1A}}{c}\right)^2\right]^2}} = \sqrt{\frac{1}{1-\left(\frac{v_{1A}}{c}\right)^2}} = \gamma_{1A} \qquad (7.17)$$

This means that using unrestricted application of the relativistic addition of velocities the momentum is smaller by the factor $\gamma_{1A}$ than required by the law of conservation of momentum.

## b) Energy

Equation Eq. (7.11) is modified to

$$C_E \cdot 2(\gamma_{3B}-1) = (\gamma_{2B}-1) - 2(\gamma_{1A}-1) \qquad (7.20)$$

With $v_{3B} = v_{1A}$ applies

$$C_E = \frac{(\gamma_{2B}-1)}{2(\gamma_{1A}-1)} - 1 \qquad (7.21)$$

To develop a simple solution, first the term $\gamma_{2B}-1$ is considered. This can be transformed using Eq. (7.04) to

$$\gamma_{2B} - 1 = \frac{1}{\sqrt{1-\left(\frac{\frac{2v_{1A}}{c}}{1+\left(\frac{v_{1A}}{c}\right)^2}\right)^2}} - 1 \qquad (7.22)$$

and

$$\gamma_{2B} - 1 = \pm\frac{1+\left(\frac{v_{1A}}{c}\right)^2}{1-\left(\frac{v_{1A}}{c}\right)^2} - 1 = 2(\gamma_{1A}^2 - 1) \qquad (7.23)$$

For this calculation it was decided to take only positive values for the results of the square root, because negative values would lead to negative $\gamma_{2B}$ and physical interpretation makes no sense in this case.

The result is inserted in Eq. (7.21)

$$C_E = \frac{2(\gamma_{1A}^2-1)}{2(\gamma_{1A}-1)} - 1 \qquad (7.24)$$

$$C_E = \frac{(\gamma_{1A}+1)(\gamma_{1A}-1)}{(\gamma_{1A}-1)} - 1 = \gamma_{1A} \qquad (7.25)$$

This calculation is leading to the same result as already obtained for the momentum.

### 7.1.5 Final approach for calculation

For final evaluation, the findings developed so far shall be summarized and reviewed first. When in case of nonelastic collision examinations concerning the conservation laws of momentum and energy with invariant mass (this means $m_3 = m_1 + m_2;\ \Delta m_3 = 0$) before and after collision are conducted, then it becomes clear that the gained results for the velocity $v_3$ are different to each other; further the calculated value using the equation of relativistic addition of velocities come to another different result. The values of $v_3$ for the combined body using conservation laws are both higher than the calculated result derived by relativistic addition.

This would mean that the concept of simple addition of mass before and after collision is no option because the basic principles concerning conservation of energy and momentum are violated. If the approach presented in Eq. (7.01) of complete conversion of kinetic energy into mass is used instead, then considering the special case $m_1 = m_2 = m$

$$\Delta m_3 = \frac{E_{1\mathrm{A}} + E_{2\mathrm{A}}}{c^2} = 2m(\gamma_{1\mathrm{A}} - 1) \tag{7.30}$$

is valid for the generated mass $\Delta m_3$ by energy conversion (see also Eq. (7.04). For momentum, the relation Eq. (7.07) remains unchanged *before* collision

$$p_{2\mathrm{B}} = m\gamma_{2\mathrm{B}}v_{2\mathrm{B}} \tag{7.07}$$

but Eq. (7.09) *after* collision is developing to

$$p_{3\mathrm{B}} = 2m\gamma_{3\mathrm{B}}v_{3\mathrm{B}} \quad \Rightarrow \quad p_{3\mathrm{B}} = m_3\gamma_{3\mathrm{B}}v_{3\mathrm{B}} \tag{7.31}$$

Because of $v_{1\mathrm{A}} = v_{3\mathrm{B}}$ derived from relativistic addition of velocities this leads to

$$p_{3\mathrm{B}} = [2m(\gamma_{1\mathrm{A}} - 1) + 2m]\gamma_{3\mathrm{B}}v_{3\mathrm{B}} = 2m\gamma_{3\mathrm{B}}^2 v_{3\mathrm{B}} \tag{7.32}$$

The consideration of complete transformation into mass can be looked at as reverse observation compared to the conditions during the disintegration of particles and may be designated as “negative mass defect”. This result is corresponding exactly to the value of the missing part of momentum and energy during collision and leads to the conclusion, that for relativistic considerations of the non-elastic collision always an increase of mass in the amount of the value presented by the transformation of kinetic energy must be presumed to prevent the occurrence of discrepancies.

This is comprehensible on an atomic scale, for macroscopic objects it is not conforming to the general understanding of processes, because e. g. during the generation of heat no transformation processes are observed. However, in this case because of the definition of heat – which means that a rising heat input is corresponding to increasing velocities of the apparent mass – the increase of energy can be interpreted as relativistic consideration of the oscillation-velocity of the participating atoms or molecules. When this issue is discussed in the literature, normally the transformation of kinetic energy into mass is placed first and then verified using the relevant equations, e.g. [47]. The approach presented here, however, provides clear evidence that the increase of mass caused by complete transformation of kinetic energy is required by the valid conservation laws.

## 7.2 Relativistic considerations of particle disintegration

As already mentioned before, the disintegration of particles can be interpreted as the reversion of the situation valid during non-elastic collision (see chapter 7.1). Because the mathematical correlations of both effects are exactly the same, it is not necessary to present the evaluations again. In this chapter the emphasis is laid on considerations of decay particles moving in different spatial directions and concerning the conditions, when the kinetic energy is not converted into mass as discussed before but is dissipated by electromagnetic radiation.

To avoid misinterpretations, it shall be generally defined that the dissipating particle is indicated with index 1, for the decay products the indices 3 and 4 (and increasing further if applicable) are used. An observer moving with a dissipating particle is additionally marked as $f'$, for an observer at rest $f$ is used (without marking).

### 7.2.1 Analysis of disintegration into 2 particles

For the investigation of the situation in arbitrary spatial directions it is necessary to use the analytical determination of aberration, which was already derived in chapter 2.3. The geometrical dependencies are presented in Fig. 7.2. The description is completely comparable and therefore the calculations will not be repeated. The only valid difference is concerning equation Eq. (2.43), where the relation between the velocity of the moving system and the speed of light is calculated. These must be replaced by the following relation

$$\text{Eq. (2.43)}: \quad \frac{b}{v} = \frac{d}{c} \qquad \Rightarrow \qquad \frac{b}{v_1} = \frac{d}{v_3} \tag{7.40}$$

where $v_1$ is the velocity of the moving system and $v_3$ is the speed of an arbitrary particle (the equations presented in the following can be derived in the same way for particle 4). It is necessary to calculate the velocity $v_3$ using Eq. (4.20) according to

$$v_3 = \frac{\sqrt{v_1^2 + v_3'^2 + 2v_1 v_3' cos\alpha_3' - \left(\frac{v_1 v_3' sin\alpha_3'}{c}\right)^2}}{1 + \frac{v_1 v_3' cos\alpha_3'}{c^2}} \tag{7.41}$$

where in this case $v_3'$ is the velocity of the particle relative to the moving system and $v_3$ is the velocity in view of the observer at rest. The calculation leads to the following result [see also Eq. (2.48)]:

$$tan\alpha_3' = \pm \frac{sin\alpha_3}{\gamma\left(cos\alpha_3 - \frac{v_1}{v_3'}\right)} \tag{7.42}$$

Here $\alpha_3$ is the angle, which an observer at rest will find between the motion of a particle relative to his system, while $\alpha_3'$ is the angle of the same particle in view of the moving observer. When the value of $\alpha_3'$ is given then the resulting value for $\alpha_3$ can also easily be calculated. The only conversion necessary is the change of the algebraic sign (for details see chapter 2.3.4) and the result is

$$tan\alpha_3 = \pm \frac{sin\alpha_3'}{\gamma\left(cos\alpha_3' + \frac{v_1}{v_3'}\right)} \tag{7.43}$$

The validity of this relation can also easily be verified by numerical comparison. In table Tab. (7.1a) some examples for the calculation of the resulting angles for different velocities $v_1$ and $v_3'$ are presented.

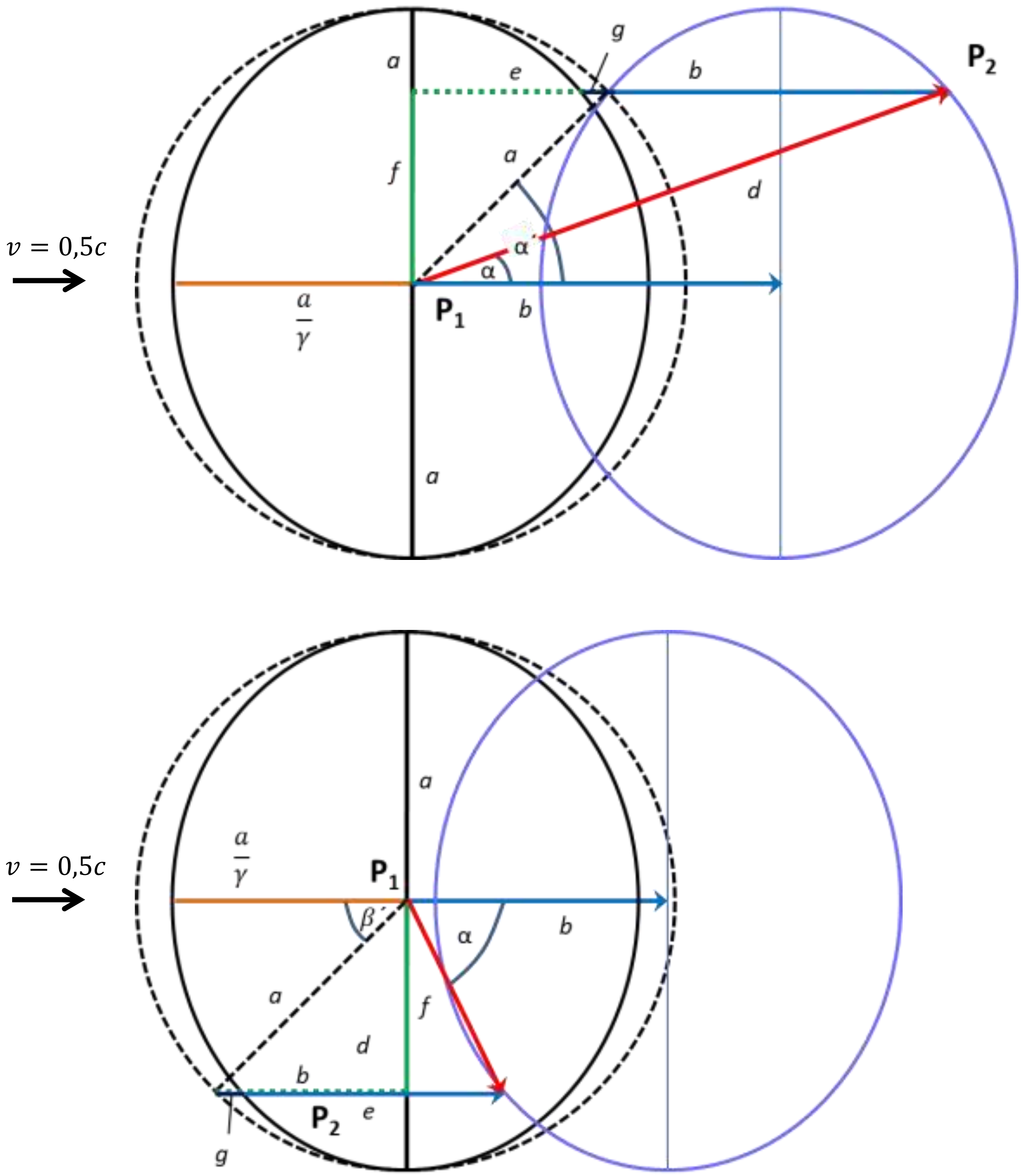



Fig. 7.2: Definition of parameters to determine the angle of an outgoing beam for a moving observer (examples for $v_1 = 0.5c$, $\alpha_3' = 45°$, $\alpha_4' = -135°$)
a) Signal emitted in moving direction, $\alpha_3 = 19{,}73°$
b) Signal emitted backwards, $\alpha_4 = -64.44°$

| $\alpha_3'$ | $v_3$ | $\alpha_3$ | $\tilde{p}_3$ | $\tilde{p}_{3X}$ | $\tilde{p}_{3Y}$ | $\alpha_4$ | $v_4$ | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0,57143 | 0 | 0,69631 | 0,69631 | 0 | -180 | 0,42105 | $v_1$= |
| 45 | 0,55439 | 37,80 | 0,66612 | 0,52636 | 0,40825 | -125,78 | 0,44953 | 0,1 |
| 90 | 0,50744 | 78,63 | 0,58890 | 0,11605 | 0,57735 | -78,63 | 0,50744 | $v_3'$= |
| 135 | 0,44953 | 125,78 | 0,50324 | -0,29425 | 0,40825 | -37,80 | 0,55439 | 0,5 |
| 180 | 0,42105 | 180,00 | 0,46421 | -0,46421 | 0 | 0 | 0,57143 | |
| $\alpha_3'$ | $\tilde{p}_4$ | $\tilde{p}_{4X}$ | $\tilde{p}_{4Y}$ | $\Sigma\tilde{p}_X$ | $\Sigma\tilde{p}_Y$ | $\tilde{E}_{kin,3}$ | $\tilde{E}_{kin,4}$ | $\Sigma\tilde{E}_{kin}$ |
| 0 | 0,46421 | -0,46421 | 0 | 0,23210 | 0 | 0,218544 | 0,102492 | 0,321035 |
| 45 | 0,50324 | -0,29425 | -0,4085 | 0,23210 | 0 | 0,201548 | 0,119487 | 0,321035 |
| 90 | 0,58890 | 0,11605 | -0,57735 | 0,23210 | 0 | 0,160518 | 0,160518 | 0,321035 |
| 135 | 0,66612 | 0,52636 | -0,40825 | 0,23210 | 0 | 0,119487 | 0,201548 | 0,321035 |
| 180 | 0,69631 | 0,69631 | 0 | 0,23210 | 0 | 0,102492 | 0,218544 | 0,321035 |
| $\alpha_3'$ | $v_3$ | $\alpha_3$ | $\tilde{p}_3$ | $\tilde{p}_{3X}$ | $\tilde{p}_{3Y}$ | $\alpha_4$ | $v_4$ | |
| 0 | 0,8 | 0 | 1,33333 | 1,33333 | 0,00000 | -90 | 0,00000 | $v_1$= |
| 45 | 0,77059 | 19,73 | 1,20908 | 1,13807 | 0,40825 | -64,44 | 0,41229 | 0,5 |
| 90 | 0,66144 | 40,89 | 0,88192 | 0,66667 | 0,57735 | -40,89 | 0,66144 | $v_3'$= |
| 135 | 0,41229 | 64,44 | 0,45254 | 0,19526 | 0,40825 | -19,73 | 0,77059 | 0,5 |
| 180 | 0 | 90 | 0 | 0 | 0 | 0 | 0,80000 | |
| $\alpha_3'$ | $\tilde{p}_4$ | $\tilde{p}_{4X}$ | $\tilde{p}_{4Y}$ | $\Sigma\tilde{p}_X$ | $\Sigma\tilde{p}_Y$ | $\tilde{E}_{kin,3}$ | $\tilde{E}_{kin,4}$ | $\Sigma\tilde{E}_{kin}$ |
| 0 | 0 | 0 | 0 | 1,33333 | 0 | 0,666667 | 0,000000 | 0,666667 |
| 45 | 0,45254 | 0,19526 | -0,40825 | 1,33333 | 0 | 0,569036 | 0,097631 | 0,666667 |
| 90 | 0,88192 | 0,66667 | -0,57735 | 1,33333 | 0 | 0,333333 | 0,333333 | 0,666667 |
| 135 | 1,20908 | 1,13807 | -0,40825 | 1,33333 | 0 | 0,097631 | 0,569036 | 0,666667 |
| 180 | 1,33333 | 1,33333 | 0 | 1,33333 | 0 | 0,000000 | 0,666667 | 0,666667 |
| $\alpha_3'$ | $v_3$ | $\alpha_3$ | $\tilde{p}_3$ | $\tilde{p}_{3X}$ | $\tilde{p}_{3Y}$ | $\alpha_4$ | $v_4$ | |
| 0 | 0,57143 | 0 | 0,69631 | 0,69631 | 0 | 0 | 0,42105 | $v_1$= |
| 45 | 0,55439 | 6,12 | 0,66612 | 0,66232 | 0,07107 | -8,12 | 0,44953 | 0,5 |
| 90 | 0,50744 | 9,83 | 0,58890 | 0,58026 | 0,10050 | -9,83 | 0,50744 | $v_3'$= |
| 135 | 0,44953 | 8,12 | 0,50324 | 0,49820 | 0,07107 | -6,12 | 0,55439 | 0,1 |
| 180 | 0,42105 | 0 | 0,46421 | 0,46421 | 0 | 0 | 0,57143 | |
| $\alpha_3'$ | $\tilde{p}_4$ | $\tilde{p}_{4X}$ | $\tilde{p}_{4Y}$ | $\Sigma\tilde{p}_X$ | $\Sigma\tilde{p}_Y$ | $\tilde{E}_{kin,3}$ | $\tilde{E}_{kin,4}$ | $\Sigma\tilde{E}_{kin}$ |
| 0 | 0,46421 | 0,46421 | 0 | 1,16052 | 0 | 0,218544 | 0,102492 | 0,321035 |
| 45 | 0,50324 | 0,49820 | -0,07107 | 1,16052 | 0 | 0,201548 | 0,119487 | 0,321035 |
| 90 | 0,58890 | 0,58026 | -0,10050 | 1,16052 | 0 | 0,160518 | 0,160518 | 0,321035 |
| 135 | 0,66612 | 0,66232 | -0,07107 | 1,16052 | 0 | 0,119487 | 0,201548 | 0,321035 |
| 180 | 0,69631 | 0,69631 | 0 | 1,16052 | 0 | 0,102492 | 0,218544 | 0,321035 |

Tab. 7.1a: Calculation for momentum and kinetic energy in a moving system.
Values marked grey: Approximation.
Values presented in frames: 180 °-angels.
Equations and dimensions: Tab. 7.1b and text.

| | | | |
|---|---|---|---|
| $$v_3 = \frac{\sqrt{v_1^2 + v_3'^2 + 2v_1v_3'cos\alpha_3' - \left(\frac{v_1v_3'sin\alpha_3'}{c}\right)^2}}{1 + \frac{v_1v_3'cos\alpha_3'}{c^2}} \cdot c$$ | | | [-] |
| $$\alpha_3 = arctan\left[\frac{sin\alpha_3'}{\gamma\left(cos\alpha_3' + \frac{v_1}{v_3'}\right)}\right] \cdot \frac{180}{\pi}$$ | [°] | $$\tilde{p}_3 = \frac{p_3}{mc} = \frac{v_3}{c}\gamma_3$$ | |
| $$\tilde{p}_{3X} = \frac{p_{3X}}{mc} = \frac{v_3}{c}\gamma_3 cos\,(\alpha_3)$$ | [-] | $$\tilde{p}_{3Y} = \frac{p_{3Y}}{mc} = \frac{v_3}{c}\gamma_3 sin\,(\alpha_3)$$ | [-] |
| $$v_4 = \frac{\sqrt{v_1^2 + v_4'^2 + 2v_1v_4'cos\alpha_4' - \left(\frac{v_1v_4'sin\alpha_4'}{c}\right)^2}}{1 + \frac{v_1v_4'cos\alpha_4'}{c^2}} \cdot c$$ | | | [-] |
| $$\alpha_4 = arctan\left[\frac{sin\alpha_4'}{\gamma\left(cos\alpha_4' + \frac{v_1}{v_4'}\right)}\right] \cdot \frac{180}{\pi}$$ | [°] | $$\tilde{p}_4 = \frac{p_4}{mc} = \frac{v_4}{c}\gamma_4$$ | |
| $$\tilde{p}_{4X} = \frac{p_{4X}}{mc} = \frac{v_4}{c}\gamma_4 cos\,(\alpha_4)$$ | [-] | $$\tilde{p}_{4Y} = \frac{p_{4Y}}{mc} = \frac{v_4}{c}\gamma_4 sin\,(\alpha_4)$$ | [-] |
| $$\sum\tilde{p}_X = \tilde{p}_{3X} + \tilde{p}_{4X}$$ | [-] | $$\sum\tilde{p}_Y = \tilde{p}_{3Y} + \tilde{p}_{4Y}$$ | [-] |
| $$\tilde{E}_{kin,3} = \frac{E_{kin,3}}{mc^2} = \gamma_3 - 1$$ | [-] | $$\tilde{E}_{kin,4} = \frac{E_{kin,4}}{mc^2} = \gamma_4 - 1$$ | [-] |
| $$\sum\tilde{E}_{kin} = \tilde{E}_{kin,3} + \tilde{E}_{kin,4}$$ | [-] | | |

Tab. 7.1b Equations and dimensions used in Tab. 7.1a

The equations used in table 7.1a and the connected dimensions are summarized in table 7.1b. To ensure a clear arrangement the values are presented in a normalized form as $\tilde{p}$ and $\tilde{E}$ with the dimension 1. This is also valid for the velocities; here the form $v/c$ was chosen.

The values marked grey were calculated using an approximation process, because for $v_3' = v_1$ the developing equations contain a division by zero. The values of $\alpha_3$ and $\alpha_4$ > 90° were calculated using first standard calculations and then the results were reduced by 180°; this is marked in the table using a frame (for further details see also chapter 2.3).

For the calculations, the following preconditions apply:

It is presumed that a particle is disintegrated into 2 decay products of equal size, of which one is removing with an arbitrary angle $\alpha'_3$. In this case the second "twin particle" will obey an angle of $\alpha'_4 = \alpha'_3 - 180°$ because of symmetry reasons. For these products, the angles $\alpha_3$ and $\alpha_4$ are calculated and the connected velocities $v_3$ and $v_4$ also. In a second step the values for momentum according to

$$p_3 = \gamma_1 m v_3 \quad bzw. \quad p_4 = \gamma_1 m v_4 \tag{7.44}$$

were determined. In a further step the fractions in moving direction $(x)$ and perpendicular to it $(y)$ according to

$$p_x = p \cdot cos(\alpha) \tag{7.45}$$

$$p_y = p \cdot sin(\alpha) \tag{7.46}$$

were calculated. When the angles $\alpha_3$ and $\alpha_4$ are added, the results in $x$-direction always show the same results, in $y$-direction they annihilate each other. Further the values for the kinetic energy were determined for particle 3 according to

$$E_{kin,3} = (\gamma_3 - 1)mc^2 \tag{7.47}$$

and for particle 4

$$E_{kin,4} = (\gamma_4 - 1)mc^2 \tag{7.48}$$

The summation of these values is producing the same result for all angles. It was possible to show with these calculations that for the disintegration into 2 decay particles the values for momentum and kinetic energy in all cases for an observer at rest and in a moving system are resulting in the same results and that it is not possible inside a system to decide whether this is moving or not.

### 7.2.2 Disintegration into 2 photons

It is well known from experimental results that a particle can disintegrate completely into photons without leaving matter. The $\pi^0$-pion for example is an extremely unstable particle with an average lifetime of approximately $10^{-18}$ s with the specific characteristic that it is disintegrating with almost 99% probability into 2 photons. When it is presumed that the disintegration is happening at a state of absolute rest the energy can be calculated using

$$E = m_0 c^2 = hf_3 + hf_4 \tag{7.50}$$

where $h$ is Planck's quantum of action and $f_3$ as well as $f_4$ are the frequencies of the emitted photons. The momentum of one photon is

$$\vec{p} = h \frac{f}{c} \vec{e} \tag{7.51}$$

with $\vec{e}$ as unit vector in moving direction. Because of the conservation laws of energy and momentum the frequencies for both photons are the same and their moving directions are exactly opposite to each other. The momentum is zero before and after disintegration.

If an observer is monitoring a velocity $v_1$ before disintegration, then because of the relativistic mass increase the total energy of the particle is

$$E = \gamma_1 m_0 c^2 \tag{7.52}$$

After disintegration, the emitted photons must carry the total energy and the momentum of the particle. The total energy of the photons is

$$E = \gamma_1 h f_3 + \gamma_1 h f_4 \tag{7.53}$$

and the momentum of one photon

$$\vec{p} = \gamma_1 h \frac{f}{c} \vec{e} \tag{7.54}$$

When these relations are analyzed according to the ratio valid in moving direction, for an observer at rest the kinetic energy of the particle and the momentum has also to be carried away completely by the emitted photons. For the energy, the following relation applies

$$\gamma_1 m_0 c^2 = \gamma_1 h f_3 + \gamma_1 h f_4 \tag{7.55}$$

and for the momentum *in moving direction*

$$\gamma_1 m_0 v_1 = \gamma_1 h \frac{f_3}{c} - \gamma_1 h \frac{f_4}{c} \tag{7.56}$$

where $f_3$ is the emission in moving direction (positive) and $f_4$ opposite to it (negative). Using subtraction resp. addition of equations Eq. (7.55) and (7.56) then the values for the frequencies are

$$f_3 = \frac{m_0(c^2 + v_1 c)}{2h} \tag{7.57}$$

$$f_4 = \frac{m_0(c^2 - v_1 c)}{2h} \tag{7.58}$$

with

$$\frac{f_3}{f_4} = \frac{c + v_1}{c - v_1} \tag{7.59}$$

This relation is exactly corresponding to the macroscopic behavior of moving emitters which will be described in chapter 8.

For the derivation of the correlations in arbitrary spatial directions first the geometric dependencies for emitter and receiver must be examined. In Fig. 7.3 it is demonstrated, in which way observer A at the time $A_1$ and $A_2$ is sending specific signals. Depending on the distance to receiver B and on the velocity different angles in relation to the moving direction will appear. For simplification it will be assumed, that the receiver B, which is at rest, is far away and the time between 2 signals is comparatively short and thus for the angles the relation $\alpha_1 = \alpha_2 = \alpha$ can be presumed.

The time between the signals send by the moving emitter A is

$$\Delta t_A = \gamma \Delta t_0 \tag{7.60}$$

compared to the relations valid for an observer at rest. Beside the extension caused by time-dilatation, receiver B will also notice a geometric influence on time, because the emitter is either coming or going relative to his position between sending out the signals. In total this adds up to

$$\Delta t_B = \gamma \Delta t_0 \left(1 - \frac{v}{c} cos(\alpha)\right) \tag{7.61}$$

This is resulting for the frequency detected by receiver B

$$f_B = \frac{f_0}{\gamma\left(1 - \frac{v}{c} cos\,(\alpha)\right)} \tag{7.62}$$

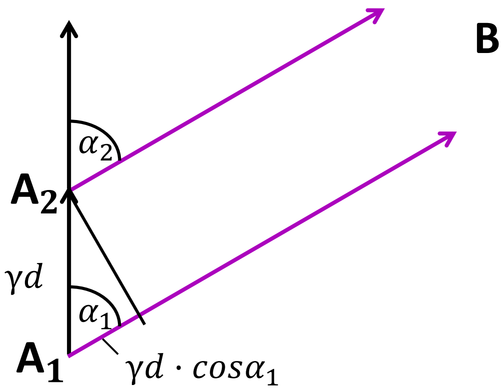



Fig. 7.3: Radiation geometry

To provide a final comparison between a particle at rest and moving, calculations for different angles for outgoing photons are made. In Tab. 7.2 different angles $\alpha_3'$ (in view of a moving observer) are defined; the corresponding angles of the "twin" photon are differing exactly by 180°, i.e. this means $\alpha_4' = \alpha_3' - 180°$. First the angles in view of the observer at rest are determined using the equations developed in chapter 2.3 and the value for $\alpha_3$ is calculated. Further the corresponding frequencies are determined, in the next step the momentum in $x$- and $y$-direction is calculated (using $cos$ resp. $sin$ of the angle according to Eq. (7.45) and (7.46) presented in chapter 7.2.1). Finally, the total energy, which is released during disintegration of the particle, is calculated for any angle.

The starting value for $f_0$ was set to 1. To ensure a clear arrangement the values for momentum end energy are again presented in normalized form as $\tilde{p}$ and $\tilde{E}$; the dimension is in this case 1. Detailed definitions and the resulting dimensions are summarized in Tab. 7.2b.

The summation of the values for momentum in $x$-direction and total energy are always identical and correspond to the expected results; the values in $y$-direction add up to zero. Further it is easy to show, that the results found for angle $\alpha_3' = 0$ correspond exactly to equation (7.59), which was derived for the simple case for emission in moving direction and opposite. Thus it was possible to show, that also in this case no differences appear whether experiments are viewed by an observer at rest or referring to a moving system and so no violations of the principles of relativity occur.

| $\alpha'_3$ | $\alpha_3$ | $\alpha_4$ | $f_3$ | $f_4$ | $\tilde{p}_{3X}$ | $\tilde{p}_{4X}$ | $\sum\tilde{p}_X$ | $\tilde{p}_{3Y}$ | $\tilde{p}_{4Y}$ | $\sum\tilde{p}_Y$ | $\tilde{E}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | -180 | 1,732 | 0,577 | 0,866 | -0,289 | 0,577 | 0 | 0 | 0 | 2,309 |
| 15 | 8,69 | -154,31 | 1,712 | 0,597 | 0,846 | -0,269 | 0,577 | 0,129 | -0,129 | 0 | 2,309 |
| 30 | 17,59 | -130,21 | 1,655 | 0,655 | 0,789 | -0,211 | 0,577 | 0,250 | -0,250 | 0 | 2,309 |
| 45 | 26,90 | -108,69 | 1,563 | 0,746 | 0,697 | -0,120 | 0,577 | 0,354 | -0,354 | 0 | 2,309 |
| 60 | 36,87 | -90,00 | 1,443 | 0,866 | 0,577 | 0,000 | 0,577 | 0,433 | -0,433 | 0 | 2,309 |
| 75 | 47,79 | -73,92 | 1,304 | 1,005 | 0,438 | 0,139 | 0,577 | 0,483 | -0,483 | 0 | 2,309 |
| 90 | 60,00 | -60,00 | 1,155 | 1,155 | 0,289 | 0,289 | 0,577 | 1 | -1 | 0 | 2,309 |
| 105 | 73,92 | -47,79 | 1,005 | 1,304 | 0,139 | 0,438 | 0,577 | 0,483 | -0,483 | 0 | 2,309 |
| 120 | 90 | -36,87 | 0,866 | 1,443 | 0,000 | 0,577 | 0,577 | 0,433 | -0,433 | 0 | 2,309 |
| 135 | 108,69 | -26,90 | 0,746 | 1,563 | -0,120 | 0,697 | 0,577 | 0,354 | -0,354 | 0 | 2,309 |
| 150 | 130,21 | -17,59 | 0,655 | 1,655 | -0,211 | 0,789 | 0,577 | 0,25 | -0,25 | 0 | 2,309 |
| 165 | 154,31 | -8,69 | 0,597 | 1,712 | -0,269 | 0,846 | 0,577 | 0,129 | -0,129 | 0 | 2,309 |
| 180 | 180 | 0 | 0,577 | 1,732 | -0,289 | 0,866 | 0,577 | 0 | 0 | 0 | 2,309 |

Tab 7.2a: Calculations of angles, momentum (moving direction: $x$, vertical: $y$), energy. Equations and dimensions see Tab. 7.2b

| | | | |
|---|---|---|---|
| $\alpha_3 = 2 \cdot arctan\left[\left(\frac{c-v}{c+v}\right)^{1/2} tan\left(\frac{\alpha'_3}{2}\right)\right] \cdot \frac{180}{\pi}$ Eq. acc. Tab. 2.4, No. 4 | | | [°] |
| $\alpha_4 = 2 \cdot arctan\left[\left(\frac{c-v}{c+v}\right)^{1/2} tan\left(\frac{\pi-\alpha'_3}{2}\right)\right] \cdot \frac{180}{\pi}$ Eq. acc. Tab. 2.4, No. 4 | | | [°] |
| $f_3 = \frac{f_0}{\gamma\left(1-\frac{v}{c}\cos(\alpha_3)\right)}$ | $[s^{-1}]$ | $f_4 = \frac{f_0}{\gamma\left(1+\frac{v}{c}\cos(\alpha_4)\right)}$ | $[s^{-1}]$ |
| $\tilde{p}_{3X} = \frac{p_{3X}}{mc} = \frac{v}{c} f_3 \cos(\alpha_3)$ | [-] | $\tilde{p}_{4X} = \frac{p_{4X}}{mc} = \frac{v}{c} f_4 \cos(\alpha_4)$ | [-] |
| $\sum\tilde{p}_X = \tilde{p}_{3X} + \tilde{p}_{4X}$ | [-] | | |
| $\tilde{p}_{3Y} = \frac{p_{3Y}}{mc} = \frac{v}{c} f_3 \sin(\alpha_3)$ | [-] | $\tilde{p}_{4Y} = \frac{p_{4Y}}{mc} = \frac{v}{c} f_4 \sin(\alpha_4)$ | [-] |
| $\sum\tilde{p}_Y = \tilde{p}_{3Y} + \tilde{p}_{4Y}$ | [-] | $\tilde{E} = \frac{E}{\gamma h} = f_3 + f_4$ | [-] |

Tab. 7.2b Equations and dimensions used for calculations in Tab. 7.2a

# 8. The constant phase-velocity of light

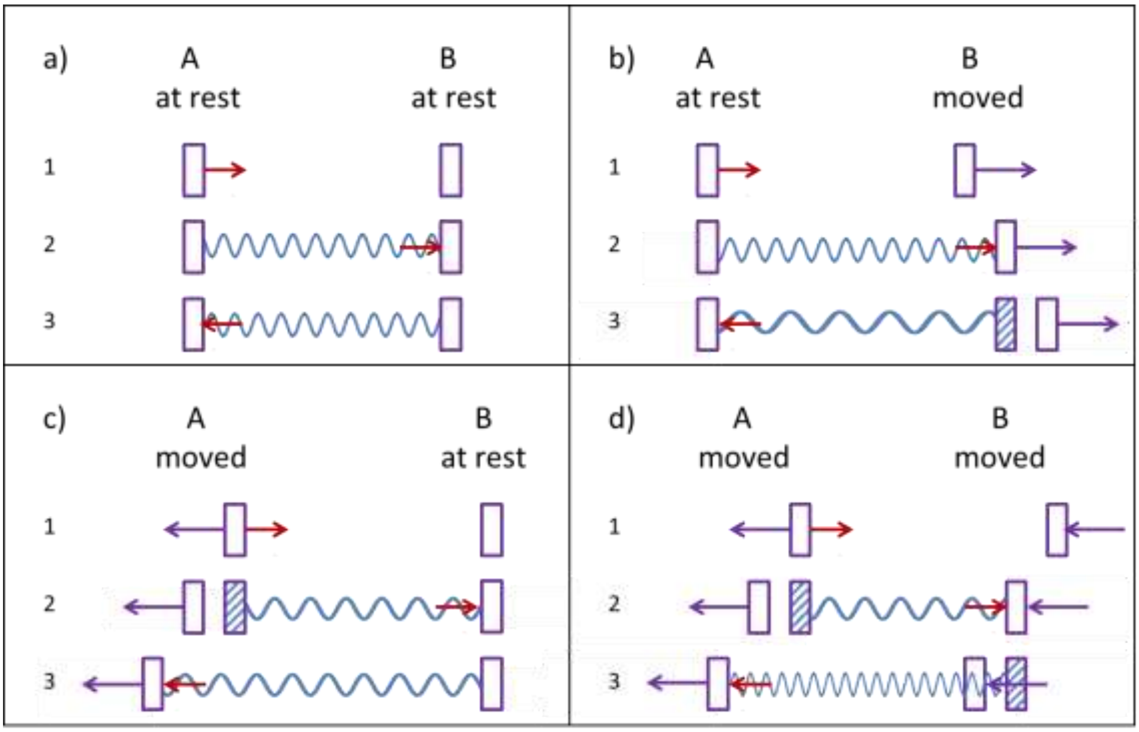
The topics discussed so far showed exact conformance with the explanations presented in many other important and undisputed publications. In the following the observations of transmitted signals with constant frequency will reveal an aspect, however, that is in contradiction to established interpretations. These can only be solved when the constant phase velocity of light is considered; this issue is therefore of great relevance for Special Relativity and the most important part of the examinations presented here. Subsequently it will become clear, that the assumption of a system at absolute rest in the universe is generally in contradiction to Special Relativity but when using the principle of constant phase velocity, it is just a special case inside the theory without violating basic experimental results.

## 8.1 Incoherency with Special Relativity using the standard derivation

In Figs. 8.1a and 8.1b the situation is illustrated, that two observers A and B exchange light signals. At the beginning (position no. 1) a signal is transmitted from observer A, and at no. 2 it is received from B and reflected immediately. At position no. 3 observer A is receiving the returning signal and the experiment comes to an end. Observers A and B are either at rest relative to each other (case a, d and g) increase the distance (case b and c) or approaching each other (case e and f). The transmitted and received signals are analyzed. It is well-known that transmitted signals with a constant frequency leaving a moving system are received with a higher frequency by a second observer when they approach each other, and the frequency is lower in the opposite direction. The relation is described by

$$f' = \frac{1}{T'} = f_0 \left( \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{1/2} = f_0 \cdot \gamma \left(1 + \frac{v}{c}\right) \tag{8.01}$$

It is considered that the frequency of a moved observer is lower by the factor $\gamma$ because of time dilatation. The values for the calculated frequency $f$, the covered distance $a$, the necessary time $t$ and the number $n$ of the oscillations in these intervals are presented in the following tables.

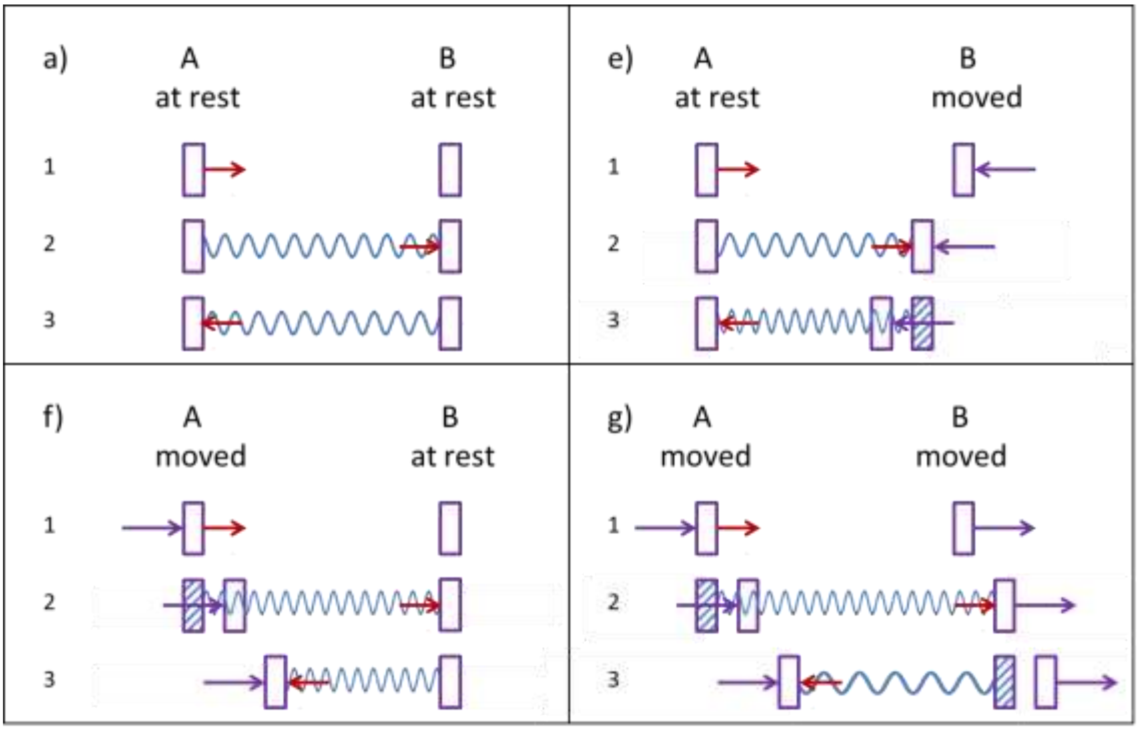



1: Emitting signal from A
2: Receiving at B, reflection to A
3: Receiving at A

| Case | $f_A$ | $f_{A\to B}$ | $f_B$ | $f_{B\to A}$ | $f_A$ |
|---|---|---|---|---|---|
| a | $f_0$ | $f_0$ | $f_0$ | $f_0$ | $f_0$ |
| b | $f_0$ | $f_0$ | $f_0\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}$ | $f_0\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)$ | $f_0\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)$ |
| c | $\frac{f_0}{\gamma}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)$ |
| d | $\frac{f_0}{\gamma}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}$ | $\frac{f_0}{\gamma}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{1/2}$ | $\frac{f_0}{\gamma}$ |

| Case | $a_{A\to B}$ | $a_{B\to A}$ | $t_{A\to B}$ | $t_{B\to A}$ | $n_{A\to B}$ | $n_{B\to A}$ |
|---|---|---|---|---|---|---|
| a | $a_0$ | $a_0$ | $t_0$ | $t_0$ | $n_0$ | $n_0$ |
| b | $a_0\frac{1}{1-\frac{v}{c}}$ | $a_0\frac{1}{1-\frac{v}{c}}$ | $t_0\frac{1}{1-\frac{v}{c}}$ | $t_0\frac{1}{1-\frac{v}{c}}$ | $n_0\frac{1}{1-\frac{v}{c}}$ | $n_0\frac{1}{1+\frac{v}{c}}$ |
| c | $a_0$ | $a_0\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)$ | $t_0$ | $t_0\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)$ | $n_0\left(1-\frac{v}{c}\right)$ | $n_0\left(1+\frac{v}{c}\right)$ |
| d | $a_0\frac{1}{1+\frac{v}{c}}$ | $a_0\frac{1}{1-\frac{v}{c}}$ | $t_0\frac{1}{1+\frac{v}{c}}$ | $t_0\frac{1}{1-\frac{v}{c}}$ | $n_0\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)$ | $n_0\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)$ |

Fig. 8.1a: Exchange of signals between observers A and B and analysis of the resulting frequencies and oscillation periods




1: Emitting signal from A
2: Receiving at B, reflection to A
3: Receiving at A

| Case | $f_A$ | $f_{A\to B}$ | $f_B$ | $f_{B\to A}$ | $f_A$ |
|---|---|---|---|---|---|
| a | $f_0$ | $f_0$ | $f_0$ | $f_0$ | $f_0$ |
| e | $f_0$ | $f_0$ | $f_0\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{1/2}$ | $f_0\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)$ | $f_0\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)$ |
| f | $\frac{f_0}{\gamma}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{1/2}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{1/2}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{1/2}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)$ |
| g | $\frac{f_0}{\gamma}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{1/2}$ | $\frac{f_0}{\gamma}$ | $\frac{f_0}{\gamma}\cdot\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}$ | $\frac{f_0}{\gamma}$ |

| Case | $a_{A\to B}$ | $a_{B\to A}$ | $t_{A\to B}$ | $t_{B\to A}$ | $n_{A\to B}$ | $n_{B\to A}$ |
|---|---|---|---|---|---|---|
| a | $a_0$ | $a_0$ | $t_0$ | $t_0$ | $n_0$ | $n_0$ |
| e | $a_0\frac{1}{1+\frac{v}{c}}$ | $a_0\frac{1}{1+\frac{v}{c}}$ | $t_0\frac{1}{1+\frac{v}{c}}$ | $t_0\frac{1}{1+\frac{v}{c}}$ | $n_0\frac{1}{1+\frac{v}{c}}$ | $n_0\frac{1}{1-\frac{v}{c}}$ |
| f | $a_0$ | $a_0\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)$ | $t_0$ | $t_0\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)$ | $n_0\left(1+\frac{v}{c}\right)$ | $n_0\left(1-\frac{v}{c}\right)$ |
| g | $a_0\frac{1}{1-\frac{v}{c}}$ | $a_0\frac{1}{1+\frac{v}{c}}$ | $t_0\frac{1}{1-\frac{v}{c}}$ | $t_0\frac{1}{1+\frac{v}{c}}$ | $n_0\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)$ | $n_0\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)$ |

Fig. 8.1b: Exchange of signals between observers A and B and analysis of the resulting frequencies and oscillation periods

It is not possible for both observers to decide based on a frequency analysis whether they belong to system a), d), g) or b), c) resp. e), f). Considering the number of oscillations between the observers, however, it should be clear that A and B according to the “principle of identity” (see chapter 1.6) in cases d) and g) for the signals coming and going (situation 2 and 3) should measure the same values. A similar situation exists for b) and c) resp. e) and f). It is obvious at first sight and without calculation that this cannot be the case. In the following this will be discussed in detail.

In the tables of Fig.8.1a and 8.1b the results for the frequencies measured by an observer at rest are shown. It is incorporated, that the generated frequencies in a moving system appear to be reduced by the factor $\gamma$ for an observer at rest. In the second part of the table the values for the distance $a$, the travelling time for the signal exchange $t$ and the number $n$ of the oscillations in these intervals are presented. The number of oscillations is calculated using

$$n = f \cdot t \tag{8.02}$$

If the light signals are passing through an interferometer and have the possibility for interaction, the observer at rest should be able to monitor interference patterns. Turning the system by a degree of 90° towards the direction of motion the interference effect should disappear.

Out of these considerations it is clear, that a discrepancy between the results of the number of oscillations between the moving system and the system at rest exists. Corresponding to the presented diagrams the observations in these systems should be completely different. According to this general theoretical approach the principle of relativity is violated here.

Not surprisingly in reality this is not the case, however. The explanation for this is that measurements by the moving observer cannot directly be compared with that of an observer at rest. Because of the dependency of measurements of electromagnetic waves on time and space, the two observers would find different results using this approach. To resolve the problem, it is therefore necessary to introduce the phase velocity, which is equal to the speed of light for both observers.

When considerations of phase velocities are used, the conformity between the numbers of oscillations detected by the two observers can be derived without difficulty. This is in particular valid for the results in the discussed cases a, d and g. Because of the impact of this important feature the effect of phase velocity is discussed in detail in the following chapter.

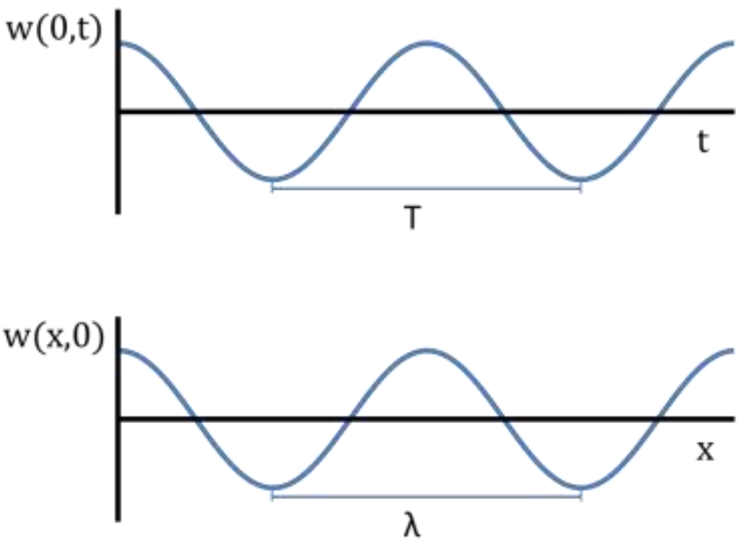
## 8.2 Concept of phase velocity to overcome the discrepancies for observers

During an exchange of signals between two observers, which are generally using light beams for transmission, in a standard case harmonic oscillation will be used. It is not possible to integrate these oscillations directly into a space-time-diagram (i.e. in a Minkowski-diagram). In short summary waves are typically considered in a way, that one of the variables (i.e. time) is looked at as constant and the other (for this example: space) is varying. Taking the simple example of a wave, which is produced when a stone is thrown into water, the investigation could be performed by taking a picture and measuring the distance of the wave peaks (in this case time is constant). If in a further measurement the distance is kept constant, e.g. by measuring a small cork moving up and down, then the frequency of the wave can be calculated by measuring the time between two defined points e.g. the maxima. Out of the combination of these measurements the velocity of the wave, which is travelling with a certain phase velocity, can be calculated. It is also possible, however, to observe the moving maxima in a direct way and measure the dependencies of time and the traveled way by taking a video.

The situation can be described as follows: The oscillation is dependent on space ($x$) and time ($t$) and is corresponding to the following equation [46a]

$$w(x,t) = A_0 \cos\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x - \alpha\right) \tag{8.10}$$

In this case $A_0$ is the amplitude, $T$ is the oscillation time (considering a stationary view), $\lambda$ is the oscillation length (considering constant time) and $\alpha$ is the angle at the starting point.




$$\alpha = 0, \qquad x = 0$$

$$w(0,t) = A_0\, cos\frac{2\pi}{T}t$$

$$\alpha = 0, \qquad t = 0$$

$$w(x,0) = A_0\, cos\frac{2\pi}{\lambda}x$$

Fig. 8.2: Oscillation diagram for constant space ($x = 0$) and constant time ($t = 0$) with starting point $\alpha = 0$

A major simplification is possible, when the variation of space *and* time of a certain point of the wave (i.e. the maximum) is defined as constant (see Fig. 8.3). In this case the cosine remains unchanged, and it applies

$$\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x - \alpha = const. \tag{8.11}$$

After differentiation of this equation

$$\frac{\Delta t}{T} - \frac{\Delta x}{\lambda} = 0 \tag{8.12}$$

the phase velocity $u$ of this point will be described by

$$u = \lim_{\Delta \to 0} \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} \tag{8.13}$$

Without the dispersion by a medium (as it is the case in a vacuum), the formula develops to

$$u = \frac{\lambda}{T} = c \tag{8.14}$$

This derivation using the mathematical concept of differential quotient and limes provides a good explanation of the physical principle [46a], more complex deductions with 4-vector and gradient are also possible and obviously come to the same solution [27].

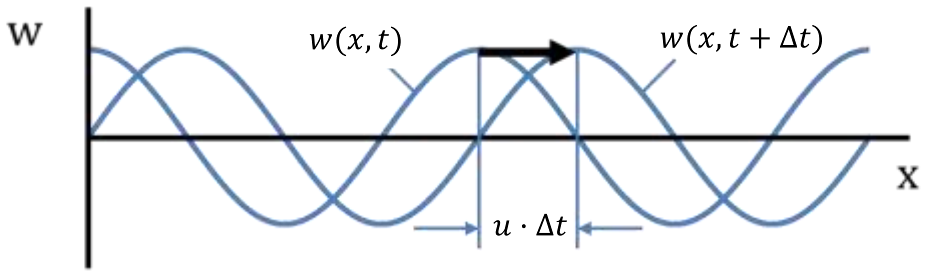



Fig.8.3: Phase velocity $u$ as propagation speed of defined parts of the oscillation (i.e. the maximum)

Thus, the main conclusion is that the phase velocity of an electromagnetic wave measured in any arbitrary inertial system is **exactly equal to the speed of light**. In Fig. 8.4 the phase velocity is presented as a function of space and time. Because it obviously shows a linear characteristic the graph will be a straight line with origin zero and, after scaling, it will display an angle of 45° to the $x$- and $t$-coordinate. The right part of the diagram is showing in addition the graphs for a moving observer with velocities of $v = 0.2c;\ 0.5c; 0.8c$.

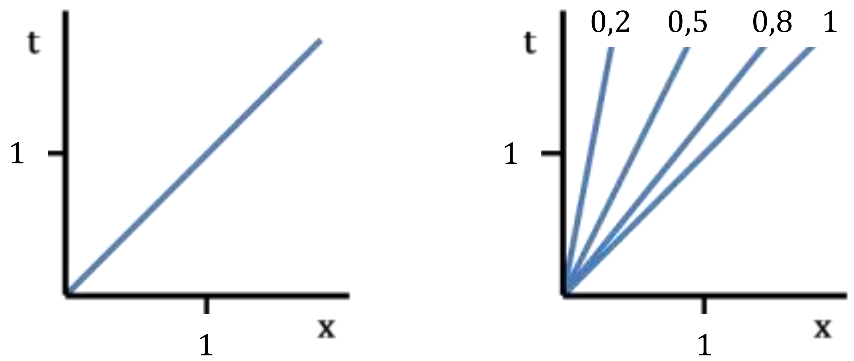



Fig. 8.4: Left: Phase velocity as a function of time and space (scaled diagram)
Right: Velocities of moved observers with different speed added

At this point it becomes clear, that this presentation is exactly coherent with a Minkowski-diagram. This means, that a phase-propagation (i.e. the maximum of a wave) can be taken as a short light pulse and therefore it can be incorporated in diagrams of this type and evaluated in the same way.

In Fig. 8.5 a situation like this is illustrated. The presentation of this diagram seems to be unusual at first sight. Having a closer look, however, some important issues can be derived from it, so that the appearance of this Minkowski-diagram will be discussed in detail in the following. Many important examinations are possible, but a clear arrangement in one diagram would not be reasonable because of the quantity of information. So, it was decided to use in Figs. 8.6 and 8.7 the same chart, covering additional information while others were skipped.

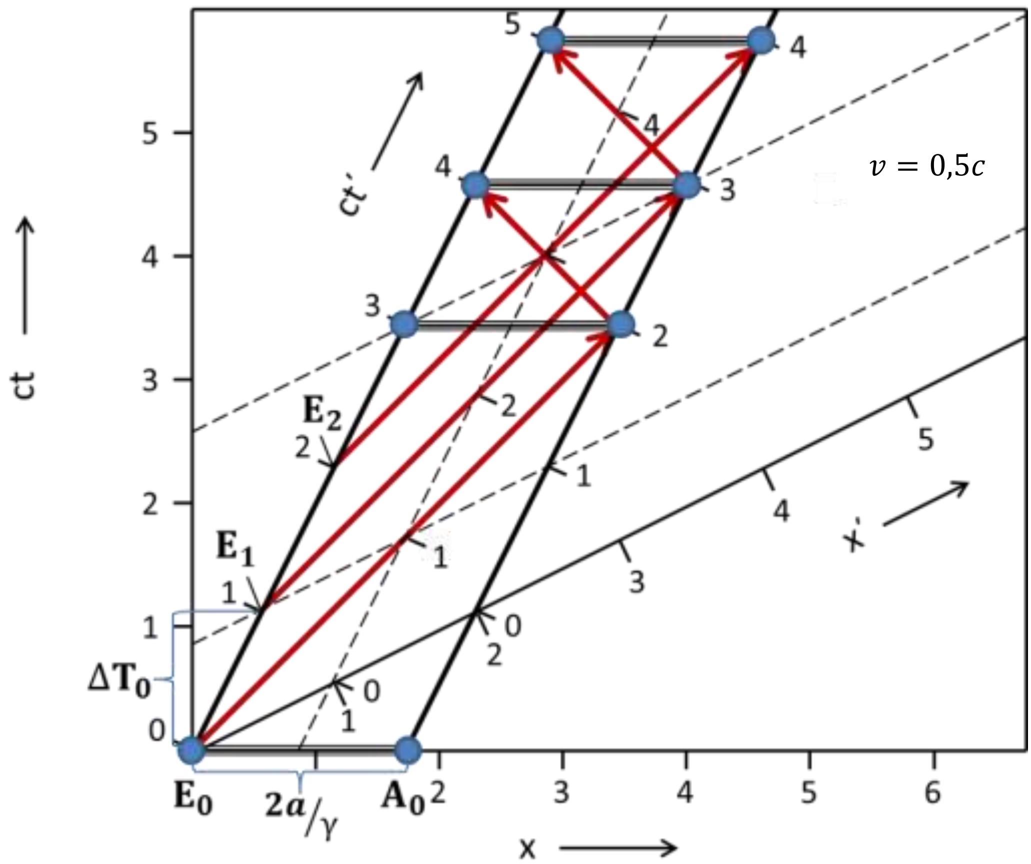



Fig. 8.5: Minkowski diagram for the exchange of signals inside a moving system

First the general setting of the chosen experiment shall be discussed: A laboratory with the length $2a$ is moving relative to another observer at rest with the speed $v = 0{,}5\ c$. The diagram is scaled to 1 concerning space and time (this means that $a = 1$ for a laboratory at rest). At time zero the moving observer starts from point $E_0$ with the transmission of a harmonic oscillation of 1Hz and is beginning with a maximum. The oscillation is reflected at point A and sent back to E.

The observer at rest will find, that the moved laboratory has a length of $2a/\gamma$. Because of his view on the time dilatation in the moved system, he will additionally find that the oscillation will end at $\Delta T_0 = \gamma$ (at point $E_1$). The following maxima will therefore start at $E_1$, $E_2$ etc. and can also be interpreted as separate pulses and so it is possible to record them in this diagram as well.

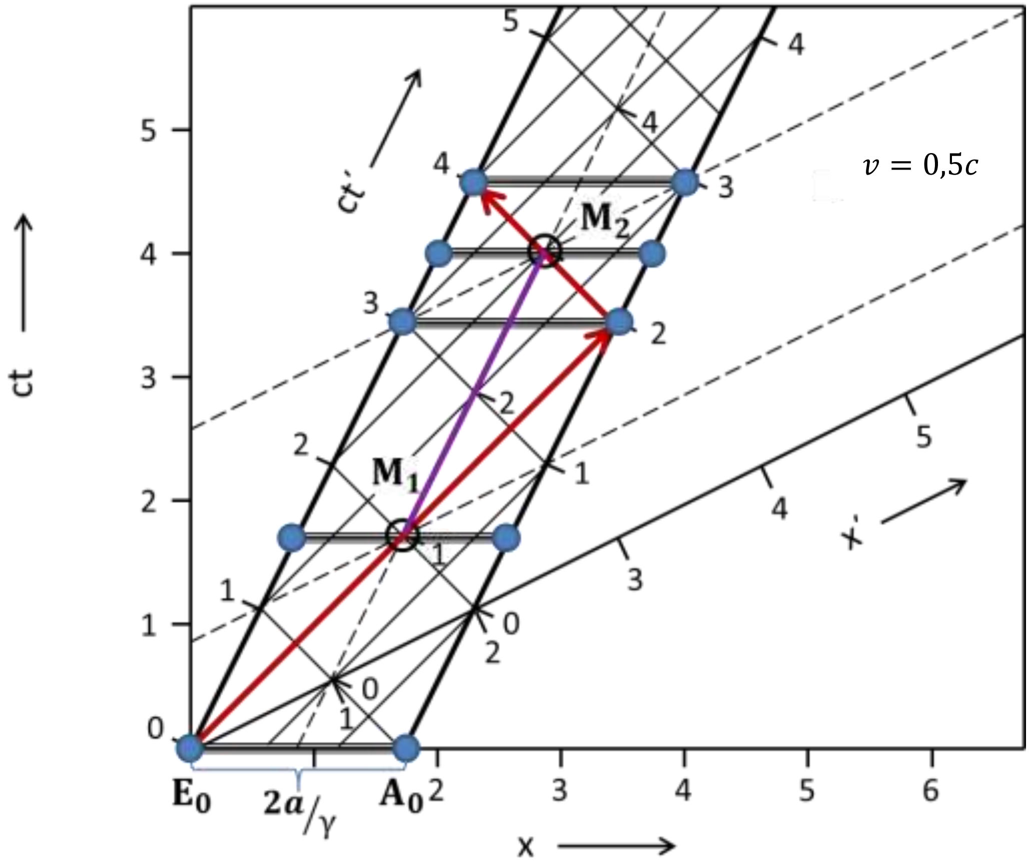



Fig. 8.6: Minkowski diagram for the exchange of signals in a moving system (middle section), variation of Fig. 8.5

The maximum of the oscillation it is moving at a speed of $v = c$ and is reaching the middle at

$$t_{M_1} = \frac{1}{\gamma \cdot \left(1 - \frac{v}{c}\right)} \tag{8.15}$$

(see Fig. 8.6) Point A will be reached after twice the time. When the wave is reflected, the point $M_2$ will be passed at

$$t_{M_2} = \frac{2}{\gamma \cdot \left(1 - \frac{v}{c}\right)} + \frac{1}{\gamma \cdot \left(1 + \frac{v}{c}\right)} = \gamma \cdot \left(3 + \frac{v}{c}\right) \tag{8.16}$$

This is exactly the value, that would be yielded by a pulse emitted from $E_2$ (equivalent to the maximum of a wave) which leads to

$$t_{M_2} = 2\gamma + \frac{1}{\gamma \cdot \left(1 - \frac{v}{c}\right)} = \gamma \cdot \left(3 + \frac{v}{c}\right) \tag{8.17}$$

This calculation shows that the situation in the middle of the moved laboratory reveals exact the same conditions compared to an observer at rest. In the latter case a signal would be emitted by $E_0$, that after reflection arrives back at $t = 3$ in the middle of the laboratory. Another signal, that is sent from $E_0$ at $t = 2$ would reach the middle at the same time. This is as already presented also valid for the moved observer when phase velocities are considered.

The relations presented here can easily be transferred to other situations, if for example frequency, geometry or other conditions are modified. This is leading to the general statement, that the measurement of the number of oscillations under no circumstances can be used to measure the state of motion of an inertial system.

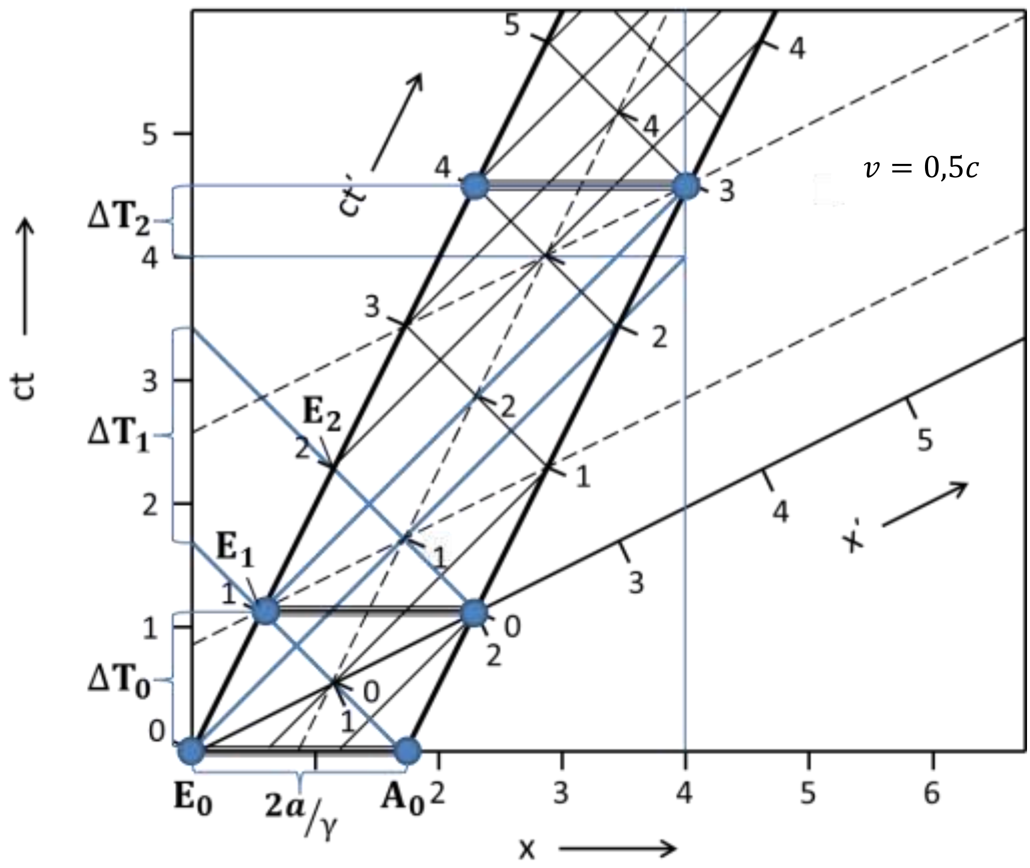



Fig. 8.7: Minkowski diagram for the exchange of signals in a moving system, variation of Fig. 8.5

Furthermore, the values for oscillation time and frequency of an observer at rest shall be derived out of this diagram. If the testing object is increasing the distance the value is

$$\Delta T_1 = \frac{1}{f_1} = \frac{1}{\gamma \cdot \left(1 - \frac{v}{c}\right)} \tag{8.18}$$

and when it is approaching

$$\Delta T_2 = \frac{1}{f_2} = \frac{1}{\gamma \cdot \left(1 + \frac{v}{c}\right)} \tag{8.19}$$

This is in accordance with standard publications (i.e. [46b]).

As main result it is possible to prove, that concerning the radiation of light in any arbitrary inertial system the phase velocity of the light emitted by one source is equal to the measurement of the speed of light in any of these systems (this is of course not the case for the simple example of surface waves on water!). The important finding derived by the considerations presented here, is that during the transition from an arbitrary inertial system to another not the speed of light, but the phase velocity remains unchanged. It was clearly shown that this is required by the theory of Special Relativity and otherwise contradictions would appear.

In the literature the importance of phase velocity in connection with Special Relativity is treated very differently. In a normal case it is not mentioned at all in books, lecture notes or publications, but there is an exception in the work of R. K. Pathria [16]. Herein the "invariance of phase velocities" between systems moved relative to each other is examined in extenso, but no further consequences concerning the theory are discussed.

The discovered relations are of great importance for the theory. It is interesting, however, that it is not possible to find this concept in the literature up to now. Because of this reason it is necessary to reconsider classical experiments, in particular those of Michelson-Morley and also Kennedy-Thorndike. It will be demonstrated that the use of the concept presented here will lead to a different understanding of the results. This will be presented in detail in chapter 9.

Finally, it is possible - before developing the theoretical background further - to present a first result of the examinations:

- It is possible, that the universe is at absolute rest and all electromagnetic waves are travelling with the speed of light $c$ inside this system.
- Observers in any inertial system with an arbitrary velocity relative it can only measure the phase velocity of these waves and doing this they will find also the same value of $c$.

At first these perceptions will be used to carry out new interpretations of classical experimental results. After further discussions finally in chapter 13 a proposal for modification of the theory of special relativity will be presented.

# 9. New interpretation of experimental results

In the following the most important experiments with impact on Special Relativity will be presented and discussed. In particular concerning the Michelson-Morley- and the Kennedy-Thorndike-Experiments new considerations will be derived when the concept of phase velocity is used. In addition, other fundamental experiments will be described in a short way.

## 9.1 Michelson-Morley-Experiment

At first the experiment conducted by A. A. Michelson and E. M. Morley will be discussed in detail. Because of the high importance, subsequently a comprehensive literature survey will be presented, and the conclusions derived from this test will be described.

### 9.1.1 Experimental layout and evaluation

The layout of the experiment presented in Fig. 9.1 is a reproduction out of the original publication in 1887 [7]. In this figure the set-up is shown, where a light beam at mirror $a$ is partly reflected in direction $ab$ and partly transmitted in direction $ac$, being returned by the mirrors $b$ and $c$, then reflected resp. transmitted to $d$ and at this point examined with an interferometer. Part 1 of Fig. 9.1 is presenting the position at rest; in part 2 the situation of a moved system (against the supposed ether) is given.

Theoretical basis of the experiment was the assumption, that the speed of light and the speed against the ether at rest could be added and that it would be possible to evaluate the latter by precise measurements. The whole time of going and coming between $a$ and $c$ can be calculated using

$$T_{\parallel} = \frac{D}{c+v} + \frac{D}{c-v} = \frac{2Dc}{c^2 - v^2} \tag{9.01}$$

where $D$ is the distance between $a$ and $c$. The distance traveled in this time is

$$D_{\parallel} = 2D\,\frac{c^2}{c^2 - v^2} \tag{9.02}$$

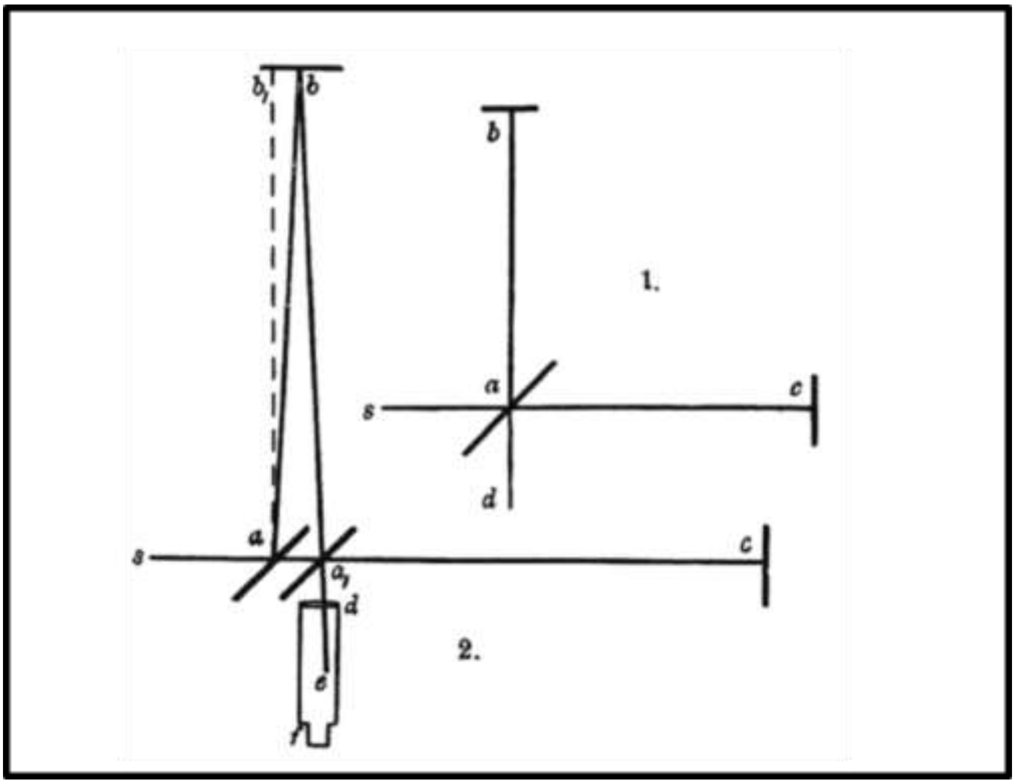



Fig. 9.1: Layout of the Michelson-Morley-Experiment, reproduction from original report [7]

In transverse direction the calculation yield

$$T_{\perp} = \frac{D_{\perp}}{v_{\perp}} \tag{9.03}$$

and

$$v_{\perp}^2 = c^2 + v^2 \tag{9.04}$$

$$D_{\perp} = 2D\sqrt{1 + \frac{v^2}{c^2}} \tag{9.05}$$

Neglecting terms of 4th order and higher the equations (9.02) and (9.05) after Taylor expansion develop to

$$D_{\parallel} \approx 2D\left(1 + \frac{v^2}{c^2}\right) \tag{9.06}$$

$$D_{\perp} \approx 2D\left(1 + \frac{v^2}{2c^2}\right) \tag{9.07}$$

Now the difference is

$$\Delta D = D_{\parallel} - D_{\perp} = 2D\left(1 + \frac{v^2}{c^2}\right) - 2D\left(1 + \frac{v^2}{2c^2}\right) = D\frac{v^2}{c^2} \tag{9.08}$$

Looking at the calculation with today's knowledge concerning the speed of light, it contains the obvious problem that the calculations predict velocities $v > c$. In the following it will be shown that this will not be the case when correct calculations are used.

First the value of $T_{\parallel}$ will be considered. For this purpose, the calculations in chapter 2 are used (see Tab. 2.1):

$$T_{\parallel} = \frac{D}{c\left(1-\frac{v}{c}\right)} + \frac{D}{c\left(1+\frac{v}{c}\right)} = \frac{2D}{c}\gamma^2 \tag{9.09}$$

It is clear at first sight that for the consideration according to this approach the time for going and coming is exactly opposite to Michelson's ideas, but that the addition of both reveals the same result.

In transverse direction the calculation is slightly different. For the calculation, the dependencies shown in Fig. 9.2 are used (see also chapters 2.1.2 and 2.2.3).

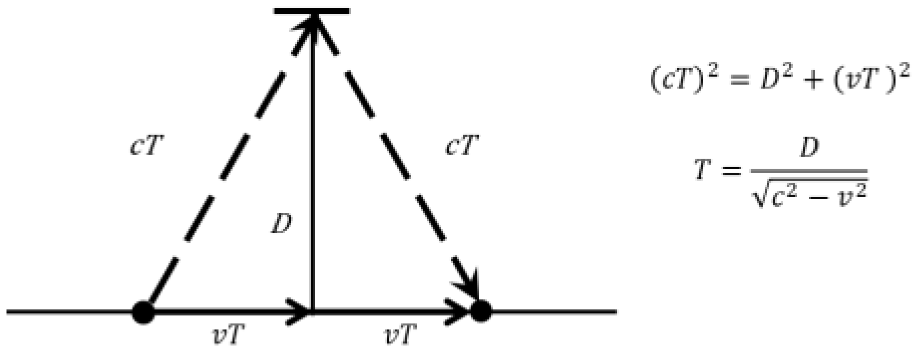



Fig. 9.2: Dependency between distance $D$ to the reflector and velocities $v$ and $c$.

So, the value for $T_{\perp}$ is

$$T_{\perp} = \frac{2D}{\sqrt{c^2 - v^2}} = \frac{2D}{c}\gamma \tag{9.10}$$

and

$$D_{\perp} = \frac{2D}{\sqrt{1-\frac{v^2}{c^2}}} = 2D\gamma \tag{9.11}$$

This result differs from the conclusion of Michelson according to Eq. (9.05). The difference is appearing with the following term and the connected Taylor expansion

$$\sqrt{1+\frac{v^2}{c^2}} = 1 + \frac{1}{2}\frac{v^2}{c^2} - \frac{1}{8}\frac{v^4}{c^4} + \frac{3}{48}\frac{v^6}{c^6} - \cdots \tag{9.12}$$

in contrast to the correct derivation

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{15}{48}\frac{v^6}{c^6} + \cdots \tag{9.13}$$

If terms of 4th order or higher are neglected the results are the same and both calculations can be used without restrictions.

Now the calculation reveals

$$\Delta D = \frac{T_{\parallel} - T_{\perp}}{c} = 2D(\gamma^2 - \gamma) \tag{9.14}$$

and for $v \ll c$ because of

$$\gamma \approx 1 + \frac{v^2}{2c^2} \tag{9.15}$$

the result is

$$\Delta D \approx 2D\left(\left(1 + \frac{v^2}{2c^2}\right)^2 - \left(1 + \frac{v^2}{2c^2}\right)\right) \approx D\frac{v^2}{c^2} \tag{9.16}$$

A. A. Michelson showed in his calculations, that the experiment was able to detect velocities of about 8km/s but a null result was received instead. The motion of the earth around the sun (without further consideration of the motion of the sun compared to the galaxy) reveals however values of approximately 30km/s.

G. F. FitzGerald proposed already in the year 1889 the idea that the length of material bodies changes, according as there are moving through the ether or across [8]. He expected an amount depending on the square of the ratio of their velocities to that of light. The same issue was also predicted independently by H. A. Lorentz three years later [13]. After the Lorentz equations were fully developed it was shown that contraction of space and dilatation of time is covered by the same parameter and the factor $\gamma$ was defined (see Eq. 1.03), i.e. [12,13].

The results of the experiment were further leading to the conclusion that the proposed “luminiferous ether" could not exist. This interpretation is correct in so far, if the radiation of light is thought to be connected in a simple way, as e.g. transporting sound through a carrier medium like gas or a liquid. If in a space, however, which is considered as absolutely at rest, time dilatation and spatial contraction in direction of a moved body belong to the characteristics of space, then a simple solution of the problem is also possible. Considering the possible effects of constant phase velocity, then during changes between different inertial systems any discrepancies will disappear.

Summing up the results of the Michelson-Morley-Experiment it can be stated, that in the direction of movement a contraction of space must take place. This contrasted with the assumptions at the end of the 19th century that ether-wind like in a gas would exist. Further interpretations, however, concerning “luminiferous ether” are not possible.

### 9.1.2 Literature review

The Michelson-Morley-Experiment is discussed in many publications. The “Conference on the Michelson-Morley Experiment” held in Pasadena at the Mount Wilson observatory in 1927 is for sure one of its highlights. Because of the paramount importance of the participating scientists and the detailed discussions laid down in the conference paper published in1928 [49] a very meaningful document of the scientific standard of that time is preserved. The basic understanding has still not changed substantially until today. Because of the particular significance in the following the detailed topics and discussions of the Conference will be presented. Beside the scientific importance there are further interesting historical highlights which will be recognized as well.

First A. A. Michelson presented a report about the historical background [49a]. He told about the first trials at the Helmholtz-Institute in Berlin, which were not successful because of disturbing traffic and were later repeated at the Observatory at Potsdam. The null result that was achieved was questioned by many scientists of that time, because of the short length of the detector arms of the device used. The important improvements realized by E. W. Morley at the University of Cleveland later led to an experiment without doubt.
The theoretical background of the experiment was presented by H. A. Lorentz [49b]. His considerations were far more complex, especially concerning the dependencies of angular measurements which were not covered by A. A. Michelson before (see chapter 7.1.1). Subsequently D. C. Miller [49c] summarized the status of the results of the experiments at that time. He also reported about measurements, which showed positive results concerning the measuring of the ether (Remark: These results could not be repeated in later experiments). R. S. Kennedy then presented information about the special measurement technique of the interferometer [49d].

E. R. Hedrick [49e] and afterwards P. S. Epstein [49f] were covering in their presentations additional theoretical aspects. The focus was directed to the difficulties which occur when a mirror is moved relative to potential ether at rest. A detailed interpretation of the work of A. Righi [50] concerning this subject was presented, although much older publications are existing (e.g. [51,52]). (Remark: Because of the sudden death of the Italian physicist A. Righi only very few and fragmentary records were available. These were summarized and edited by J. Stein S. J. from the observatory of the Vatican [50]). E. R. Hedrick presented a list of 15 publications, which are dealing with theoretical interference-problems concerning the Michelson-Morley-Experiment [49e].

It is noteworthy that some of the results of the cited authors differ significantly. A. Righi expressed the opinion that the Michelson-Morley-Experiment, because of the angular measurement of the mirror through the ether, could not reveal any result; E. R. Hedrick however concluded that these effects exist but could be neglected.

Without further going into detail, it can be stated, that in all cases only the shifting of the mirror in relation to the ether is considered but not the movement of the whole system with the interferometer. H. A. Lorentz made a statement in the discussion, that the presented calculations regarding a moved mirror showed deviations to his results and encouraged an additional survey [49g]. A consideration of phase velocity was not taken into account. H. A. Lorentz died in the year after the conference and no reports exist, whether he ever again dealt with the problem. This is also valid for other authors and no statement of the participants concerning this matter is available.

## 9.2 Kennedy-Thorndike-Experiment

In this experiment performed by R. J. Kennedy and E. M. Thorndike also, like in the Michelson-Morley experiment, an interferometer as testing equipment was used. The chosen set-up is different from the Michelson-Morley-Experiment mainly using measuring arms with different length in the design of the interferometer. In addition, the very stable construction and the extreme accuracy of the temperature control made it possible to conduct long-term measurements. The measuring device was surrounded by a water system that made it possible to keep the temperature deviations at a level lower than 1/1000 °C. With this equipment experiments were conducted that lasted weeks or even months.

In principle the experiment follows the idea, that the measurement device is not moved or tilted but that the deviations in direction to the ether are supposed to be executed by the rotation of the earth and the circulation around the sun, and thus tilting and also accelerations are caused by the movements of the earth [16]. In the literature interpretations exist, where the rotation of the device relates to the Michelson-Morley experiment and only the acceleration is referred to as the original Kennedy-Thorndike experiment [54]. Because of the general situation, that both effects (rotation, acceleration) are always connected to each other, they shall be discussed here together as well.

During this experiment and in following trials with an extraordinary increase of the precision – like for the Michelson-Morley-Experiment – a null-result was achieved.

In the following it shall be demonstrated first that the interpretation by the authors [16] in the year 1932 because of some conceptual shortcomings was not correct. Because of this reason actual considerations follow modified approaches like presented for example by D. Giulini [19]. It shall be demonstrated, however, that these new concepts also contain weak points and that it is possible to overcome this problem by a modified interpretation. This will now be discussed in detail and afterwards a final examination will be presented.

### 9.2.1 Interpretation according to the original publication

A system S is considered, where a light beam during time $dt$ is traveling the distance $ds$ [16]. Then it applies

$$ds = cdt \tag{9.20}$$

If now a system S′ is introduced which is moving relative to S with a velocity of $v$ then this leads to

$$c^2(dt')^2 = (ds')^2 + v^2(dt')^2 + 2vds'dt'cos\theta' \tag{9.21}$$

where $\theta'$ is the angle between the radiation of the light and the moving direction. For $\theta' = 0$ it applies

$$cdt' = ds' + vdt' \tag{9.22}$$

If the results derived by the Michelson-Morley-Experiment are considered, the following relations in longitudinal direction ($\theta$ resp. $\theta' = 0°$) and further in transverse direction ($\theta$ resp. $\theta' = 90°$) will be found.

| | Moved system $S'$ | | System at rest $S$ |
|---|---|---|---|
| $\theta' = 0°$ | $dt' = \dfrac{ds'}{c\left(1 - \frac{v}{c}\right)}$ | $\theta = 0°$ | $dt' = \dfrac{ds}{\gamma c\left(1 - \frac{v}{c}\right)}$ |
| $\theta' = 90°$ | $dt' = \dfrac{ds'}{c}\gamma$ | $\theta = 90°$ | $dt' = \dfrac{ds}{c}\gamma$ |

The integration according to [16] show the following result

$$t'_{\parallel} - t'_{\perp} = \frac{s'_{\parallel} - s'_{\perp}}{c}\gamma \tag{9.23}$$

Here conceptual problems become apparent, because only the path in direction to the reflecting mirror is considered, but not the way back. For the distance to the mirror in moving direction and back different values will appear. Further it is assumed, that spatial contraction and time dilatation are exactly the same, what is not possible at this stage of the interpretation without further assumptions. Therefore, the interpretation of the original publication shall be stopped here and thus switched to modern descriptions of the experiment.

### 9.2.2 Concept according to actual publications

In recent publications (e.g. [19]) the presentation of the experiment is different. It is only possible to derive the equation

$$t'_{\parallel} - t'_{\perp} = B\,\frac{2\left(s'_{\parallel} - s'_{\perp}\right)}{c}\,\gamma \tag{9.24}$$

where the constant $B$ will be measured later, for example by using the Ives-Stilwell-experiment (see chapter 9.3), and then shows a value of $B = 1$.

The real problem concerning the interpretation of the Kennedy-Thorndike-experiment using this concept is the principle of evaluation. Hereby the situation occurs that according to equation

$$\Delta N = f \cdot B\,\frac{2\left(s'_{\parallel} - s'_{\perp}\right)}{c}\,\gamma \tag{9.25}$$

with $f$ as frequency a dependency is established between the number of oscillations of a light beam going and coming the way the from a light source to an interferometer and the connected frequency. If the different length of the measuring arms is taken into account, however, it is clear at first sight that a light beam, which is split and sent out in different directions obviously after reflection does not rejoin at the same place, and that a certain delay will be observed. In the following an alternative interpretation of the experiment will be derived where this condition is respected.

### 9.2.3 New interpretation of the experiment

As already stated, one of the most important conditions of the experimental set-up is the fact, that measuring arms with different lengths are used. Because of this situation it makes no sense to compare the total amount of oscillations of the light beams between these arms. The concept of constant phase velocity of light discussed in chapter 8 opens a different possibility on the apparent effects of interference. When an interaction between light pulses is observed and a comparison at an interferometer is conducted it becomes clear, that the pulse, which is running the way of going and coming at the shorter measuring arm must be considered as delayed compared to the other one.

When the lengths of the measuring arms are defined as $L_C$ (long) and $L_B$ (short) then the time for the delay $T_0$ until the transmitting of a pulse in a system at rest is

$$T_0 = \frac{2L_C}{c} - \frac{2L_B}{c} = \frac{2L_C(1-k_A)}{c} \tag{9.30}$$

with

$$k_A = \frac{L_B}{L_C} \tag{9.31}$$

defined as the constant for the ratio of the arm lengths. The total time for going and coming of a light beam is therefore

$$T_B = \frac{2L_C(1-k_A)}{c} + \frac{2L_C k_A}{c} \tag{9.32}$$

$$T_C = \frac{2L_C}{c} \tag{9.33}$$

where Eq. (9.32) and Eq. (9.33) are obviously equal. If now the experimental set-up is moving with measuring arms longitudinal and transverse to the moving direction, and for the arm in longitudinal direction a spatial reduction of $\gamma$ derived by the results of the Michelson-Morley experiment is valid, then the following calculations will be derived for the different situations

$$T_{\parallel B} = T_{\perp B} = a\frac{2L_C(1-k_A)}{c} + b\frac{2L_C k_A}{c}\gamma \tag{9.34}$$

$$T_{\parallel C} = T_{\perp C} = b\frac{2L_C}{c}\gamma \tag{9.35}$$

where $a$ is an initially unknown constant for the correction of the starting time of the signal at the shorter arm. The additional constant $b$ is introduced because the result of the Michelson-Morley experiment shows that just the ratio between the contraction in longitudinal and transverse direction can be derived but not the exact dimension.

Now equations Eq. (9.34) and Eq. (9.35) are set equal

$$b\frac{2L_C}{c}\gamma = a\frac{2L_C(1-k_A)}{c} + b\frac{2L_C k_A}{c}\gamma \tag{9.36}$$

$$b\frac{2L_C}{c}(1-k_A)\,\gamma = a\frac{2L_C(1-k_A)}{c} \tag{9.37}$$

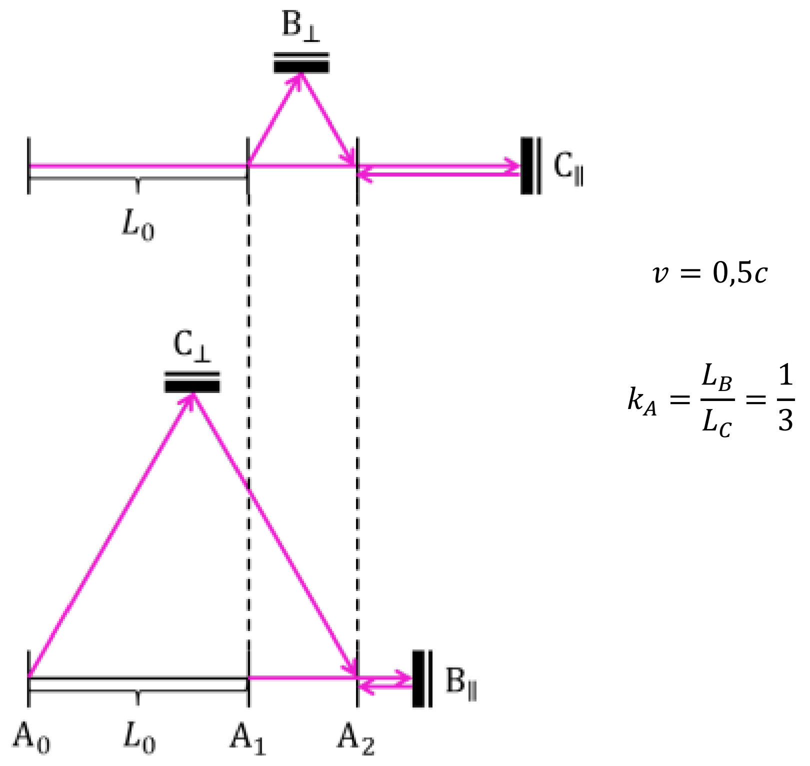



Fig. 9.3: Kennedy-Thorndike experiment: Rotation

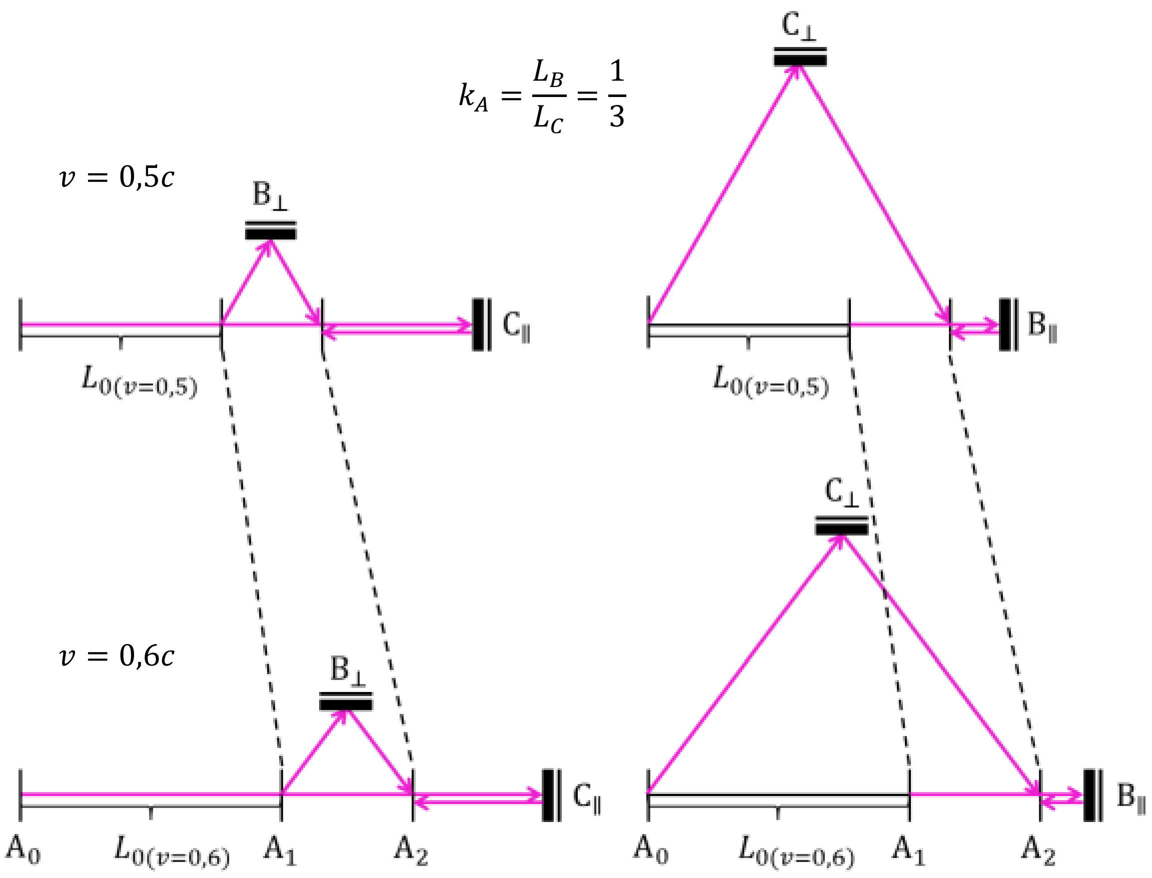



Fig. 9.4: Kennedy-Thorndike experiment: Acceleration

with

$$\frac{a}{b} = \gamma \tag{9.38}$$

The calculation shows that a zero result for the measurements is only possible when the ratio between the constant for the starting time of the shorter measuring arm and the factor for the spatial contraction are exactly equal to the constant $\gamma$ of the Lorentz Equations. Today we know, derived by additional experiments concerning time dilatation (e.g. by Ives-Stilwell, see chapter 9.3), that $b = 1$. During the time of the first execution of the experiment in the year 1932, however, this was not the case.

It is possible to illustrate the experimental set-up as presented in Fig. 9.3 and Fig. 9.4, were for the relation between the lengths of the measuring arms a ratio of 1/3 was chosen. At first the behavior during a rotation is presented (Fig. 9.3) afterwards the situation for acceleration (Fig. 9.4). In reality, it will be generally the case that both situations cannot be separated and will appear together, so that the effects will superimpose each other. Light pulses are transmitted and reflected at mirrors B and C; the denomination $C_{\parallel}$, $C_{\perp}$, $B_{\parallel}$ and $B_{\perp}$ shows, whether the reflection will be in longitudinal or in transverse direction relative to the movement of the system.

The coordinates of the relevant points were calculated and are presented in table 9.1. The format $x, y, t$ was chosen; in the direction of $z$ no movement takes place and so those values were not included (this means $z = 0$). For the relation between longitudinal and transverse direction the spatial contraction was considered by the factor of $\gamma$ according to the results of the Michelson-Morley experiment.

| **Coordinate** | ***x*** | ***y*** | ***t*** |
|---|---|---|---|
| $C_{\parallel}$ | $\frac{L_C}{\gamma\left(1-\frac{v}{c}\right)}$ | 0 | $\frac{L_C}{\gamma c\left(1-\frac{v}{c}\right)}$ |
| $C_{\perp}$ | $\frac{\gamma L_C v}{c}$ | $L_C$ | $\frac{\gamma L_C}{c}$ |
| $A_1$ | $\frac{2\gamma L_C v}{c}(1-k_A)$ | 0 | $\frac{2\gamma L_C}{c}(1-k_A)$ |
| $B_{\parallel}$ | $\frac{2\gamma L_C v}{c}(1-k_A)+\frac{L_C k_A}{\gamma\left(1-\frac{v}{c}\right)}$ | 0 | $\frac{2\gamma L_C}{c}(1-k_A)+\frac{L_C k_A}{\gamma c\left(1-\frac{v}{c}\right)}$ |
| $B_{\perp}$ | $\frac{2\gamma L_C v}{c}(1-k_A)+\frac{\gamma L_C v}{c}k_A$ | $L_C k_A$ | $\frac{2\gamma L_C}{c}(1-k_A)+\frac{\gamma L_C}{c}k_A$ |
| $A_2$ | $\frac{2\gamma L_C v}{c}$ | 0 | $\frac{2\gamma L_C}{c}$ |

Table 9.1: Presentation of coordinates according to Fig. 9.3

The values in Table 9.1 for $A_2$ can be derived according to the effective length of the beam in 4 different ways which are all leading to the same results. This is presented in Table 9.2. It was presented here, that a rotation of the apparatus and the comparison between a moved system and a system at rest show the same results. The calculation reveals further that correction factors for the starting time and for the spatial contraction are equal and must be exactly $\gamma$ in both cases. This will be discussed further in more detail in the next chapter.

The presentation for acceleration in Fig. 9.4 shows the same correlations already presented in chapters 4 and 5 and so there is no need for further evaluation. There are in principle no discrepancies when transitions between systems with different velocities are analyzed.

| **Path over:** | $x$ | $t$ |
|---|---|---|
| $C_\parallel$ | $\frac{L_C}{\gamma\left(1-\frac{v}{c}\right)}-\frac{L_C}{\gamma\left(1+\frac{v}{c}\right)}$ | $\frac{L_C}{\gamma c\left(1-\frac{v}{c}\right)}+\frac{L_C}{\gamma c\left(1+\frac{v}{c}\right)}$ |
| $C_\perp$ | $\frac{\gamma L_C v}{c}+\frac{\gamma L_C v}{c}$ | $\frac{\gamma L_C}{c}+\frac{\gamma L_C}{c}$ |
| $B_\parallel$ | $\frac{2\gamma L_C v}{c}(1-k_A)+\frac{L_C k_A}{\gamma\left(1-\frac{v}{c}\right)}-\frac{L_C k_A}{\gamma\left(1+\frac{v}{c}\right)}$ | $\frac{2\gamma L_C}{c}(1-k_A)+\frac{L_C k_A}{\gamma c\left(1-\frac{v}{c}\right)}+\frac{L_C k_A}{\gamma c\left(1+\frac{v}{c}\right)}$ |
| $B_\perp$ | $\frac{2\gamma L_C v}{c}(1-k_A)+\frac{\gamma L_C v}{c}k_A+\frac{\gamma L_C v}{c}k_A$ | $\frac{2\gamma L_C}{c}(1-k_A)+\frac{\gamma L_C}{c}k_A+\frac{\gamma L_C}{c}k_A$ |

Table 9.2: Calculations of value $A_2$ using paths over the positions $C_\parallel$,$C_\perp$, $B_\parallel$ and $B_\perp$. All calculations reveal the same result.

### 9.2.4 Evaluation of results

When the results of the experiments of Michelson-Morley and Kennedy-Thorndike with the apparent zero results are viewed closely it becomes clear, that a final definition of the constants $a$ and $b$ is not possible without further information and a statement about the validity of the Lorentz equations will remain incomplete. Usually the Ives-Stilwell-experiment will be used for this purpose, which is described shortly in chapter 9.3. It is in addition also possible to use other simple possibilities to validate the results.

If for example the assumption is made that $a = 1$ and thus $b = 1/\gamma$, this would mean that a moving system is not subject to any time dilatation at all, but on the other hand the spatial contraction in moving direction would be of the factor $\gamma^2$ and in addition in transverse direction the factor $\gamma$ must be taken.

Tests concerning effects like these are not complex and could be subject to several different experiments. This would be possible for example for the exchange of signals between moving observers (see chapter 2.1) or for the frequency measurements of moving bodies (chapter 8). Beside the differences in the measurements, furthermore the principle of relativity will be violated, and different results must appear whether a situation is monitored by an observer at absolute rest or from a moving system. The deviations are not only valid for this example but for all other configurations when the constants are not chosen as $a = \gamma$ and $b = 1$.

The title of the publication from Kennedy and Thorndike was “Experimental Establishment of the Relativity of Time”. Because of the dependencies between the constants $a$ and $b$ expressed above it is today generally rejected, that the approach expressed in the headline was successful, see e.g. [19]. However, if in the year 1932 the authors would have carried out a correct interpretation by using the principle of constant phase velocity, then already at that time the statement would have been possible. Independent from this, this experiment with all the improvements in accuracy carried out in the meantime, is an important tool for the understanding of the nature of signal exchange between moving observers.

## 9.3 Further important experiments

There are many further pioneering experiments connected with the nature of light and radiation. Those with high importance concerning the evaluations presented here will be discussed shortly in the following.

### *a) Rømer-Experiment*

This was the first experiment with the attempt to measure the speed of light. Most important was, that it was proved for the first time (in the year 1676!) that the speed of light is limited. The detection was conducted by O. C. Rømer measuring the occultation of the Jupiter moon Io that occurs earlier when the planet is close to earth and later when the distance is larger. With his measurement results C. Huygens in 1678 was able to calculate the speed of light and he found a value of ca. 213.000 km/s which is approximately 71% of the correct result.

### *b) Aberration of light*

This experimental effect was established the first time by J. Bradley in the year 1725. He discovered that the star Gamma Draconis showed a small deviation of its position in the sky during the progress of a year and supposed that this was caused by the finiteness of the speed of light. His measurements already achieved a precision of 2% (for further details see also chapter 1.3).

### *c) Double star experiment*

The examination of double star systems provided evidence for the first time that the speed of light is independent of the speed of the object that is transmitting the signals. These considerations were mainly carried out by W. de Sitter, who was able to verify by spectroscopic examinations that the addition of the speeds of light and the emitting source would lead to a violation of Kepler’s laws [55].

*d) Ives-Stilwell-Experiment*

This experiment is confirming directly that time is running more slowly for a moved observer compared to a reference system [17,18]. To prove this the transversal Doppler Effect of light was investigated using canal rays that were approaching or increasing the distance to the installed measuring equipment. The values found are showing impressively the value of the Lorentz-Factor $\gamma$.

*e) Trouton-Noble-Experiment*

In this case a charged capacitor was taken, which could turn free around an axis. In case of evidence of the ether it would tilt around this axis because of a reaction which would be caused by the rotation of the earth. The basic principle of this experiment is comparable to the Michelson-Morley-Experiment. Although electromagnetic effects are not part of the considerations presented here the mentioning of this important experiment shall not be missed [56].

If additional information is required further experiments can be found in other publications (e.g. [19,21,57,58]).

## 9.4 Final examination of the experiments

In the year 1949 H. P. Robertson was the first to establish a summarizing classification of the different types of measurements and created a concept that is still in use today [59]. The following measuring methods and the significance connected with these are defined:

1. *Michelson-Morley:*
   The total time required for light to traverse a certain distance and return is independent of its direction.

2. *Kennedy-Thorndike*
   The total time required for light to traverse a closed path is independent of the velocity of the system compared to an arbitrary reference system.

3. *Ives-Stilwell*
   The frequency of a moving atomic source is reduced by the factor $\gamma$ compared to an arbitrary reference system.

Modern presentations of the experiments are sometimes using slightly different interpretations, but the description shown here is very close to the first classification by Robertson.

When the relations of the invariance of phase velocity presented before are considered, it can be stated that the new improved interpretation of the experiments is leading to a better understanding of the processes, but that the fundamental results of the experiments are still valid. The Michelson-Morley and the Kennedy-Thorndike experiment are not able to describe the situations appearing in moving systems in full detail. It is possible, however, to use other simple experiments to validate the results (see chapter 9.2.4).

The question remains, why the great importance of the phase invariance of light between systems moving relative to each other was not in focus and is not part of the discussion till now. The fundamental principle belongs to the standard knowledge of today's physics, e.g. [46a]. The effect of the movement of an experimental set-up was also discussed quite often (see e.g. [49,50,51,52]). Further comprehensive theoretical examinations concerning the "invariance of phase-velocity" exist [27]. Despite of the great importance of the discussed matter for modern physics up to now no approach was made to combine these findings. It seems to be highly likely, that the results presented here are caused mainly by the consequent approach regarding the observation of a system at rest compared to moving systems and following the resulting relations.

Finally, some examples shall be presented, how the precision of measurements was developing in the last decades.

- In the year 1960 the definition of the length of 1 meter was defined in the following way using the wavelength: "The metre is the length equal to 1650763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p10$ and $5d5$ of the krypton 86 atom" [87].
- This definition was valid for many years and was then replaced by a new regulation with the time as basis. Since 1967 the second has been defined as the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of cesium 133 atoms [88].

Without the principle of invariance of the phase velocity both definitions are not possible, because already smallest movements relative to reference systems containing one of these experiments would have led to discrepancies in observations.

There is a further aspect referring to the invariance of phase velocity. This is the "frequency comb", where pulses of extreme shortness are produced with a femtolaser and then reflected in a mirror-system to interfere. Thus, a standing wave is produced that is also referred to as "comb" (to background and historical development see e.g. [53]). It is interesting that this technique is a "hybrid" type of generation of light; the extreme short pulses can be interpreted in their collectivity as waves. Here a classical interpretation with the comparison of frequencies makes definitely no sense at all.

# 10. Electromagnetism and Gravity

In the 19th century, electrical and magnetic effects were intensively researched. As already described in chapter 1.4, the invention of the first functional battery by Alessandro Volta made experimental investigations possible. The chapter also summarizes further details on the key developments and the many people involved.

The most important result is that all electromagnetic processes can be summarized in the representation of Maxwell's equations. These are listed in chapter 10.1, followed by a formal comparison with the conditions concerning gravity. To understand these relationships, a basic knowledge of vector calculus is required, the most important elements of which are summarized briefly in Appendix E.

## 10.1 Maxwell's equations

The system of Maxwell's equations consists of 4 laws. Their names and a brief explanation are given below. The formulaic representation and the basic meanings are summarized in Table 12.1. Table 12.2 shows the designation of the formula symbols and the associated dimensions.

1. *Gauss's law*
   In the physical fields of electrostatics and electrodynamics, Gauss's law describes the electrical flow through a closed surface. It is named after the mathematician Carl Friedrich Gauss, who developed the integral theorem named after him for a vector field.
2. *Gauss's law for magnetism*
   Analogous to the electric field, this describes the magnetic flux through a closed surface.
3. *Faraday's law*
   The law of induction, discovered by Michael Faraday, describes the structure of electric fields.
4. *Ampère's law with Maxwell's addition*
   Based on André-Marie Ampère's law, this describes the structure of a magnetic field.

For a better understanding of the relationships, the 4 Maxwell equations in Table 10.1 have been arranged in such a way that the sequence from the static electric field via the dynamic changes in electric and magnetic properties leads to the static magnetic field. The coupling of the electric and magnetic field constants represents the connection between the two parts. In the right half of the table, the respective meaning of the relationships has been added in short words.

| | |
|---|---|
| (1) $\vec{\nabla}\cdot\vec{E} = \frac{\rho_{el}}{\varepsilon_0}$<br>$\vec{F} = \frac{Q_1\,Q_2}{4\pi\varepsilon_0 s^2}\cdot\vec{s}_0$<br>$\varepsilon_0 = 8{,}8542\cdot 10^{-12}\,\frac{C^2}{Nm^2}$ | **Electric Field**<br>Source: Electric Charge<br>Charges of the same type repel each other |
| (3) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ | The change of a magnetic field $\vec{B}$ causes the build-up of an electric field $\vec{E}$ (in the form of a closed loop) |
| $\varepsilon_0\mu_0 = \frac{1}{c^2}$ | The field constants are coupled with the speed of light |
| (4) $\vec{\nabla}\times\vec{B} = \mu_0\vec{j}_{el} - \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$ | The flow of an electric current $\vec{j}_{el}$ and the change of an electric field $\vec{E}$ cause the build-up of a magnetic field $\vec{B}$ |
| (2) $\vec{\nabla}\cdot\vec{B} = 0$<br>$\vec{M} = \vec{m}\times\vec{B}$<br>$\mu_0 = 1{,}2566\cdot 10^{-6}\,\frac{Ns^2}{C^2}$ | **Magnetic Field**<br>Free of sources (closed loop)<br>Similar poles repel each other |

Tab. 10.1: Maxwell's equations and their interpretation (definition of symbols in Table 10.3) The numbers of the laws precede the respective formula.

## 10.2 Comparison between electric field and gravity

Due to the formal similarity between the electric field and the gravitational field, it was assumed early on that Maxwell's equations should also apply here. Heaviside was the first to put forward this thesis in 1895. Today, there is a general consensus that this assumption is correct but only applies to the limit range of small masses and velocities [94]. For other conditions, especially when processes with large masses, such as black holes, are considered, other relationships apply and space curvature etc. must be taken into account, as is the case in the general theory of relativity, for example.

There is a formal difference between the representations of the electric field and gravity. Because the relationships for the electric field were derived for a homogeneous situation (as it is the case in a capacitor, for example, with Q as charge and s as plate distance), but gravitation for a spatial distribution (with m as mass, r as radius), the calculations for the forces differ:

| Force in electric fields | Force in gravitational fields |
|---|---|
| $\vec{F} = \frac{Q_1\,Q_2}{4\pi\varepsilon_0 s^2} \cdot \vec{s}_0$ (10.01) | $\vec{F} = -\,G\,\frac{m_1\,m_2}{r^2} \cdot \vec{r}_0$ (10.02) |

Since the following task is a comparison of the fields, it makes sense to standardize the representation and convert one of the quantities accordingly. If the gravitational constant is chosen for this, the result is

$$G' = \frac{1}{4\pi G}$$

If this modified formula is used, a comparison results in the form shown in Table 10.2. Further the Maxwell equations are presented here in a modified form, in which the magnetic field constant is not used and the coupling with the speed of light is considered instead (see Tab 10.1). In this way, it is not necessary to redefine corresponding quantities for the gravitational field. The resulting analogy to Maxwell's equations leads to the definition of a system of equations whose physical meaning is generally interpreted today as "Gravitoelectromagnetism (GEM)" [94].

| Maxwell's equations for Electromagnetism | Maxwell's equations for Gravitoelectromagnetism |
|---|---|
| (1) $\vec{\nabla} \cdot \vec{E} = \frac{\rho_{el}}{\varepsilon_0}$ | (1) $\vec{\nabla} \cdot \vec{E}_g = -\frac{\rho_g}{G'}$ |
| (3) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ | (3) $\vec{\nabla} \times \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$ |
| (4) $\vec{\nabla} \times \vec{B} = \frac{\vec{J}_{el}}{\varepsilon_0 \cdot c^2} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$ | (4) $\vec{\nabla} \times \vec{B}_g = -\frac{\vec{J}_g}{G' \cdot c^2} + \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t}$ |
| (2) $\vec{\nabla} \cdot \vec{B} = 0$ | (2) $\vec{\nabla} \cdot \vec{B}_g = 0$ |

Tab. 10.2: Application of Maxwell's equations to the gravitational field. The numbers of the laws precede the respective formula.

It is assumed here that the speed of light c and the propagation speed of gravity are the same.

The equations shown correspond to each other with the difference that the prefixes for equations 1 and 4 are different. For equations 1, this firstly has the simple meaning that masses attract each other, while similar charges repel each other. With regard to equation 4, it follows in the same way that poles of the same direction in the GEM field do not repel each other as in electromagnetism but attract each other.

In this context, there is the interesting question of whether there is an equivalent for gravity for positive and negative charges. This could apply to the pair of matter/antimatter. Of great importance for theoretical considerations is whether matter and antimatter attract, repel or, as some theories predict, attract each other more weakly than pure matter. There was a first breakthrough in this regard in 2023, when investigations at CERN on antihydrogen atoms showed that they are attracted by the Earth's gravity [95]. This is one of the most interesting current experiments, the accuracy of which is to be further increased in order to clarify fundamental questions.

| | Physical Variable | Dim. | | Physical Variable | Dim. |
|---|---|---|---|---|---|
| $\vec{E}$ | Electric field | $\frac{N}{C}$ | $\vec{E}_g$ | Gravitational field | $\frac{m}{s^2}$ |
| $\vec{B}$ | Magnetic flux | $\frac{Ns}{Cm}$ | $\vec{B}_g$ | Gravitomagnetic field | $\frac{1}{s}$ |
| $\vec{M}$ | Moment | Nm | $\vec{m}$ | Magnetic dipole moment | $\frac{Cm^2}{s}$ |
| $\vec{j}_{el}$ | Electric current flow | $\frac{C}{m^2 s}$ | $\vec{j}_g$ | Mass flow | $\frac{kg}{m^2 s}$ |
| $\rho_{el}$ | Electrical charge density | $\frac{C}{m^2}$ | $\rho_g$ | Mass density | $\frac{kg}{m^2}$ |

Tab. 10.3: Definition of the used physical variables with dimensions.
For the definition and application of the Nabla operator $\vec{\nabla}$ see Appendix E

Despite the formal similarity between the variables shown in Table 10.2 and Table 10.3, there are substancial differences in their characteristics. This will first be considered for the electric field and gravitational field. If the difference in the respective attractive forces between a proton and an electron is calculated in a simple example, the formulae (10.01) and (10.02) can be used. The values for the specific quantities are listed in Table10.4. If the values are used, the result for this case is an extreme difference between the electric and gravitational forces of attraction, namely by a factor of $2{,}27 \cdot 10^{39}$!

| | | | |
|---|---|---|---|
| Mass Proton | $m_P = 1{,}6726 \cdot 10^{-27}\text{kg}$ | Mass Electron | $m_E = 9{,}1094 \cdot 10^{-31}\text{kg}$ |
| Electr. Charge Proton | $\text{Q}_\text{P} = 1{,}6022 \cdot 10^{-27}\text{C}$ | Electr. Charge Electron | $\text{Q}_E = -1{,}6022 \cdot 10^{-27}\text{C}$ |
| Gravitational constant | $G = 6{,}6743 \cdot 10^{-11}\,\frac{\text{m}^3}{\text{kg s}^2}$ | Electric constant | $\varepsilon_0 = 8{,}8542 \cdot 10^{-12}\,\frac{\text{C}^2}{\text{Nm}^2}$ |

Tab. 10.4: Values of the physical quantities used for the calculations.

Note on Table 10.4: The values for $\varepsilon_0$ are often given in the literature with the dimension As/Vm. This can be easily converted using the power $P$ in watts [W] and results with the charge C (Coulomb) as As (Ampere-seconds)

$$\left[1\text{W} = 1\,\frac{\text{kg m}^2}{\text{s}^3} = 1\,\frac{\text{Nm}}{\text{s}} = 1\ \text{VA}\right]$$

The difference between the electric field and the gravitational field lies not only in the magnitude of the attractive force, but above all in the fact that electric charges compensate each other in everyday life, i.e. every positive atomic nucleus is opposed by a negative electron. In addition, electric charges can be shielded. In the case of gravity, on the other hand, all masses add up and, according to current knowledge, the effective attractive forces cannot be influenced in any way.

Other important differences are that electric charges always occur as multiples of the elementary charges, whereas gravity has no known smallest indivisible unit. In addition, the kinetic energy of masses is dependent on the state of motion, whereas this does not apply to electric charges. Furthermore, permeability effects are unknown for gravity.

Despite the small effects, the gravitational balance developed by the Englishman H. Cavendish [1731-1810] made it possible to determine density differences in the earth and calculate the gravitational constant as early as 1798.

Direct experimental proof of the existence of gravitomagnetism in the form shown here has not yet been achieved on the Earth's surface due to the extremely small effects that occur. According to calculations by D. Giulini, a gyroscope set up at the North Pole would cause a precession at a speed of 0.6 milliarcseconds per day; given the current experimental conditions, this is still 1 to 2 orders of magnitude outside today’s detection limits [96].

In cosmic dimensions, on the other hand, larger effects occur, whereby the shape of such a field can be determined by calculations. Fig. 10.1 shows the characteristics of a gravitomagnetic dipole field in a graphical representation, evaluated at points at an angular distance of 30°, which lie on a circle around the center [96]. In the center is the rotating star, whose angular momentum is symbolized by an upward-pointing arrow (vector). It generates the dipole field.

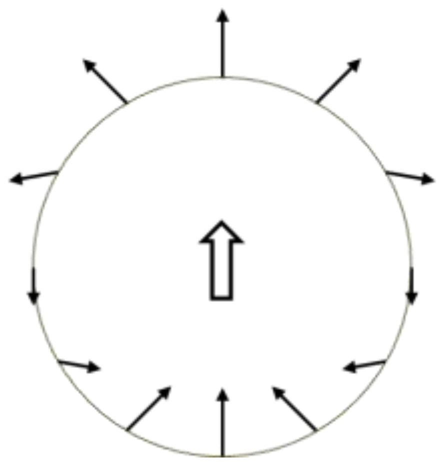


Fig. 10.1: Expression of a gravitomagnetic dipole field generated by a rotating star in the center [96].

Experimental proof was only possible with the launch of the Gravity Probe B into space in 2004, with which the interactions between 4 counter-rotating gyroscopes and the rotating Earth were investigated. After lengthy and complicated evaluations due to interference effects that occurred, results were published in 2011 that were obtained as part of investigations to verify the general theory of relativity [97]. These are the effects of spacetime curvature and the Lense-Thirring effect. For details, please refer to further literature [96, 97].

Finally, another interesting aspect should be considered. For years there have been investigations into the amplification of gravitomagnetic effects, similar to those observed when the permeability of magnetic fields is increased (e.g. by feeding an iron core into a magnetic coil). Such evidence would have enormous implications for the foundations of the theory of general relativity and is therefore subject to special observation. In one of the experiments, for example, a large quantity of rotating liquid helium was used in a superconducting Nb tube, and a gyroscope was placed in it. However, after initial positive results for increasing the gravitomagnetic effect, it became apparent that these could not be reproduced [98]. None of the experiments carried out so far have been successful and therefore no effects on the theory are recognizable.

# 11. Limits of the Theory of Special Relativity

It was already demonstrated at lengths that an impressive number of examples exist, which are conforming to the Theory of Special Relativity. This was shown e.g. for kinematic considerations of moving observers, further it was proved for the processes during clock transport and also for the relations between mass, momentum, force, energy and for elastic or non-elastic collisions of moved bodies and further the relativistic observation of rocket acceleration. It was shown for a large number of configurations that using the Lorentz-Transformation no differences can be found for a system at rest or for moving observers and that no possibility exists to decide inside a system whether it is moving or at rest. This is in accordance with the postulates of the Theory of Special Relativity which stipulates that all observers are considered as equal and so no evidence could be found that the principles of relativity are not valid.

All these examples share the basis that the transport of signals is occurring with the speed of light. However, when superluminal velocities are considered, which were discovered during tunneling processes, it can be shown that – provided that also information are transported with superluminal speed (a concept which is still controversially discussed) – the appearing effects are not in accordance with Special Relativity. This will be reviewed in detail. Finally, the situation concerning synchronization after acceleration will be discussed and it will be shown that in this case conflicts will appear.

## 11.1 Superluminal effects during tunneling processes and their significance

Optical examinations with prisms were conducted already since a very long time. It is well known that Newton, Huygens, and many other scientists focused their work on the fundamental relations.

With the development of modern research methods, the examination of effects based on quantum mechanics started. Fritz Goos (1883-1968) and Hilda Hänchen (1919-2013) were the first to find that a linear polarized light-wave during the transition from a medium with a higher to a lower optical thickness is not reflected at the boundary layer but at a virtual surface with an orientation parallel to it situated inside the medium with the lower optical thickness. It is not possible to explain this observation with a standard model and quantum mechanics are used instead. The investigations were made during the 2nd world war in Berlin and published partly not before 1947 [60,61].

Further examinations revealed that optical boundaries generate tunneling effects, which are independent of their thicknesses [62]. This led to intensive discussions concerning the appearing of superluminal velocities.

### 11.1.1 Tunneling effects

Tunneling effects and connected measurements of velocities of electromagnetic waves during passing of an optical boundary were already part of numerous examinations. For a better understanding a comprehensive survey about the investigations using prisms and other optical devices carried out with waves of different frequencies, published by H. G. Winful, is recommended [63].

Out of the multitude of possibilities an example shall be chosen, where double prisms are used for experiments. A typical experimental set-up is presented as shown in Fig. 11.1.

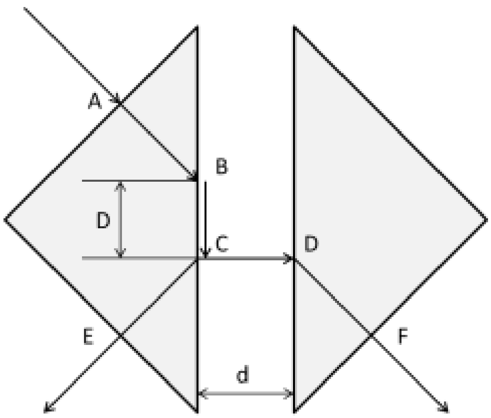



Fig. 11.1: Experimental set-up for measuring of tunneling effects (after [64])

An electromagnetic wave is reaching point A of a prism and is transmitted into the body. If an appropriate angle is taken (see e.g. [64]) the wave will be reflected at point B. When another identical prism is situated opposite to it, a tunneling effect will be observed which can only be explained using quantum mechanics. In this case the paths $\overline{\mathrm{BC}}$ and $\overline{\mathrm{CD}}$ will be passed without delay. The largest part of the wave will reach point E, a much smaller part is detected at F. The exit of both will be exactly at the same time. Experiments of this type allow the use of set-ups with large dimensions, though the intensity of the beam on the way $\overline{\mathrm{DF}}$ is strongly dependent on the distance $b$ of the prisms. Experiments with $d = 280$ mm were already performed and the corresponding effects could be observed. Because of the multitude of possible experiments, it is referred for further details to publications with a general survey [63,64].

At present there is no consensus concerning the interpretation of the observed results at all. Very often the argument is used that superluminal velocities occur, but that it is impossible during these experiments to transport information faster than light. The reason for this is that the results of the measurements are interpreted not as the velocity of a single pulse but as an effect caused by the group velocity of a signal. Complex information (e.g. speech) can only be transported by a wave-packet and the velocity of this is supposed to be the speed of light. Because of the importance of this argument in the following a short introduction concerning this matter shall be presented before a final investigation is made.

To describe the effect of group velocity in a simple way in publications dealing with this matter some analogies are found like the comparison between fly and elephant, the interpretation of a tortoise race or the consideration of the behavior of a very long train [63,65]. Specially the last example is very suitable to understand the circumstances and shall be discussed shortly:

A train needs for the travel between 2 points a defined time span. If this train is considered as extremely long, then a simple definition of departure and arrival time is no longer suitable and differences will occur, whether the locomotive, the middle of the train or the end is observed. If in a second tour a train with the same length travels with the same speed the same distance, and during the trip wagons are uncoupled than the middle and of the train, which is consequently moving forward during the trip, is arriving earlier than in the example discussed before. However, independent of this the locomotives of both trains are reaching the destination at the same time. Following this interpretation, the velocity of the middle of the train (the group velocity) is faster than the speed of the locomotive.

Transferred to the discussed example it is obvious, that during the tunneling of the wave no even damping occurs but that the end of the wave-packet must be perpetually cut off. In this case the group velocity is faster than light although this is not valid for the front and so in this case no violation of the Theory of Relativity would occur.

The authors dealing with superluminal velocities measured at prisms and other optical devices are using quite different interpretations for the results. Beside the argument concerning group velocities described before this covers a total denial of superluminal effects because of complete misinterpretation of the experimental results [63], assumed contamination effects which demands an infinite size of the prisms when a reasonable signal transfer is required [66] or the final discussion is left completely open [67,68]. Some authors still today have the opinion that it is possible during these experiments to transport information with a speed faster than light [65,69]. The main reason for this is the observation, that a tunneled wave after amplification has the same shape compared to a reflected wave and that it shows no cut off like it must be assumed when the above-mentioned example of group velocity would be valid.

However, for clearly documented evidence it is not necessary to transport complex information, but a single pulse would be sufficient (like using the Morse alphabet). Considering this, the thesis that measurements are not possible because of lack of information transport, is assessed as not plausible. If the distinct detection of a transmitted pulse with superluminal velocity would be possible, then this result would cause severe consequences for Special Relativity which will be discussed in the following.

### 11.1.2 Significance of superluminal velocities for Special Relativity

Whereas all considerations discussed so far have led to the perception that observers during the exchange of signals in a system at rest or when moving will find the same measuring results, this will definitely not be the case when information is transmitted using superluminal velocities. This can be derived easily when the situation presented in Fig. 11.2 is analyzed.

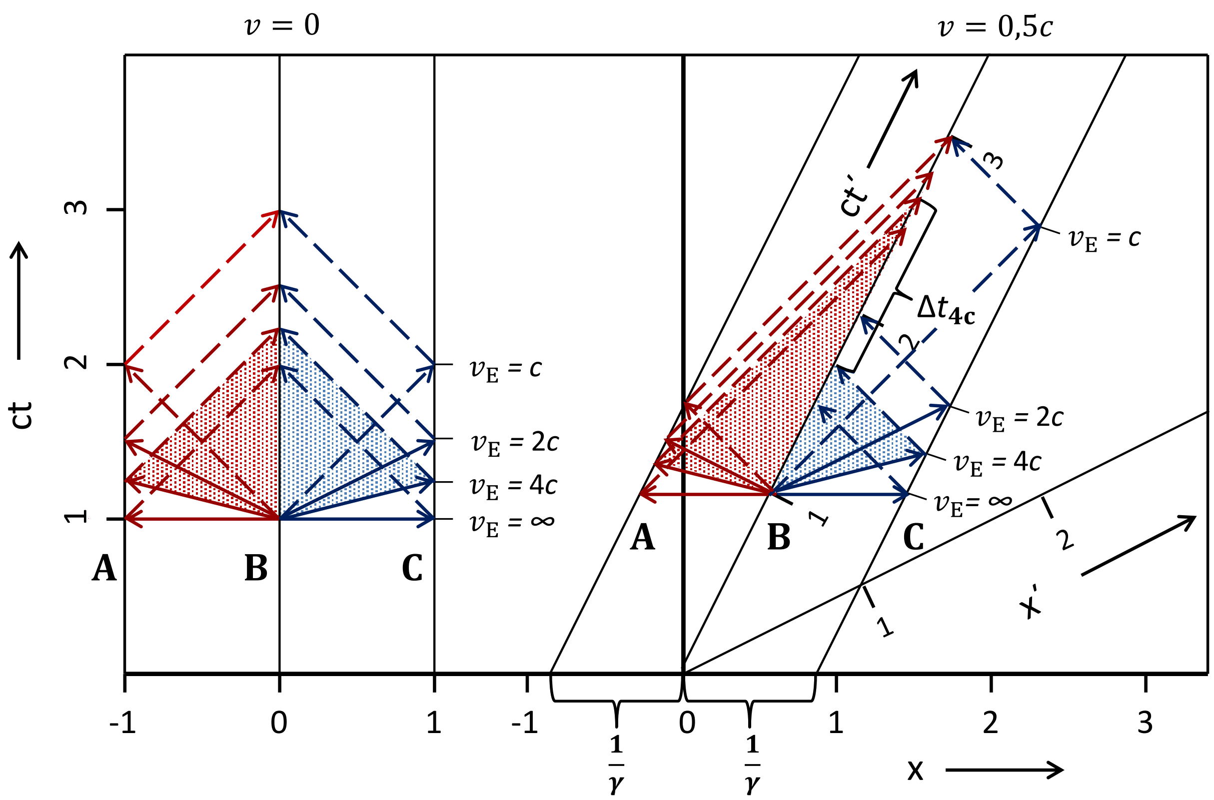



Fig. 11.2: Differences between a system at rest and a moving observer when information is transmitted with superluminal velocity.

On the left-hand side as usual a system at absolute rest is presented. The transmission of signals is carried out with superluminal velocity $v_E$ between observer B to the points A and C. Immediately at arrival a responding light signal ($v = c$) is triggered and sent back to B. Because the experimental set-up is symmetrical the arrival at B will be at the same time.

On the right-hand side the same situation is presented for a moving system. Because observers A and C have different positions, the light signal will arrive at different times at B. The time span is depending on the superluminal velocity (values for $v_E = 2c, 4c$ and $\infty$ are shown) and also on the speed of the system $v_S$ (in this case values of $v_S = 0$ and $0{,}5c$ were chosen). This diagram also includes the values for the time difference $\Delta t_{4C}$ that would appear when a superluminal velocity of $v_E = 4c$ would be achieved.

The time span relevant for different superluminal velocities can easily be derived using simple geometric considerations as presented in Fig. 11.3.

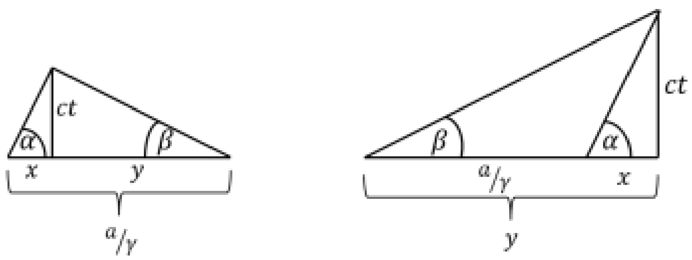



Fig. 11.3: Geometric dependencies of used dimensions for $v_E = 2c$

The following general dependencies apply

$$tan\alpha = \frac{ct}{x} = \frac{c}{v_S} \qquad tan\beta = \frac{ct}{y} = \frac{c}{v_E} \qquad \Rightarrow \qquad xv_E = yv_S \tag{11.01}$$

The cases for the signal transmission in moving direction and opposite to it must be treated separately. It applies

| *Signal opposite to the moving direction* | *Signal in moving direction* | |
|---|---|---|
| $\frac{a}{\gamma} = x + y$ | $\frac{a}{\gamma} = y - x$ | (11.02) |
| $\Rightarrow \quad xv_E = \frac{a}{\gamma}v_S - xv_S$ | $\Rightarrow \quad xv_E = \frac{a}{\gamma}v_S + xv_S$ | (11.03) |
| $t_1 = \frac{a}{\gamma(v_E+v_S)}$ | $t_3 = \frac{a}{\gamma(v_E-v_S)}$ | (11.04) |

To calculate the entire time for the signal exchange the part for the way back must be added. Thus, the total time for the path B→A→B is:

$$t_T(C) = t_1 + t_2 = \frac{a}{\gamma(v_E+v_S)} + \frac{a}{\gamma(c-v_S)} \tag{11.05}$$

The path B→C→B leads to

$$t_T(A) = t_3 + t_4 = \frac{a}{\gamma(v_E-v_S)} + \frac{a}{\gamma(c+v_S)} \tag{11.06}$$

To discuss the influence of the signal velocity on the measuring effect finally the difference must be determined

$$t_T = t_T(C) - t_T(A) = \frac{a}{\gamma(v_E+v_S)} + \frac{a}{\gamma(c-v_S)} - \frac{a}{\gamma(v_E-v_S)} - \frac{a}{\gamma(c+v_S)} \tag{11.07}$$

and be compared with $v_E \rightarrow \infty$. Hence

$$t_D = \frac{t_T}{t_\infty} \tag{11.08}$$

In Fig 10.4 the results for different velocities for the signal and the used reference systems are presented. Generally, it can be stated that the speed of the system has only limited influence on the results and a noteworthy effect appears at remarkably high values. Further it becomes clear that the signal velocity of $v_E = 2c$ is already reaching half values which are calculated for $v_E \to \infty$. The relations show, that it is not necessary to suppose signal velocities of extreme magnitude because the sensitivity of the measurement is extraordinarily strong.

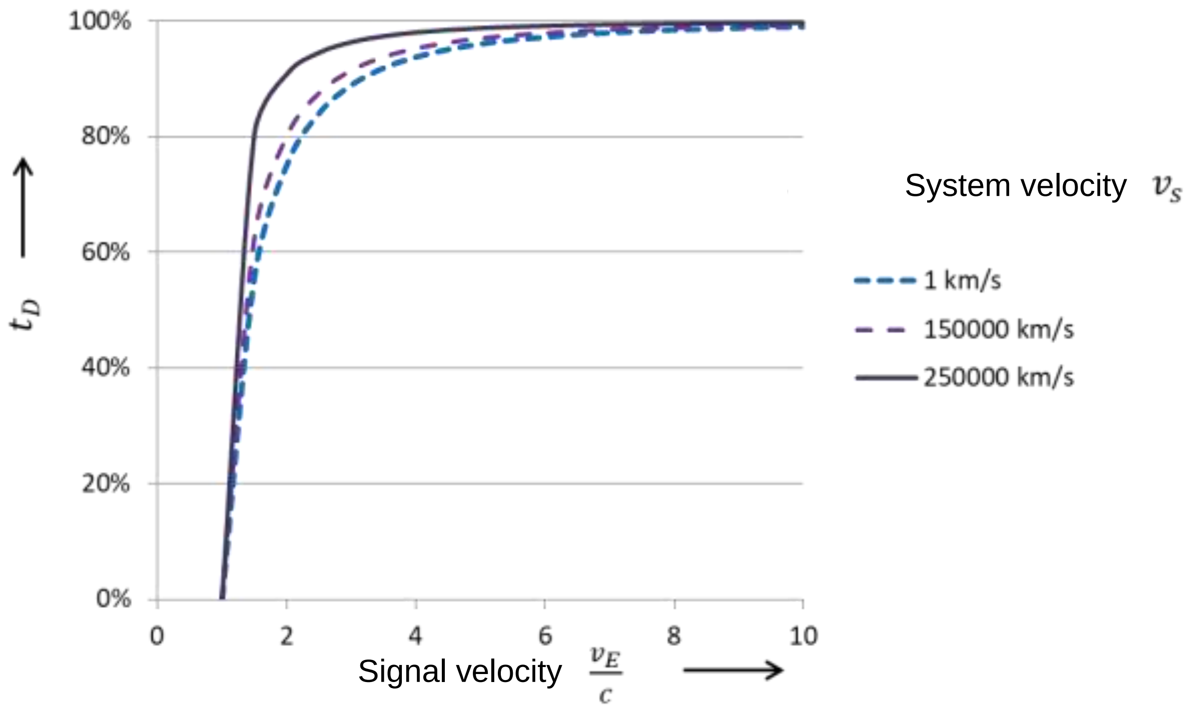



Fig. 11.4: Expected measuring effect $t_D$ in relation to signal velocity $v_E$ and system velocity $v_S$

Further additional considerations concerning the existence of superluminal velocities exist, where it is assumed that in this case the principle of causality would be violated [63]. Other publications are denying effects like this [64,65].

In general, the violation of the principle of causality would stand for the fact, that an incoming signal would be received earlier than the outgoing signal. This would mean that a negative time must be assumed, for which no experimental evidence exists. It is clear, however, that inside a system with high velocity compared to a system at rest (as shown at the right-hand side of Fig. 11.2) the incoming signal will arrive earlier (case B → A → B) or later (case B → C → B) as expected according to the synchronization procedure before. In this case no violation of the principle of causality will occur because the signal measured is earlier or later (depending on the speed of the system) than expected due to synchronization but in no case before the start of the procedure.

It shall be mentioned that the existence of superluminal velocities for the transport of signals would lead to severe conflicts with the principle of relativity which cannot be solved. Differences in measurements between systems would occur, which travel at different speed. An undisputed measuring effect would provide evidence that a system of absolute rest must exist. In chapter 13.1 a possible experiment to prove this will be presented and the dimensions of values which can be expected will be discussed in detail.

## 11.2 Synchronization after acceleration

In the past many scientists tried to detect the one-way speed of light in a moved system in a direct way. Concerning this problem different concepts were taken into consideration; one of these is the "slow clock transport". The principal idea in this case is that in a moved laboratory a clock is slowly transported from one end (e.g. the back end) to the other side and then compared with a second clock at that place which was synchronized before. It was already shown, that during this transport, irrespective of the chosen speed, the synchronization remains unchanged, and a zero result will be achieved (see also chapter 5).

Another possible alternative, which was first considered by E. Dewan and M. Beran [70] and later reviewed in detail by J. S. Bell [71] and also by D. J. Miller [72] and F. Fernflores [73] is the investigation of changes in systems before and after acceleration. In this case observers, which are transporting synchronized clocks, are accelerated homogenous in a way, that they show the same speed compared to each other before and after. It is required that the acceleration for all observers shall be the same; further preconditions are not necessary.

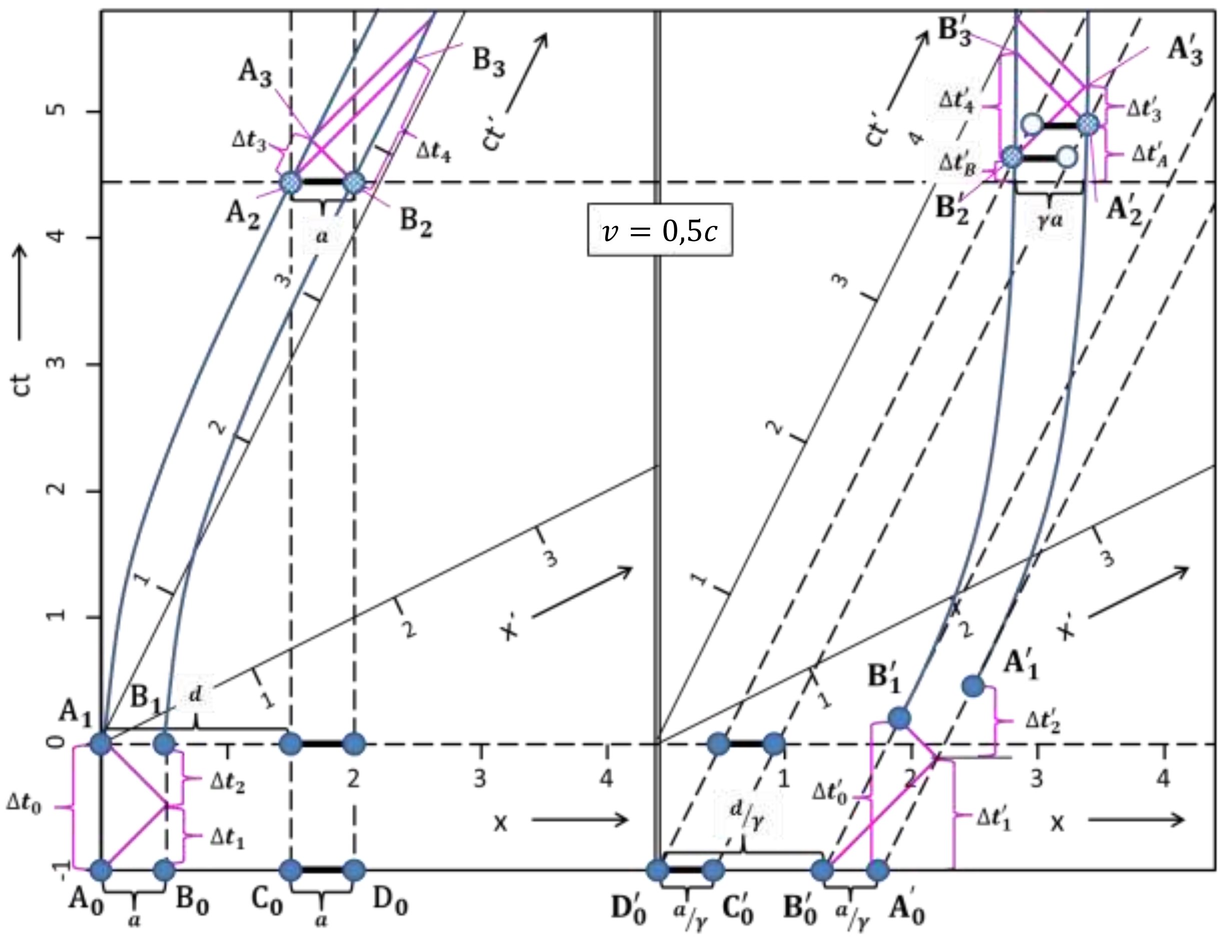



Fig. 11.5: Exchange of signals before and after an acceleration ($v = 0{,}5c$)
a) Left: System at rest to moved system
b) Right: Moved system to system at rest

At first the situation shall be examined, when the observers are lined up in direction of acceleration. The configuration of this experimental set-up is presented in Fig. 11.5.

The left-hand side is showing the case, that in a system at rest the observers A and B first synchronize their clocks and then start at the same time with acceleration. Concerning this it was determined before that A is starting directly after receiving a signal from B, but B is first calculating the starting time and takes $\Delta t_2$ for his start of acceleration (see diagram). The time $\Delta t_2$ is exactly half of the time $\Delta t_0$, that a signal is taking for travelling the distance between B and A and then back. The acceleration is running homogenously until the points C and D, which are fixed to each other, are met (A is contacting C, B reaches D). Here acceleration is stopped, and a signal is transmitted to the other observer.

A and B will now find that

1. the distance between each other has (subjectively) increased to $\gamma a$,
2. time $\Delta t_3$ is larger and $\Delta t_4$ is smaller compared to $\Delta t_2$

The issue presented in point 1 is also named “Bell’s Spaceship Paradox”. J. S. Bell supposed the existence of a thread between these spaceships and assumed, that this would also be contracted.

In a further investigation a moved system is considered, in which the participants A and B are (from their point of view) subject to the same conditions (right side of Fig. 11.5). In this case an observer at rest will find, that $\Delta t_1$ is larger compared to the value monitored before. For this reason, A will start acceleration later than B, because he will start $\Delta t_2 = \Delta t_0/2$ after receiving the signal from A. Therefore, participant A will reach C later than B is reaching D. After the end of this trial, the distance and the times will be checked again and it will be proved, that all values are the same compared to the case looked at before. In the following the calculations of the space- and time-coordinates are presented in detail.

## a) From a system at rest to a moved system

In this case the calculation is easy. Because of the accelerations running parallel it is obvious, that (from the point of view of an observer at rest) the distance $a$ will be constant in the moved system as well. Furthermore, the following calculations apply

$$\Delta t_0 = \frac{2a}{c} \tag{11.11}$$

$$\Delta t_1 = \Delta t_2 = \frac{a}{c} \tag{11.12}$$

$$\Delta t_3 = \frac{a}{c\left(1 - \frac{v}{c}\right)} \tag{11.13}$$

$$\Delta t_4 = \frac{a}{c\left(1 + \frac{v}{c}\right)} \tag{11.14}$$

## b) From a moved system to a system at rest

In this case some additional calculations are necessary.

$$\Delta t_0' = \frac{2a\gamma}{c} \tag{11.15}$$

$$\Delta t_1' = \frac{a}{c\gamma\left(1-\frac{v}{c}\right)} = \frac{a\gamma}{c}\left(1+\frac{v}{c}\right) \tag{11.16}$$

$$\Delta t_2' = \frac{a\gamma}{c} \tag{11.17}$$

$$\Delta t_B' = \Delta t_0' - \Delta t_0 + t_2 = \frac{2a}{c}(\gamma-1) + t_2 \tag{11.18}$$

$$\Delta t_A' = \Delta t_1' + \Delta t_2' - \Delta t_0 + t_2 = \frac{a}{c}\left(2\gamma + \gamma\frac{v}{c} - 2\right) + t_2 \tag{11.19}$$

$$x(\mathrm{B}_2') = \Delta t_B' \cdot v = \left(\frac{2a}{c}(\gamma-1) + t_2\right)v \tag{11.20}$$

$$x(\mathrm{A}_2') = \Delta t_A' \cdot v + \frac{a}{\gamma} = \left(\frac{a}{c}\left(2\gamma + \gamma\frac{v}{c} - 2\right) + t_2\right)v + \frac{a}{\gamma} \tag{11.21}$$

$$\begin{aligned}\Delta x\left(\frac{\mathrm{A}_2'}{\mathrm{B}_2'}\right) &= \left(\frac{a}{c}\left(2\gamma + \gamma\frac{v}{c} - 2\right) + t_2\right)v + \frac{a}{\gamma} - \left(\frac{2a}{c}(\gamma-1) + t_2\right)v \\ &= \frac{av}{c}\gamma\frac{v}{c} + \frac{a}{\gamma} = a\gamma \end{aligned} \tag{11.22}$$

$$\Delta t_3' = \frac{a\gamma}{c} + \Delta t_B' - \Delta t_A' = \frac{a}{\gamma c\left(1-\frac{v}{c}\right)} = \frac{1}{\gamma}\Delta t_3 \tag{11.23}$$

$$\Delta t_4' = \frac{a\gamma}{c} + \Delta t_A' - \Delta t_B' = \frac{a}{\gamma c\left(1+\frac{v}{c}\right)} = \frac{1}{\gamma}\Delta t_4 \tag{11.24}$$

These calculations show, that $a$, $\Delta t_3$ and $\Delta t_4$ in a moved system and a system at rest are connected by $\gamma$ and that the observers A and B from their point of view cannot decide after the end of the trial whether they changed their position from a system at rest to a moved system or vice versa.

However, concerning the behavior of "Bell's thread", which is situated between the spaceships, initially a difference can be observed in the considerations between the cases a) and b). While in a) the distance and caused by this the strain on the thread increases constantly, the case b) will lead to a considerable change in the beginning of the experiment. This is caused by the fact, that observer B starts before A with the acceleration and therefore uneven strain occurs. However, this effect is only appearing seemingly and not real because the thread has a limited rigidity. Like already discussed in connection with the triggering of engines after synchronization in chapter 4.3, the strain in the thread will be transported with limited velocity and so all differences will disappear.

The validity of this argument shall be demonstrated in the following by using a simple example. The beginning of the experiment relates to the fact that in a system of absolute rest both spaceships are starting at the same time. If no total rigidity of the thread is assumed, but the transport of tension with arbitrary velocity is considered, then a thin and almost massless thread will behave like a rope and this is resulting in the fact, that a loop will be formed near the second observer. This would cause an extremely complicated situation and so a simple model is considered here instead were

1. the force will be induced into the thread not only by traction (by observer B, see Fig. 10.5) but also by compression (observer A) into a stable thread (no rope),
2. a buckling or bending of the thread will not occur.

For the start of the spaceships from the state of absolute rest it is obvious, because of symmetry conditions, that any arbitrary velocity will lead to the situation that traction and compression will reach the middle within the same time. For the moving system, the conditions already discussed in chapters 4.1 and 4.3 are valid. The relativistic addition of velocities in combination with appearing synchronization differences will also cause the effect that traction and compression will appear in the middle simultaneously. Thus, for the observers no differences will be measurable.

In publication concerning this matter different perceptions can be found, whether the thread will be contracted or not after acceleration or, in simple words, whether it is breaking or not. (This discussion for obvious reason contains the precondition that the thread is of infinite small mass and has no influence on the behavior of the spaceships). The calculations presented here lead to the clear opinion that the thread is strained, which means it will break. This is simply derived out of the fact that the acceleration phases for both spaceships can also be performed and monitored separately and in this case, when the spaceships act autonomously, the same results must appear.

Before closing the discussion, the additional issue shall be reviewed, that the observers are not lined up in acceleration direction but transverse to it. In this case the quite simple effect occurs, that during an exchange of signals after acceleration the distance between the observers is increased by the factor $\gamma$ compared to the situation at rest. This must be valid because of geometrical reasons; the observer at rest will find that the signal is following a triangle with a side length larger by factor $\gamma$ compared to its height. This effect is compensated exactly by the time-dilatation and so in this case no change in synchronization is observed.

Summing up the discussion two points are worth mentioning. First the chosen experimental conditions are causing tensions between two observers, which are independently accelerated under the same conditions, and this could be part of experimental observations. Obviously in this case differences in measurement results can be expected, dependent on the situation whether the observers are considered as point-shaped or spatially expanded. Second the calculations show that in case of clocks lined up in the direction of acceleration differences in synchronization will occur; this is valid for independent observers and in addition for extended spatial bodies. This effect will not be found if the observers are arranged transverse to acceleration direction. This is required by Special Relativity because of the "Relativity of Simultaneity" and represents a fundamental test regarding the principles of the theory. Details concerning this are discussed in chapter 13.2.

# 12. Conclusions and proposals for modification

The Theory of Special Relativity postulated by A. Einstein in combination with the transformation equations derived by Larmor, Lorentz and Poincaré and further the relativistic increase of mass makes it possible to describe all conceivable relations between moving bodies in arbitrary inertial systems without contradictions. To prove this a wide selection of examples concerning this issue was already discussed in detail in the chapters presented before.

However, this concept is not sufficient to describe all observed cosmological cases. At the beginning of the second half of the 20th century it was found that a cosmic microwave background radiation exists, which is isotropic and constant in all directions. Therefore, based on the “Ether-theories” already developed at the end of the 19th century, new attempts were made to bring special relativity in accordance with a state of absolute rest. However, none of these theories were able to show results without severe discrepancies to experimental findings. The most important theories will be discussed briefly in the following. In addition, the Einstein synchronization already discussed in chapter 3.4. will be evaluated again.

Furthermore, it is proved that by using light pulses for a signal exchange between two observers moving arbitrarily to each other, additionally a superordinate system of absolute rest can be incorporated. With the use of the Lorentz transformation as only precondition this system can be integrated without contradiction. This is done first for the case that two observers are on a straight line in orientation to the system at rest, then for arbitrary constellations.

## 12.1 Alternative theories

In the following theories shall be presented, which are not in accordance with the calculus of the Lorentz-Transformation (LT). They were developed to avoid the principle of “relativity of simultaneously”, which is integral part of LT. The main difference is the introduction of an absolute time which is concurrent valid in any arbitrary inertial system. Although all these theories in their initial form are not in compliance with experimental results, they are historically important and, because of the basic approach concerning violations of LT, are still basis for current research programs.

### 12.1.1 Simple addition of velocities

At the early beginning of discussion concerning speed of light and "ether-drift" it was generally assumed, that the velocity of an observer (together with the measuring device carried with him) and the speed of light must be simply added [12c]. Also, the theoretical approach connected with the Michelson-Morley-Experiment is based on this assumption, and for the calculation of light beams coming and going to mirrors the value was either higher or lower than the speed of light $c$.

Already in the year 1913, however, the examination of double star systems by W. de Sitter provided evidence, that the speed of light is independent of the speed of the object that is transmitting the signals [55]. It was now proven for the first time that this assumption is not in accordance with the facts.

### 12.1.2 Theory of „Neo-Lorentzianism“

Following a similar idea of H. Ives and developed further by J. S. Prokhovnik [74] it is assumed that in all parts of the universe a reference system S exists, which is at absolute rest. When a different inertial system is moving relative to it, the only related attribute valid for this system is, that space is contracting according to

$$x_A = \frac{x_S}{\gamma} \tag{12.01}$$

Consequently, for the coming and going of a light signal inside this system the following different velocities will appear

$$c_1 = c + u_A \tag{12.02}$$

$$c_2 = c - u_A \tag{12.03}$$

The characteristics of time can be calculated by the consideration of a closed loop for a signal

$$t_A = \frac{x_A}{c_1} + \frac{x_A}{c_2} = \frac{x_S(c - u_A + c + u_A)}{\gamma(c + u_A)(c - u_A)} = \frac{2x_S}{c}\gamma = \gamma t_S \tag{12.04}$$

This means that time dilatation is only a seemingly effect which is not real. Effects connected with this theory should be found easily using e.g. synchronization experiments and, because this is not the case, the theory must be rejected. However, the involved persons, mainly Herbert E. Ives (1882-1953), are still today of historical interest. He was all his life in strict opposition to Einstein and, apart from his different theoretical approach, tried hard to discredit him in any possible way. He denied his contribution to Special Relativity and even tried to show that the equation

$$E = mc^2 \tag{6.17}$$

was not originally developed by Einstein [75]. Nevertheless, he provided evidence with the Ives-Stilwell-experiment (co-working with G. R. Stilwell) that time-dilatation for moved bodies exists [17,18] and thus supported, surely without intention, the validity of the Lorentz-equations.

### 12.1.3 RMS-Test theory

The development of another alternative theory started with a proposal by H. Robertson [59] and was finalized by R. Mansouri and R. Sexl [24] and is today usually referred to as Robertson-Mansouri-Sexl- or RMS-Theory. In this case it is assumed, that a system of absolute rest (called “ether system”) exists. For the notation of this ether-system capital letters and for any arbitrary initial reference system small letters are used for calculation. The following general transformation equations are valid:

$$t = aT + \varepsilon x \tag{12.10}$$

$$x = b(X - vT) \tag{12.11}$$

where the factors $a$ and $b$ can be determined by measurements (e.g. Michelson-Morley- and Kennedy-Thorndike-experiments) and $\varepsilon$ out of synchronization effects as

$$\frac{1}{a} = b = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \qquad \varepsilon = -v \tag{12.12}$$

Hence

$$t = \frac{T}{\gamma} - vx \tag{12.13}$$

$$x = \gamma(X - vT) \tag{12.14}$$

Equation (12.14) is obviously corresponding to the Lorentz-Transformation according to Eq. (1.08). Eq. (12.13) can be transformed to

$$t = \frac{T}{\gamma} - vx = \gamma T(1 - v^2) - vx = \gamma T - \gamma T v^2 - vx \tag{12.15}$$

If Eq. (12.11) is converted, then

$$T = \frac{X - \frac{x}{\gamma}}{v} \tag{12.16}$$

with

$$t = \gamma T - \gamma \frac{X - \frac{x}{\gamma}}{v} v^2 - vx = \gamma T - \gamma v X + vx - vx \tag{12.17}$$

and

$$t = \gamma(T - vX) \tag{12.18}$$

This means that the calculations follow exactly the Lorentz-Transformation. The RMS-Theory now predicts that during passing of a moving system a comparison of clocks inside both systems shows the result

$$\Delta t = -vx \tag{12.19}$$

Eq. (12.13) is transforming to

$$t = \frac{T}{\gamma} \tag{12.20}$$

A graphic presentation leads to the diagram shown in Fig. 12.1.

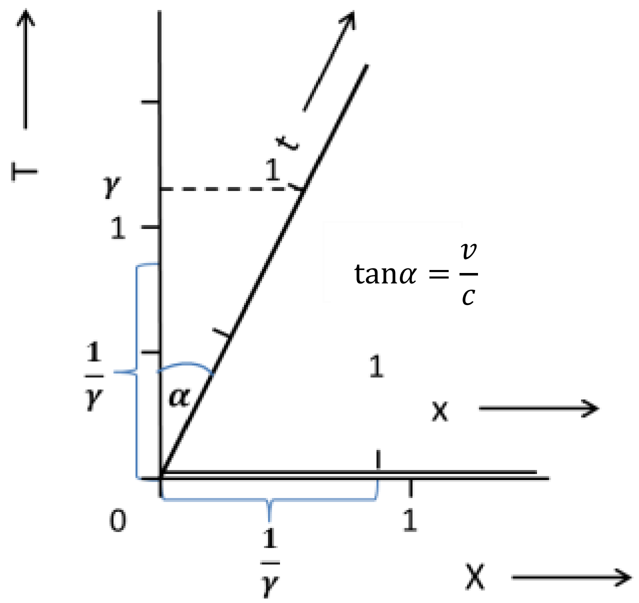



Fig. 12.1: Space-time diagram following Eq. (12.18) (according to [24])

It is obvious that in this case during a synchronization performed with light signals differences inside the moving system should occur; however, no experimental evidence could be provided up to now [54,75]. Although the theory shows severe shortcomings, it is further developed until today [54]. The reason is that new approaches using quantum gravitation resp. string theory are suggesting violations of the Lorentz-Transformation. In combination with the equation

$$y = d \cdot Y \qquad\qquad z = d \cdot Z \tag{12.21}$$

now effort is made to find small differences to the equations given by the Lorentz-Transformation

$$\frac{1}{a} = b = \left[1 - \left(\frac{v}{c}\right)^2\right]^{-\frac{1}{2}} \qquad\qquad d = 1 \tag{12.22}$$

The intention is that with increasing accuracy of experiments following the methods of Michelson-Morley, Kennedy-Thorndike, and Ives-Stilwell these differences will be detected and that it will be possible to integrate the results into a generally valid overall picture. Examples for new measurements with highest precision are given e.g. [76,77,78,79], however, up to now no violations of the Lorentz-Invariance could be detected.

### 12.1.4 Further alternatives

In the last years many alternative theories were developed, which are demanding variations of the Lorentz-Equations. These approaches are usually connected with a further development of the "Theory of General Relativity", trying to find a general unifying theory

(GUT), which can possibly bridge the gap to quantum mechanics. These new theories are generally of high complexity, but despite of fierce struggle it was not possible to find a reasonable formalism during the last decades. Here the question is allowed, why such an effort is made and how this can be justified. To answer this, a remarkable statement shall be cited out of a publication by C. M. Will [64]. This is in principle dealing with the position of General Relativity, but because further developments of this theory are mainly connected with the search for violations of the Lorentz-Invariance, it is also valid for the relations discussed before:

"We find that general relativity has held up under extensive experimental scrutiny. The question then arises, why bother to continue to test it? One reason is that gravity is a fundamental interaction of nature, and as such requires the most solid empirical underpinning we can provide. Another is that all attempts to quantize gravity and to unify it with the other forces suggest that the standard general relativity of Einstein is not likely to be the last word. Furthermore, the predictions of general relativity are fixed; the theory contains no adjustable constants so nothing can be changed. Thus, every test of the theory is either a potentially deadly test or a possible probe for new physics. Although it is remarkable that this theory, born 80 years ago out of almost pure thought, has managed to survive every test, the possibility of finding a discrepancy will continue to drive experiments for years to come."

## 11.2 Interpretation of Einstein-synchronization

In chapter 3.4 the Einstein synchronization was already discussed shortly. Because of the paramount importance it shall be investigated again and a close look at this topic will be taken. In a first step the theoretically appearing synchronization differences for an observer at rest and in a moved system are established.

In the following space-time-diagram the synchronization differences $\Delta S$ and $\Delta S'$ experienced by an observer at rest A in view of a moved observer B are presented (Fig. 12.2). The diagram is standardized (which means a scaling of $\Delta t = \Delta x = 1$). In a diagram scaled this way, light pulses show a graphic orientation of 45° to $t$ and $x$ axis. The velocity used for B in this diagram is $v = x/t = 0{,}5c$.

The cases appear, that:

a) A is sending a signal which is reflected by B,

b) B is sending a signal which is reflected by A.

The equations necessary for the calculation of the synchronization differences are compiled in Tab. 11.1. For A the interpretation of diagram a) is simple and because of the appearing symmetry $\Delta t_0 = \Delta t_2 = \Delta t_1$ is valid.

The situation of part b) is different and the calculation more complex. Observer A is monitoring in his view, that the signal will be sent later from B, because time is running slower by factor $\gamma$, but that it is arriving earlier compared to the signal sent by him. The latter is caused by the effect, that B is increasing distance to A during the transmission of the signal (for exact definition and modes for calculation see chapter 2.).

The synchronization difference for A can be calculated as follows

$$\Delta S = \Delta t_S \frac{1 - \frac{1}{\gamma}}{1 - \frac{v}{c}} \qquad (12.30)$$

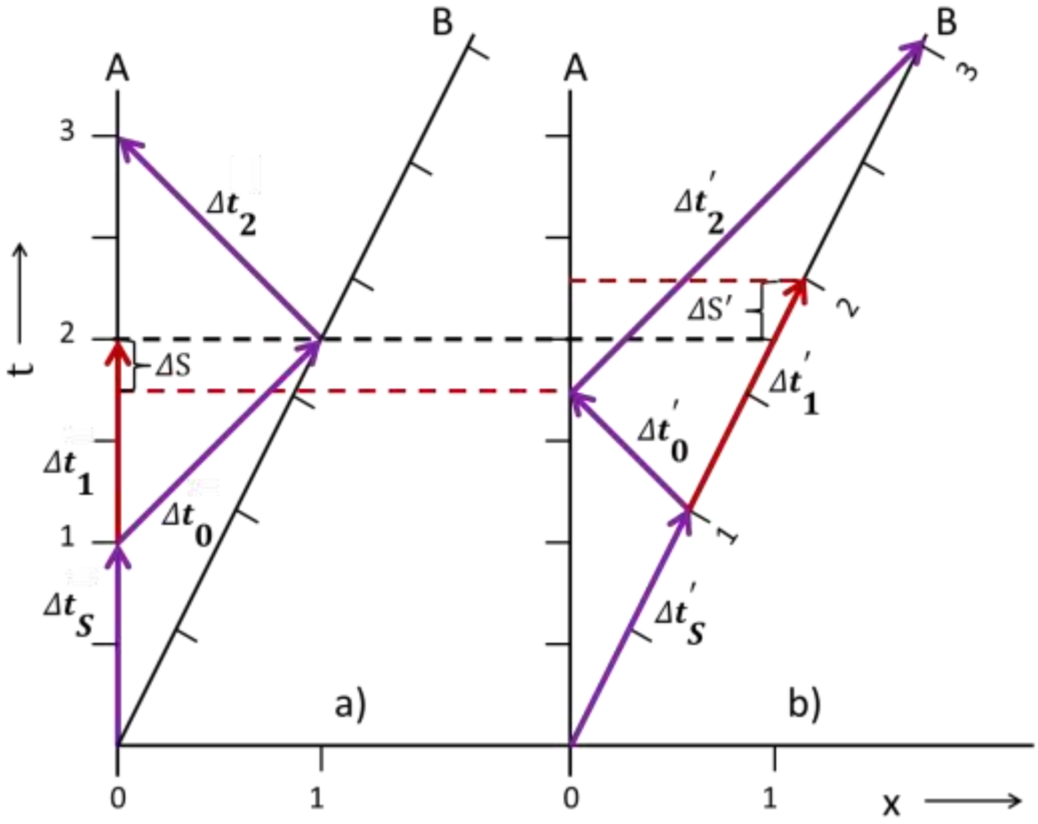



Fig. 12.2: Definition of synchronization differences $\Delta S$ resp. $\Delta S'$

| | |
|---|---|
| $\Delta t_S$ | $\Delta t'_S = \gamma \Delta t_S$ |
| $\Delta t_0 = \Delta t_S \left[\frac{v}{c\left(1 - \frac{v}{c}\right)}\right]$ | $\Delta t'_0 = \Delta t_S \gamma \frac{v}{c}$ |
| $\Delta t_2 = \Delta t_S \left[\frac{v}{c\left(1 - \frac{v}{c}\right)}\right]$ | $\Delta t'_2 = \Delta t_S \gamma \left[\frac{v\left(1 + \frac{v}{c}\right)}{c\left(1 - \frac{v}{c}\right)}\right]$ |
| $\Delta S = \Delta t_S + \left[\frac{\Delta t_0 + \Delta t_2}{2}\right] - [\Delta t'_S + \Delta t'_0]$ $= \Delta t_S \frac{1 - \frac{1}{\gamma}}{1 - \frac{v}{c}}$ | $\Delta S' = \Delta t'_S + \left[\frac{\Delta t'_0 + \Delta t'_2}{2}\right] - [\Delta t_S + \Delta t_0]$ $= \Delta t_S \frac{\gamma - 1}{1 - \frac{v}{c}}$ |

Tab. 12.1: Equations for the calculation of $\Delta S$ resp. $\Delta S'$

In view of the moving observer B in b) a similar situation appears. In this case also the signal will arrive too early with

$$\Delta S' = \Delta t_S \frac{\gamma - 1}{1 - \frac{v}{c}} \tag{12.31}$$

Because time is running slower for the moving observer the subjective values for both are equal and it applies

$$\Delta S' = \gamma \Delta S \tag{12.32}$$

The Einstein-Synchronization now specifies the following:

At time $t_S$ resp. $t'_S$ a signal will be transmitted by observers A and B. When the signals are received by B resp. A, the clocks are considered as synchronized, when the following conditions apply:

$$t_1 = t_S + \frac{t_2 - t_0}{2} \tag{12.33}$$

and

$$t'_1 = t'_S + \frac{t'_2 - t'_0}{2} \tag{12.34}$$

For system a), the validity of the determination results directly from the representation in the diagram and there are no differences to the calculations carried out. For b), however, there are serious changes.

An essential statement is first that $\Delta t'_1$ is hereby uniquely determined and the division between the single times $\Delta t'_0$ and $\Delta t'_2$ does not play any role. Together with the statement that the speed of light is the same in all inertial systems, in this way the synchronization difference becomes a virtual quantity which cannot be determined from the moving system. Since this value would be measurable for a resting observer, however, at the transmission of impulses with superluminal velocities, there must be no information transmission faster than the light and also no system of absolute rest on the basis of these determinations. Herewith, a central statement of the special relativity theory is described.

So, it becomes clear that the Einstein synchronization is a definition and not covered by an observation.

The use of the Einstein synchronization has beside the possibility for the calculation of the Lorentz equations still another meaning. As already described in detail, from the point of view of an observer at rest it is not possible to describe the course of oscillation of an electromagnetic wave (e.g. light) without contradiction without using the principle of constant phase velocity in a moving system.

To avoid this, it is a simple means to use the definition of the Einstein synchronization in such a way that oscillation considerations are permitted in principle only within the respective inertial system. If one proceeds according to this principle, it follows that a state of absolute rest cannot be inserted; this leads to apparent contradictions, and then the principle, that a system of absolute rest can exist, must be rejected as erroneous. This will be an important consideration in a final study of the speed of light in chapter 13.1.

In the following, another important aspect on the subject of the speed of light will be dealt with. The statement: "The speed of light is the same in all inertial frames" must be considered and interpreted carefully.

However, if several test participants from different inertial systems moving against each other observe the *same* event, e.g. the signal exchange between different spatially separated points, different observations must occur. If the speed of light of the own system is taken as a basis for measurements and if the times and distances necessary for the signal exchange are determined for the way there and back, they come to different results. Path and time are *not* divided symmetrically. This effect is caused by the "relativity of simultaneity".

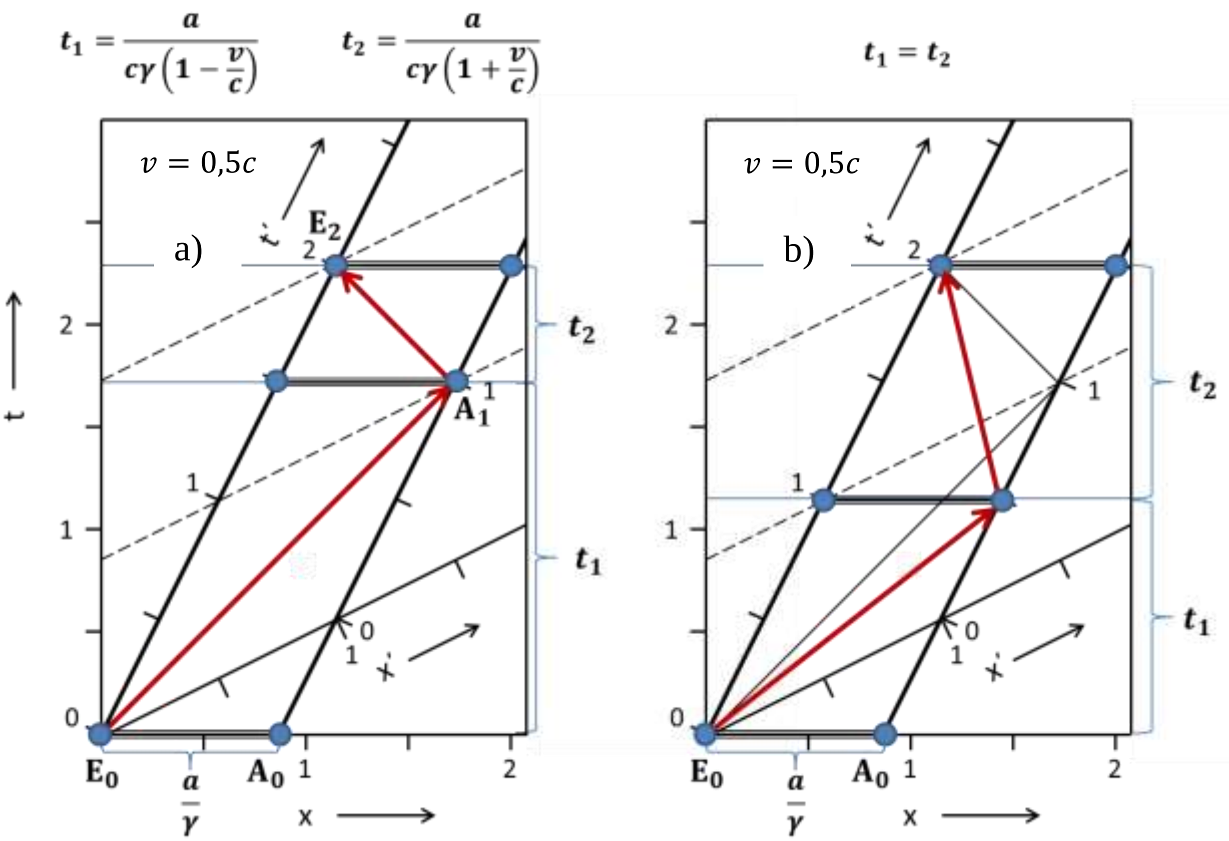



Fig. 12.3: Schematic presentation of a signal in a laboratory L between E and A from the point of view of an inertial system S moving relative to it ($v = 0,5c$).
a) Correct: $c =$ const. referred to S.
b) Not correct: $t_1 = t_2$ referred to S

To make this clear, the situation is shown in fig. 12.3. While the situation is always clear for an observer at rest (the outward and return paths are of equal length and the individual times are also equal), this does not apply to an observer from an inertial system S moving relative to the lab.

The determination of the Einstein synchronization, i.e. at the outward and return path for the signal exchange between two points (e.g. the ends of a laboratory A and E) time and path are in each case divided to the half, is valid only subjectively for the system L which is in rest to the laboratory. If from another inertial system S moving relative to it this determination would also apply and the times $t_1 = t_2$ would be equal, the situation would arise as shown in the right part of the diagram with signal velocities larger or smaller than $c$ as well as measurable synchronization differences. Moreover, according to these considerations, a situation where the path is constant in both directions cannot even theoretically occur because the lab end moves away from the original point immediately after the signal is emitted and is at a different location on the return path. Instead, the situation as shown in the left partial picture applies. This means that the determination of a reference system can always only be subjective.

## 12.3 Integration of a system at absolute rest into the Lorentz-Equations

The approaches to combine a system of absolute rest with the Lorentz-equations presented in chapter 11.1 were obviously not successful. In the following it will be examined whether it is possible for two observers moving in arbitrary directions against each other to integrate an additional superior system which is at absolute rest. In this case the use of the Lorentz equations must lead to a consistent connection without discrepancies. First a simple comparison reveals the fact, that this must be possible because the discussed equations can be considered as a mathematical group. The implementation of the Lorentz equations in a system A→ B can therefore easily being carried out using A→ S→ B, where S could be a system with a basis at absolute rest.

Because of the importance of this proposition the validity of this correlation will be presented here in detail. To show this, the possible constellations between the observers will be treated separately in the following.

1. Observers A and B are moving on a straight line in relation to S

In the following the experimental relation shall be examined in an analytical way, where the system at rest S and an arbitrary reference system 1 with observer A, which is moving in relation to it with $v_0$ and the investigated system 2 with observer B (moving with $v_1$ compared to the reference system) are lined up and $v_0 < v_1$ applies. To simplify the calculation the values of the velocities shall be replaced by their quotient to the speed of light $c$.

The Lorentz equations between Reference System 1 and the investigated System 2 are given by

$$x_2 = \gamma_1(x_1 - v_1 t_1) \tag{12.40}$$

$$t_2 = \gamma_1(t_1 - v_1 x_1) \tag{12.41}$$

where $x_1$ and $t_1$ are coordinates of the Reference System 1 and $x_2$ and $t_2$ coordinates of the investigated System 2, which is traveling with speed $v_1$ compared to system 1. If a system which is at absolute rest is introduced, then system 1 will generally show a movement compared to this. In view of the system at rest the following relations apply

$$x_1 = \gamma_0(x_0 - v_0 t_0) \tag{12.42}$$

$$t_1 = \gamma_0(t_0 - v_0 x_0) \tag{12.43}$$

$$x_2 = \gamma_2(x_0 - v_2 t_0) \tag{12.44}$$

$$t_2 = \gamma_2(t_0 - v_2 x_0) \tag{12.45}$$

where $v_0$ is the speed between system at rest and Reference System 1, whereas $v_2$ represents the speed between the system at rest and system 2. Furthermore, the equation for relativistic addition of velocities applies

$$v_2 = \frac{v_0 + v_1}{1 + v_0 v_1} \tag{12.46}$$

Equations Eq. (12.42) and (12.43) are leading to the following relationship for the coordinates $x_0$ and $t_0$

$$x_0 = \gamma_0(x_1 + v_0 t_1) \tag{12.47}$$

$$t_0 = \gamma_0(t_1 + v_0 x_1) \tag{12.48}$$

In combination with (12.44) and (12.45) this yields

$$x_2 = \gamma_2\big(\gamma_0(x_1 + v_0 t_1) - v_2\gamma_0(t_1 + v_0 x_1)\big) \tag{12.49}$$

$$x_2 = \gamma_2\gamma_0\big((1 - v_0 v_2)x_1 - (v_2 - v_0)t_1\big) \tag{12.50}$$

$$t_2 = \gamma_2\big(\gamma_0(t_1 + v_0 x_1) - v_2\gamma_0(x_1 + v_0 t_1)\big) \tag{12.51}$$

$$t_2 = \gamma_2\gamma_0\big((1 - v_0 v_2)t_1 - (v_2 - v_0)x_1\big) \tag{12.52}$$

The equations (12.40) and (12.41) shall be identical with equations (12.50) resp. (12.51). To prove this a comparison of coefficients is carried out and the following equations apply

| | | | |
|---|---|---|---|
| (12.40) ⇔ (12.50) | $x_1$: | $\gamma_1 = \gamma_2\gamma_0(1 - v_0 v_2)$ | (12.53) |
| (12.40) ⇔ (12.50) | $t_1$: | $v_1\gamma_1 = \gamma_2\gamma_0(v_2 - v_0)$ | (12.54) |
| (12.41) ⇔ (12.51) | $t_1$: | $\gamma_1 = \gamma_2\gamma_0(1 - v_0 v_2)$ | (12.55) |
| (12.41) ⇔ (12.51) | $x_1$: | $v_1\gamma_1 = \gamma_2\gamma_0(v_2 - v_0)$ | (12.56) |

Obviously, the equations (12.53) and (12.55) as well as (12.54) and (12.56) are identical. Because of

$$v_1 = \frac{v_2 - v_0}{1 - v_0 v_2} \tag{12.57}$$

Eq. (12.54) can be replaced by Eq. (12.53) since

$$(v_2 - v_0)\gamma_1 = \gamma_2\gamma_0(v_2 - v_0)(1 - v_0 v_2) \tag{12.58}$$

It is now proved that all 4 equations are identical. To show the validity of the complete system it is necessary to validate only one of these equations.

If now both sides of the Eq. (12.54) are squared and the respective values for $\gamma$ are inserted, it follows

$$\frac{v_1^2}{(1 - v_1^2)} = \frac{(v_2 - v_0\,)^2}{(1 - v_2^2)\cdot(1 - v_0^2)} \tag{12.59}$$

and

$$(1 - v_2 v_0)^2 v_1^2 = (v_2 - v_0)^2 \tag{12.60}$$

If for $v_2$ the equation (12.46) is used, then

$$\left(1 - \frac{v_0 + v_1}{1 + v_0 v_1} v_0\right)^2 v_1^2 = \left(\frac{v_0 + v_1}{1 + v_0 v_1} - v_0\right)^2 \tag{12.61}$$

If this equation is expanded completely, then 20 terms will occur which will add up to zero. It was thus shown for this case that the integration of a system at rest will not lead to any violations or to mathematical inconsistencies by using the Lorentz equations. Modified conditions taking $v_0 > v_1$ into consideration lead to the same result, because in any case only linear conditions are present which can be combined without restrictions.

When an arbitrary dependency between the combinations of velocities for the movement of observers in different directions is considered, however, the calculation will be more difficult. In this case the observers will not contact each other but approaching to a minimum before they increase the distance again. It was already shown in chapter 2.1.2, that for any observer in a system at rest (A) or in a moved system (B) there is no difference in their observation of the situation and that it is not possible for both of them to decide with measurements during a signal exchange, whether they are moving or at rest. If a system of absolute rest is integrated, which is different from zero to an observer A which was stipulated to be as at rest before, then the calculation will be more complex, but the situation can be simplified considerably if a suitable point of origin for the calculation is defined.

For simple calculation the fact is used that the direction vectors of both observers are passing along a straight line. If the vectors are moving along these lines the correlation between them are changing as a linear quantity, which means in a mathematical sense a constant is added which can be subtracted later after the calculation is finalized. Two different cases must be dealt with:

2. The straight lines of the direction vectors are intersecting

For this purpose, the fact is used that if a system at rest is assumed then no further requirements concerning the point of origin are necessary from which the examination would have to start. This means that out of the unlimited possibilities the point of origin can be defined in a way that A is distancing to it and is part of the directional vector; this line is defined as corresponding to the $x$-axis. Further the vectors of both observers are moved in such a way that they are intersecting. These are the conditions to determine the point of origin as zero-coordinates of $x, y, t$ in view of the system at rest S, the values for the $z$ axis are always zero due to the definition of the coordinates. In this case the correlations must follow the Lorentz equations.

For verification the following experiment shall be discussed: Starting from observer A observer B is departing with an arbitrary angle in relation to the $x$-axis. After a certain time $\Delta t$ this observer is emitting a signal. The related coordinates will be determined by observer A and in the system at rest S. When these are identical after use of the Lorentz equations then the system at rest can be integrated without discrepancies.

The following calculations apply:

Observer A finds that the signals transmitted by observer B distancing with the velocity $v_1$ are arriving with the delay

$$t_1 = \gamma_1 \Delta t \tag{12.62}$$

The connected coordinates are

$$x_1 = v_1 t_1 cos\ \alpha' \tag{12.63}$$

$$y_1 = v_1 t_1 sin\ \alpha' \tag{12.64}$$

In view of system S the velocity of observer B is calculated according to Eq. (4.20), see also chapter 4.1:

$$v_2 = \frac{\sqrt{(v_0^2 + v_1^2 + 2v_0 v_1 cos\alpha') - (v_0 v_1 sin\alpha')^2}}{1 + v_0 v_1 cos\alpha'} \tag{12.65}$$

where in his view the velocity of A is equal to $v_0$. The angle $\alpha$ measured by S is following equation Eq. (7.43)

$$\alpha = arctan\left[\frac{sin\ \alpha'}{\gamma_0\left(cos\alpha' + \frac{v_0}{v_1}\right)}\right] \tag{12.66}$$

(For details see chapter 7.2). Analogous to the coordinates found before it is

$$t_2 = \gamma_2 \Delta t \tag{12.67}$$

$$x_2 = v_2 t_2 cos\alpha \tag{12.68}$$

$$y_2 = v_2 t_2 sin\alpha \tag{12.69}$$

Finally, the coordinates are calculated which can be found using the Lorentz equations and it applies

$$t_1' = \gamma_0(t_2 - v_0 x_2) \tag{12.70}$$

$$x_1' = \gamma_0(x_2 - v_0 t_2) \tag{12.71}$$

The following correlations must apply:

$$t_1' = t_1 \tag{12.72}$$

$$x_1' = x_1 \tag{12.73}$$

$$y_2 = y_1 \tag{12.74}$$

Eq. (12.74) shows that the values in $y$ direction are the same in all systems, what is a direct requirement of the Lorentz transformation.

| $\alpha'$ | $t_1$ | $x_1$ | $y_1$ | $v_2$ | $\alpha$ | $t_2$ | $x_2$ | $y_2$ | $t_1'$ | $x_1'$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1,154701 | 0,577350 | 0,000000 | 0,571429 | 0,00000 | 1,218544 | 0,696311 | 0,000000 | 1,154701 | 0,577350 |
| 15 | 1,154701 | 0,557678 | 0,149429 | 0,569508 | 12,45513 | 1,216566 | 0,676539 | 0,149429 | 1,154701 | 0,557678 |
| 30 | 1,154701 | 0,500000 | 0,288675 | 0,563786 | 25,01756 | 1,210770 | 0,618571 | 0,288675 | 1,154701 | 0,500000 |
| 45 | 1,154701 | 0,408248 | 0,408248 | 0,554386 | 37,79756 | 1,201548 | 0,526357 | 0,408248 | 1,154701 | 0,408248 |
| 60 | 1,154701 | 0,288675 | 0,500000 | 0,541551 | 50,91089 | 1,189531 | 0,406181 | 0,500000 | 1,154701 | 0,288675 |
| 75 | 1,154701 | 0,149429 | 0,557678 | 0,525691 | 64,48031 | 1,175536 | 0,266234 | 0,557678 | 1,154701 | 0,149429 |
| 90 | 1,154701 | 0,000000 | 0,577350 | 0,507445 | 78,63457 | 1,160518 | 0,116052 | 0,577350 | 1,154701 | 0,000000 |
| 105 | 1,154701 | -0,149429 | 0,557678 | 0,487753 | 93,50218 | 1,145500 | -0,034130 | 0,557678 | 1,154701 | -0,149429 |
| 120 | 1,154701 | -0,288675 | 0,500000 | 0,467905 | 109,19583 | 1,131505 | -0,174078 | 0,500000 | 1,154701 | -0,288675 |
| 135 | 1,154701 | -0,408248 | 0,408248 | 0,449528 | 125,78294 | 1,119487 | -0,294253 | 0,408248 | 1,154701 | -0,408248 |
| 150 | 1,154701 | -0,500000 | 0,288675 | 0,434472 | 143,24177 | 1,110266 | -0,386467 | 0,288675 | 1,154701 | -0,500000 |
| 165 | 1,154701 | -0,557678 | 0,149429 | 0,424533 | 161,41618 | 1,104469 | -0,444435 | 0,149429 | 1,154701 | -0,557678 |
| 180 | 1,154701 | -0,577350 | 0,000000 | 0,421053 | 0,00000 | 1,102492 | -0,464207 | 0,000000 | 1,154701 | -0,577350 |
| | | | | | | | | | | |
| 0 | 1,154701 | 0,577350 | 0,000000 | 0,800000 | 0,00000 | 1,666667 | 1,333333 | 0,000000 | 1,154701 | 0,577350 |
| 15 | 1,154701 | 0,557678 | 0,149429 | 0,796896 | 6,50446 | 1,655309 | 1,310617 | 0,149429 | 1,154701 | 0,557678 |
| 30 | 1,154701 | 0,500000 | 0,288675 | 0,787340 | 13,06431 | 1,622008 | 1,244017 | 0,288675 | 1,154701 | 0,500000 |
| 45 | 1,154701 | 0,408248 | 0,408248 | 0,770588 | 19,73390 | 1,569036 | 1,138071 | 0,408248 | 1,154701 | 0,408248 |
| 60 | 1,154701 | 0,288675 | 0,500000 | 0,745356 | 26,56505 | 1,500000 | 1,000000 | 0,500000 | 1,154701 | 0,288675 |
| 75 | 1,154701 | 0,149429 | 0,557678 | 0,709783 | 33,60502 | 1,419606 | 0,839213 | 0,557678 | 1,154701 | 0,149429 |
| 90 | 1,154701 | 0,000000 | 0,577350 | 0,661438 | 40,89339 | 1,333333 | 0,666667 | 0,577350 | 1,154701 | 0,000000 |
| 105 | 1,154701 | -0,149429 | 0,557678 | 0,597477 | 48,45800 | 1,247060 | 0,494121 | 0,557678 | 1,154701 | -0,149429 |
| 120 | 1,154701 | -0,288675 | 0,500000 | 0,515079 | 56,30993 | 1,166667 | 0,333333 | 0,500000 | 1,154701 | -0,288675 |
| 135 | 1,154701 | -0,408248 | 0,408248 | 0,412289 | 64,43855 | 1,097631 | 0,195262 | 0,408248 | 1,154701 | -0,408248 |
| 150 | 1,154701 | -0,500000 | 0,288675 | 0,289259 | 72,80788 | 1,044658 | 0,089316 | 0,288675 | 1,154701 | -0,500000 |
| 165 | 1,154701 | -0,557678 | 0,149429 | 0,149449 | 81,35612 | 1,011358 | 0,022716 | 0,149429 | 1,154701 | -0,557678 |
| 180 | 1,154701 | -0,577350 | 0,000000 | 0,000000 | 0,00000 | 1,00000 | 0,000000 | 0,000000 | 1,154701 | -0,577350 |
| | | | | | | | | | | |
| 0 | 1,005038 | 0,100504 | 0,000000 | 0,571429 | 0,00000 | 1,218544 | 0,696311 | 0,000000 | 1,005038 | 0,100504 |
| 15 | 1,005038 | 0,097079 | 0,026012 | 0,569508 | 2,15163 | 1,216566 | 0,692356 | 0,026012 | 1,005038 | 0,097079 |
| 30 | 1,005038 | 0,087039 | 0,050252 | 0,563786 | 4,22175 | 1,210770 | 0,680763 | 0,050252 | 1,005038 | 0,087039 |
| 45 | 1,005038 | 0,071067 | 0,071067 | 0,554386 | 6,12440 | 1,201548 | 0,662320 | 0,071067 | 1,005038 | 0,071067 |
| 60 | 1,005038 | 0,050252 | 0,087039 | 0,541551 | 7,76517 | 1,189531 | 0,638285 | 0,087039 | 1,005038 | 0,050252 |
| 75 | 1,005038 | 0,026012 | 0,097079 | 0,525691 | 9,03827 | 1,175536 | 0,610295 | 0,097079 | 1,005038 | 0,026012 |
| 90 | 1,005038 | 0,000000 | 0,100504 | 0,507445 | 9,82643 | 1,160518 | 0,580259 | 0,100504 | 1,005038 | 0,000000 |
| 105 | 1,005038 | -0,026012 | 0,097079 | 0,487753 | 10,00607 | 1,145500 | 0,550222 | 0,097079 | 1,005038 | -0,026012 |
| 120 | 1,005038 | -0,050252 | 0,087039 | 0,467905 | 9,46232 | 1,131505 | 0,522233 | 0,087039 | 1,005038 | -0,050252 |
| 135 | 1,005038 | -0,071067 | 0,071067 | 0,449528 | 8,11836 | 1,119487 | 0,498198 | 0,071067 | 1,005038 | -0,071067 |
| 150 | 1,005038 | -0,087039 | 0,050252 | 0,434472 | 5,97964 | 1,110266 | 0,479755 | 0,050252 | 1,005038 | -0,087039 |
| 165 | 1,005038 | -0,097079 | 0,026012 | 0,424533 | 3,18024 | 1,104469 | 0,468161 | 0,026012 | 1,005038 | -0,097079 |
| 180 | 1,005038 | -0,100504 | 0,000000 | 0,421053 | 0,00000 | 1,102492 | 0,464207 | 0,000000 | 1,005038 | -0,100504 |

Tab. 12.2: Comparison of calculations using Lorentz-Transformation. Values marked grey: Approximation (otherwise division by zero); Presentation in frames: 180 °+angle Equations for $t_1 \rightarrow$ Eq. (12.33) to $x_1' \rightarrow$ Eq. (12.42) see text.

An analytical solution of these equations is complex, a direct numerical comparison not. In tab. 12.2 the results for the calculation of different angles between A and B and varying velocities are presented. No differences occur and Eq. (12.72), (12.73) and (12.74) are unrestrictedly valid.

3. The straight lines of the direction vectors are not intersecting

In the case where the direction vectors of observers A and B are not intersecting, this means in the terminology of analytical geometry, that the straight lines are "out of square". For the solution of this problem first the position must be determined where the distance between the straight lines for both observers reach a minimum. In this case, here (and only here) the angle of the connecting line is matching the value of 90° in relation to the straight lines for both observers.

This connecting line is now selected as basis for the $z$-axis, which played no role in the interpretation up to now. The intersection point with the $x$-axis is now defined as origin of the coordinate system and the direction of the $y$-axis is perpendicular to both. When observer B has reached the minimum distance to the center of origin with distance $z_{min}$ on the $z$-axis then $x = y = 0$ applies. Now the fact is used that the values in $z$-direction do not change during Lorentz transformation and that therefore a projection by factor $z_{min}$ is possible. The situation appearing now is identical to the case, where the direction vectors showed intersection. So, in a final statement it can be noticed, that it was possible to prove that a system of absolute rest can be integrated in any arbitrary inertial system without violation of the Lorentz equations or showing any other discrepancies.

# 13. Possible experiments

In the following it shall be discussed, which possibilities exist to clarify the situation created by the survey presented in this elaboration. For this purpose, the set-up of possible experiments is introduced, and an approach will be made to evaluate output data on the basis of realistic input. The proposals for these experiments are based on the considerations presented in chapter 10, were major subjects of the theory of Special Relativity were discussed.

A new approach to the subject is, when during quantum mechanical tunneling experiments it is assumed that information – considered as a simple pulse – could be transported with superluminal velocity. This would only be possible, when in contrast to the well-known preconditions of Special Relativity a system at absolute rest is assumed as general frame.

Further an experiment will be proposed to clarify, whether differences in the synchronization within a system in motion before and after acceleration really exist. With this experiment it could be possible to find distinct evidence about the statements concerning Relativity of Simultaneity as already discussed in chapter 11.3, which is classified as not valid by some new theories. Further an experiment is described that could measure the relativistic mass increase of a non-elastic collision in an indirect way.

## 13.1 Measurement of tunneling in different spatial directions

It was already presented in chapter 10.1 that transport of information with superluminal velocities and Special Relativity are leading to a severe conflict. If such an effect could be verified it would be possible to solve the appearing discrepancies by assuming a state of absolute rest in the universe as general frame. In the following an experiment will be described, which would allow to detect a relative motion relative to a resting frame using quantum mechanical tunneling and the connected superluminal velocity of a pulse transport.

First the principle and limits of the experimental set-up shall be discussed in detail. As already presented in chapter 10.1 the principle to conduct measurements is that a pulse is induced into a double-prism and afterwards the reflected and the tunneled pulses are compared relating to their transit time. The reflected beam is leaving the prism with almost unchanged intensity and in contrast the intensity of the tunneled beam is much smaller. It is therefore part of the experiment to amplify the tunneled beam with an extremely high intensity.

Starting an analysis, the measured values must first be amplified to the same size, i.e. they have to be normalized. One of the most important difficulties during the evaluation of these normalized values of reflected and tunneled pulses is the fact that the results are not obeying the form of sharp rectangular pulses but appear as bell-shaped Gaussian distribution curves and must be interpreted in a correct way. As an example, for this effect in Fig. 13.1 experimental values published in the literature for a reflected and a tunneled beam after normalization are presented [64]. To show the difficulty for evaluation the “original” value of the tunneled pulse – already with high amplification – was added.

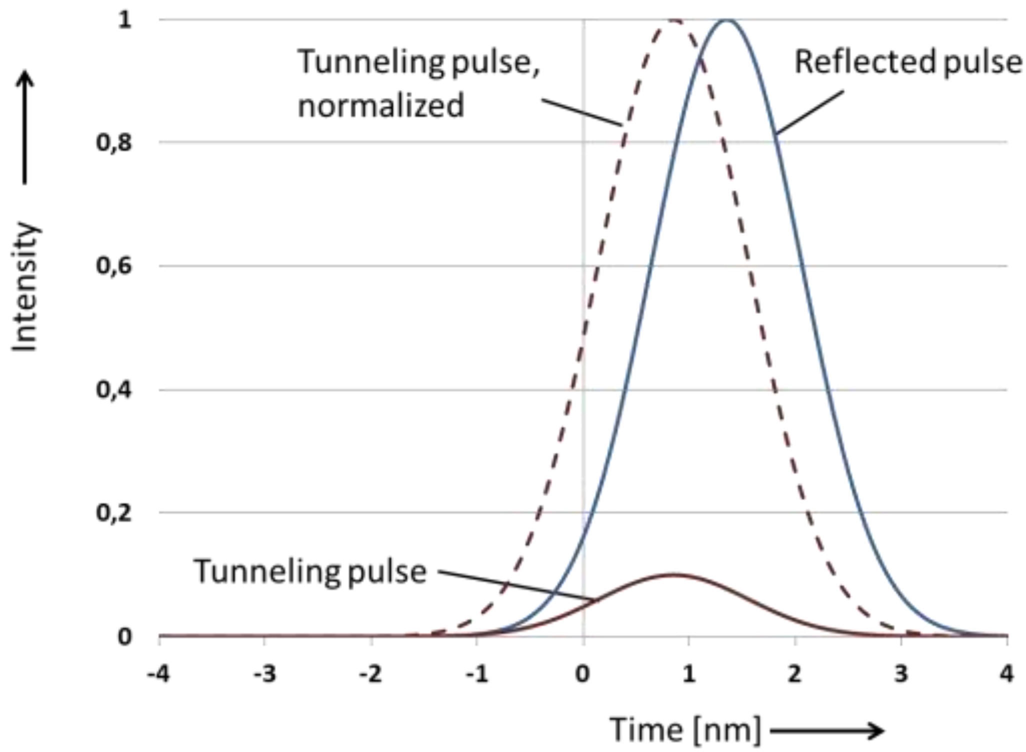



Fig. 13.1: Published data [64] of normalized values during tunneling experiments
Presentation of reflected and tunneled pulses
“Original” tunneling pulse (already with high amplification) was added.

According to G. Nimtz [64] the evaluation of these experiments showed values for the reflected beam $v_R = 0.665c$ and for the tunneled beam $v_T = 4.6c$. Although measurements like these, which were verified during several other experiments, are not generally questioned, it is argued in many cases that in fact superluminal velocities occur, but it is not possible to transport information faster than light during these trials. The general background was already discussed in chapter 10.1. Independently of considerations concerning Special Relativity, the appearing measuring effects are of general interest, and it would be worth finding out whether a single pulse, which can also be taken as small part of information, is travelling faster than light or not.

Because of the experimental challenges an unambiguous verification is very difficult. The function profile is generally expressed by

$$f(t) = \frac{exp\left[-\left(\frac{t-\Delta t}{k}\right)^2\right]}{\sqrt{k\cdot\pi}} \tag{13.01}$$

where for normalization the original values relate to the maximum of the function at point

$$f_{max} = f(t - \Delta t) \tag{13.02}$$

In this relation $k$ describes the width of the bell-shaped curve (which is appearing smaller for increasing values of $k$) and $\Delta t$ the distance of the maximum of the function compared to the initial value $t = 0$.

In the past already several experiments using double prisms were carried out. The largest dimensional set-up used a measuring distance of approximately 280mm. As already discussed, a superluminal transport of information is only possible when the existence of a system at absolute rest is assumed. It is well known that our solar system is moving with a speed of about 369km/s against the isotropic cosmic background radiation. When it is supposed that the latter is connected to a frame of absolute rest, then it could be possible to detect a measuring effect using an apparatus with a double prism and taking measurements in different spatial directions.

However, the effects to be expected are exceedingly small. To show this, based on the considerations in chapter 10.1 the expected values are calculated and presented in Tab. 13.1. The calculations for the measuring effects are valid for a distance of 280mm and a signal velocity of 4.6c as taken from [64]. Inserting these values in Eq. (10.07) the calculation will give the results presented in Tab. 13.1 for the orientation in moving direction ($t_1 + t_2$) and opposite to it ($t_3 + t_4$). It is instantly clear that the resulting differences in time are quite small and approximately 2-3 orders of magnitude smaller than the differences using the original experiment.

| | |
|---|---|
| $t_1 = \frac{a}{\gamma(v_E + v_S)} = \frac{0{,}28m}{\gamma(4{,}6 + 0{,}00123)c}$ | $2{,}02844 \cdot 10^{-10}$s |
| $t_2 = \frac{a}{\gamma(c - v_S)} = \frac{0{,}28m}{\gamma(1 + 0{,}00123)c}$ | $9{,}34482 \cdot 10^{-10}$s |
| $t_3 = \frac{a}{\gamma(v_E - v_S)} = \frac{0{,}28m}{\gamma(4{,}6 - 0{,}00123)c}$ | $2{,}02953 \cdot 10^{-10}$s |
| $t_4 = \frac{a}{\gamma(c + v_S)} = \frac{0{,}28m}{\gamma(1 + 0{,}00123)c}$ | $9{,}32186 \cdot 10^{-10}$s |
| $Eq. (10.07)$: $t_T = t_1 + t_2 - t_3 - t_4$ | $2{,}19 \cdot 10^{-12}$s |

Tab.13.1: Maximum of expected values using prisms according to Fig. 10.1 with $a = 280\ mm;\ v_E = 4.6c;\ v_S = 369$ km/s

To increase the informational value of an experiment it is therefore necessary to adjust one of the parameters. This could be achieved by a tight decrease of the length of the pulse, i.e. using a femtolaser. However, this approach would be limited by the absorption capability of the beam at the surface of the prism and by the increasing complexity of the measurement technique. Further it is theoretically possible to enlarge the distance of the measuring device to increase the value of $\Delta t$; in this case it must be respected that an extreme reduction of the tunneling effect will appear.

An experimental set-up on basis of the discussed parameters is therefore not reasonable and has to be optimized considerably by suitable modifications. To respect this, the proposal presented in Fig. 13.2 shall be brought into discussion. In this case instead of the typically used single beam and the comparison between reflected and tunneled pulse a second beam is symmetrically passing the device. For examination only the tunneled parts of the pulses are amplified and compared with each other. Using this concept all problems with the interpretation of the experiment as discussed before, where the comparison between reflected and tunneled pulses was necessary, will be avoided.

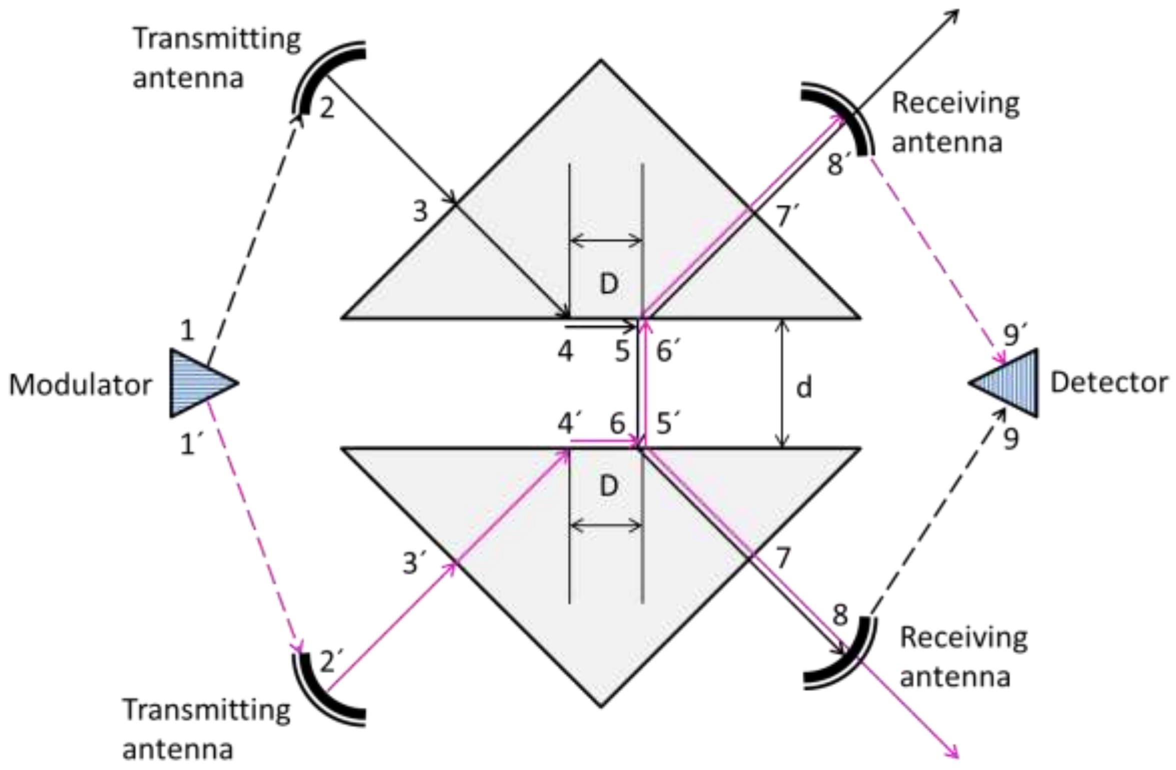



Fig. 13.2: Possible experimental set-up for the measurement of tunneling effects in different spatial directions.

Using this set-up, the experiment will start when the modulator is sending signal S and S′ to the transmitting antennas situated at opposite directions. The generated pulses will pass the device according to the presentation of Fig. 13.2.

For a reasonable evaluation, the use of differential analysis should be preferred. In this case the apparatus must be gauged in an arbitrary direction in such a way that the tunneled pulses of both prisms are exactly matching; measurements of a time-difference will in this case show by definition a zero-result. When in a second step the apparatus is turned and an effect like discussed before exists, then between both prisms a time difference for the passing pulses will appear. The height will be dependent on the direction to the state of absolute rest, the velocity of the signal and on the total length of the used apparatus.

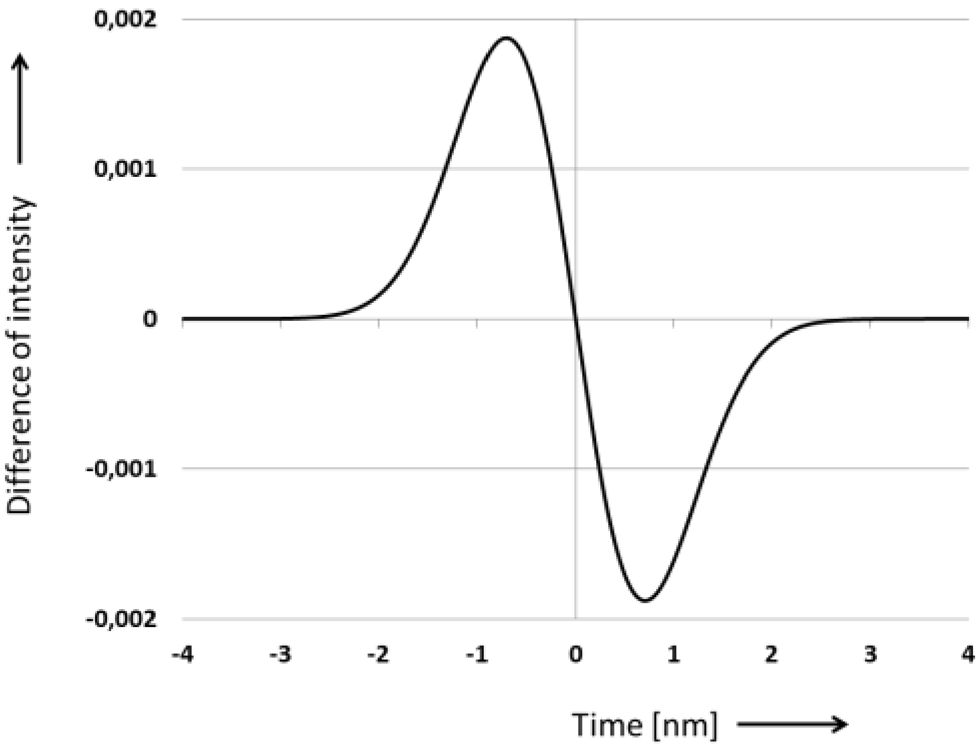



Fig. 13.3: Expected values for an apparatus with a length of 280 mm, $v_S = 369$ km/s and $v_T = 4.6c$

To amplify the signals, the enlargement of the prisms or the distance between them is no suitable option, because in these cases the measuring effects will be considerably reduced. However, it is possible to detect the signals of the prisms and after amplification to transmit these into a secondary set-up to repeat the measurements. The converting of the signals will most probably result in small differences of the measured time which will have an influence on the related values. However, these effects are not detrimental for the experiment and can be neglected because in principle only differences between both parts are measured.

It is noticeable that the expected values are exceedingly small, but that the proposed experiment has a realistic chance to provide reliable data. Particularly important is the mechanical stability of the set-up. This must be placed on a turning table to realize measurements in different spatial directions. Further on if a positive result could be achieved, the differences between values measured during the realization over a day and the connected change of the position of earth to a system of absolute rest will appear.

With the presented experiment it could be possible to provide evidence about basic physical aspects. Either a positive effect will be detected and then the already discussed consequences for Special Relativity must be considered, or, if it is not the case, the possibility of superluminal information transport during tunneling experiments is finally answered in a negative way.

## 13.2 Measurement of synchronization differences

As already described in chapter 10.2 the Lorentz Transformation is causing differences in synchronization because of the relativity of simultaneously for systems with different velocities. There are possibilities for measurements, when between two clocks, which are placed in a certain distance in a laboratory, synchronization is realized first, the lab is then accelerated in direction of their orientation and finally the procedure is repeated. In this case according to laws of the Lorentz Transformation synchronization differences at both clocks must appear.

In Fig. 13.4 the relations discussed before are presented. To ensure proper graphical reproduction exceedingly high velocities were chosen ($v = 0{,}5c \pm 0{,}25c$, this is corresponding to values of $v_1 = 0{,}667c$ and $v_2 = 0{,}286c$ when the correlations for relativistic addition of velocities are used).

When $t_0$ is the time for a signal running between positions A and B in a system at rest then for the left part of the diagram the following measuring effects will be achieved:

$$t_{AB} = \frac{t_0}{\gamma_1\left(1 - \frac{v_1}{c}\right)} \qquad t'_{AB} = \frac{t_0}{\gamma_2\left(1 - \frac{v_2}{c}\right)} \tag{13.10}$$

$$t_{BA} = \frac{t_0}{\gamma_1\left(1 + \frac{v_1}{c}\right)} \qquad t'_{BA} = \frac{t_0}{\gamma_2\left(1 + \frac{v_2}{c}\right)} \tag{13.11}$$

and

$$\Delta t_{AB} = t'_{AB} - t_{AB} = \frac{t_0}{\gamma_2\left(1 - \frac{v_2}{c}\right)} - \frac{t_0}{\gamma_1\left(1 - \frac{v_1}{c}\right)} \tag{13.12}$$

$$\Delta t_{BA} = t'_{BA} - t_{BA} = \frac{t_0}{\gamma_2\left(1 + \frac{v_2}{c}\right)} - \frac{t_0}{\gamma_1\left(1 + \frac{v_1}{c}\right)} \tag{13.13}$$

Because of

$$c = \frac{a}{t_0} \tag{13.14}$$

this leads for $v_1, v_2 \ll c$ to

$$\Delta t_{AB} \cong \frac{a[v_1 - v_2]}{c^2} \tag{13.15}$$

and

$$\Delta t_{BA} \cong \frac{a[v_2 - v_1]}{c^2} \tag{13.16}$$

Further the difference to the situation in chapter 10.2 is that in this case not 2 independent observers perform the test, but 2 clocks in just one integrated laboratory.

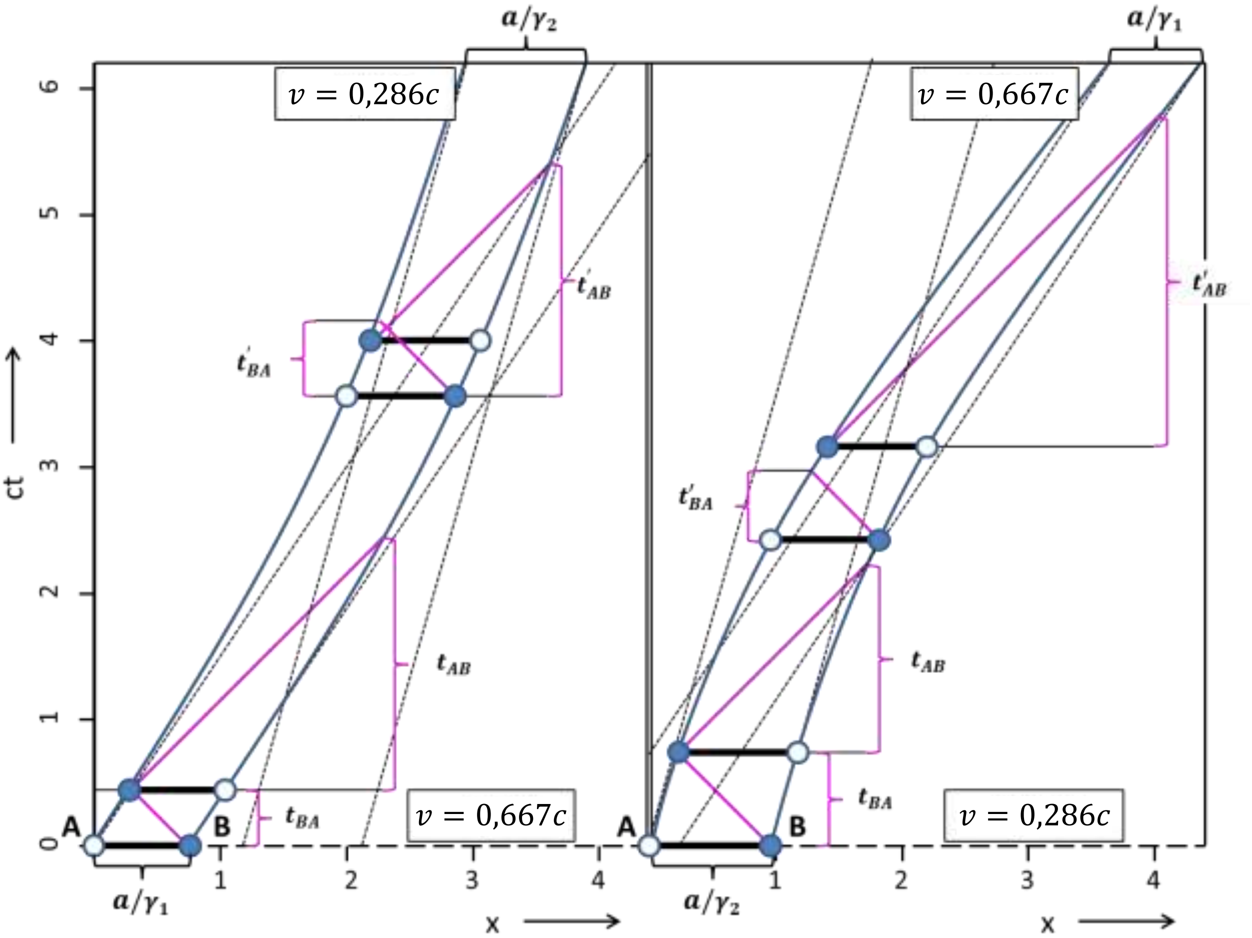



Fig. 13.4: Space-time-diagram for systems after changing velocities
Left: Reducing speed
Right: Increasing speed

For the right-hand side of the diagram the same relations apply, with the difference that $\gamma_1$ and $\gamma_2$ are changed. For the measurement of these differences the following experiment is proposed:

## a) Experimental set-up

For the experiment 2 clocks are placed in a distance $a$ at the positions A and B. In a moving system (see Fig. 13.4) the distance changes to $a/\gamma$. After the exchange of signals for each clock a synchronization procedure is carried out. It is important that the signals are not reflected to a central station for comparison because – as it is the case for the Michelson-Morley or Kennedy-Thorndike-Experiment – a null result would appear. Afterwards the laboratory is accelerated in orientation direction of the clocks and after another exchange of signals the synchronizations are repeated. Following this procedure, then because of the Lorentz Transformation a synchronization difference between the positions before and after acceleration must appear which reads $\Delta t_{BA}$ for clock A and $\Delta t_{AB}$ for clock B.

When an experiment like proposed before is conducted it would make sense to consider the differences between $\Delta t_{AB}$ and $\Delta t_{BA}$ (in this case one of the values will be positive, the other negative). First this will result in the fact that the measuring value is doubled, second distortions caused by deviations in the length of the device (i.e. by temperature changes or effects due to acceleration) of the dimension $\Delta t_S$ are eliminated because effects of increasing or reducing length would have the same influence. In this case the following equation is obtained:

$$\Delta t \cong \Delta t_{AB} + \Delta t_S - (\Delta t_{BA} + \Delta t_S) = \frac{2a[v_1 - v_2]}{c^2} \tag{13.17}$$

The result is depending on the distance between the clocks $a$ and the velocities $v_1$ and $v_2$ only.

## b) Estimations of the size of possible generated results

The best and most accurate method to perform a measurement like this would be to place the whole experimental device in a rocket and drive it to space, but without doubt the effort in this case would be extremely high. On the other hand, the speed differences that could be realized would be also high and so standard 87Rb-clocks, which are already in use for the GPS satellite navigation system with a standard deviation of approx. $3 \cdot 10^{-12} s$ could create very reasonable results.

When terrestrial experiments are considered, the requirements concerning accuracy would increase significantly. An experiment like this could be e.g. conducted using an airplane. A synchronization procedure at the ground and a comparison with data after the start is useless, however, because differences in the height above ground would lead to a distortion of the values. Instead, measurements after the start using a constant height are proposed. Reasonable values are e.g. differences between 300 km/h and 900 km/h. The experiment should be repeated in several directions relative to the rotation of the earth to eliminate distortions (i.e. by the Sagnac-effect).

When a difference of 600 km/h for the velocities and a length of 30m for the set-up is assumed, then values of approximately $1{,}1 \cdot 10^{-13} s$ will be obtained according to Eq. (13.17). An experiment like this could reveal reasonable data, because using advanced atomic clocks measurements in the range of $10^{-17} s$ are possible. This is of course not a simple operation and needs careful verification processes, because it must be shown first, that the clocks needed for the experiment are sufficiently stable for the use in an accelerated system.

In alternative considerations the use of a train or magnetic levitation train transporting the experiment could be possible. Because of lower speed differences the measuring sensitivity would be reduced but the necessary budget is smaller. In an alternative experimental set-up, the complete equipment could be placed in a container, tested on the ground and then loaded into a plane. If a usual commercial 40 feet container is used values of approximately $5 \cdot 10^{-14}$s could be expected which are, with the limitations already discussed, also sufficient to create a significant result.

At this stage of the discussion, it is possible to make the objection that in principle measurements like these are not feasible inside the gravity field of earth. As a counterargument it can be stated, however, that important experiments with a positive and meaningful result were executed in this way. In particular the trials of J. C. Haefele and R. E. Keating [81,82] shall be mentioned. In this case high precision atomic clocks were transported by plane around the earth and their values were compared afterwards with reference clocks which were not transported. The flight in direction of earth's rotation showed, that the transported clocks run slow and in opposite direction they were faster than the clocks on the ground. The results were in good compliance with the values predicted by the theory. So, with these experiments it was possible to identify a condition of rest not including the rotation of the earth.

However, if a terrestrial measurement is not possible then the use of a rocket is the only alternative left for the execution of the proposed experiment.

If any of these experiments whether on ground, in air or in space will show a positive result, experimental evidence is provided, that the "Relativity of Simultaneously", which is a necessary condition when the Lorentz-Transformation is valid, reveals the expected differences in local time after acceleration. It shall be pointed out again that this experiment must generate values possible to measure. This is in contradiction to many other experiments where the theory of Special Relativity is predicting a zero result. This experiment could therefore deliver the final answer, whether the proposed Relativity of Simultaneously, which is a major and necessary part of the Lorentz-Transformation, does really exist.

## 13.3 Measurement of velocity after non-elastic collision

In chapter 7.1 it was already demonstrated that an increase of mass must appear during non-elastic collision to avoid conflicts with the laws of conservation for momentum and energy. When this is the case the speed of a combined body after collision can be easily derived by using the relativistic addition of velocities. If this would not be the case, or partly not, then the measurement of the speed of a joined body after non-elastic collision would provide interesting new information.

To verify this, the following experiment is proposed: Mass $m_2$ is accelerated to the exactly defined speed $v_2$. When it is hitting a mass at rest $m_1$, both objects form a composite body, and the resulting velocity is subject to exact measurement. This experiment could verify that during a nonelastic collision the potential energy of $m_2$ is completely transformed into mass. Although this conversion is verified on microscopical scale, however, for objects with large dimensions it could be possible that during deceleration a part of the energy is transformed into thermal energy and carried out of the system by radiation and not be available for reduction of the speed (concerning radiation see also chapter 7.2). This behavior would violate the principles of relativity and could be measured.

Example:

An object with mass $m_1$ is considered, which is at absolute rest ( $v_1 = 0$), an identically second mass (i.e. $m_2 = m_1$) is hitting it with velocity $v_2$, both objects are joining and moving on with the speed $v_3$.

According to the discussions in chapter 7.1 the following values for the different concepts can be calculated:

*a) Nonrelativistic*

In this case the Galilei-Transformation is valid

$$v_3 = \frac{v_2}{2} \tag{13.20}$$

*b) Relativistic*

This requires a transformation analog to Eq. (7.04) which leads to

$$v_2 = \frac{2v_{3\mathrm{R}}}{1 + \left(\frac{v_{3R}}{c}\right)^2} \tag{13.21}$$

$$v_{3R}^2 - \frac{2v_{3R}c^2}{v_2} = -c^2 \tag{13.22}$$

and finally

$$\frac{v_{3R}}{c} = \frac{c}{v_2} - \sqrt{\frac{c^2}{v_2^2} - 1} \tag{13.23}$$

An examination of this equation shows that positive results of the square root are leading to values $v_{3R} > c$ and therefore cannot be permitted because of plausibility reasons. If this square root in Eq. (13.23) is solved by Taylor expansion (for $v_2 \to 0$) then the result

$$\sqrt{\frac{c^2}{v_2^2} - 1} = \frac{c}{v_2} - \frac{v_2}{2c} - \frac{v_2^3}{8c^3} - \cdots \tag{13.24}$$

appears. Values of higher order can be neglected. Eq. (13.23) is changing accordingly to

$$\frac{v_{3R}}{c} = \frac{c}{v_2} - \sqrt{\frac{c^2}{v_2^2} - 1} \cong \frac{v_2}{2c} + \frac{v_2^3}{8c^3} \tag{13.25}$$

In table 13.2 calculated results for impact-velocities between 1 and 100.000 km/s are shown. To allow a better comparison, only the differences to the non-relativistic case $\Delta v$ according to Eq. (13.26) are presented. The value of $\Delta v$ is always positive, i.e. the calculation of $v_{3R}$ is leading in all cases to results higher than that of $v_3$.

$$\Delta v = v_{3R} - v_3 \tag{13.26}$$

| $v_2$ | 1 | 10 | 100 | 1000 | 10.000 | 100.000 |
|---|---|---|---|---|---|---|
| $\Delta v$ | 1,391 $10^{-12}$ | 1,391 $10^{-9}$ | 1,391 $10^{-6}$ | 1,391 $10^{-3}$ | 1,392 | 1,474 $10^{3}$ |

Tab. 13.2: Calculation of differences for end velocity after nonelastic collision. Initial value: Galilei-Transformation Eq. (12.20). Velocities in km/s.

The results for velocities $v_2 \geq 1000$ km/s related to the relativistic approach were calculated using the basic equation Eq. (13.23). For smaller values, the precision of a standard computer with 15 digits accuracy is no longer useful, and Eq. (13.25) must be used instead. This equation, however, must be extended with higher order terms using velocities of more than 10.000 km/s, so, a combination of both approaches was chosen.

For the realization of the proposed experiment, it would be reasonable to use a massive and compact body for the moving part, e.g. a sphere. For the not moving object it is proposed to use a ring with high plasticity. The ring should have an inner diameter slightly smaller than the diameter of the sphere. A set-up like this should allow precision measurements of the velocity directly on the surface of the sphere and would avoid problems which appear, when a plate or a deformable foil, which is wrapping around the sphere during the execution of the experiment, is used instead for the body at rest. Because of the expected small effects, the experiment must be conducted using a vacuum.

An evaluation of the expected results clearly shows that with increasing velocity by one order of magnitude the measuring effect will be boosted by 3 orders (with other words: factor 10 compared to factor 1000). It is therefore reasonable to increase the speed as much as possible. On the other hand, the demands concerning the precision of the required testing equipment will rise considerably with increasing speed so that it is necessary to find a reasonable compromise. When for example the value of 1 km/s is chosen, which is corresponding to the speed of a projectile of firearms, then according to the calculations presented here, a result of $10^{-9}$ s per meter of the measuring length would appear. It should be possible to detect values like this with a suitable experimental set-up.

For experiments like this an exact monitoring would be essential. It could for example happen, that because of the high accelerations at the start of the sphere and also during deceleration of the connected body the applied stresses on the material will be quite high and so vibrations could occur which could affect the results of the measurements. In this case maybe the use of composite materials with a soft inner core is necessary. The experiment must be conducted in different spatial directions. Although as pointed out in chapter 7.1 it is not likely that the result will differ from the relativistic addition of the velocities, this experiment is a reasonable addition to provide evidence about the relativistic increase of mass for non-elastic collisions on a macroscopic scale.

Finally, the question may be raised why an experiment like this should be performed at all, when theoretical considerations conclude that the result must be in accordance with the relation of relativistic addition of velocities. However, as already shown in chapter 11.3 effort is made since many years to provide evidence that Lorentz invariance can be violated and thus expand the theoretical basis. An experiment like it is presented here could therefore extend the range of possibilities in an interesting way.

# 14. Final evaluation of Special Relativity

At the end of the presented investigations, the various presentations of special relativity (SRT) available in the literature are discussed and evaluated in brief form. For this purpose, first the two central preconditions "principle of relativity" and "constancy of the speed of light" are examined. To represent the occurring range used in the literature, the possible representations were divided into "objective observation criterion" and "axiom". In recent publications very often the axiomatic approach is chosen. The earlier presentations, e.g. of Einstein, were mostly using the objective observation concept.

The common interpretation of the SRT today includes the aspect that there can be no system of absolute rest. The chains of reasoning used in the literature concerning this matter are quoted and evaluated. It is shown that none of these approaches can deliver a generally valid proof.

Einstein has chosen a top-down approach for the formulation of the SRT. For this purpose, the principle of relativity and the constant speed of light were defined as basics and the Lorentz transformation and later also the relativistic mass increase were derived from them. Now, with an "Extended Lorentz theory", a bottom-up concept is presented where the relativity principle is the result. The validity was proved by a multitude of examples.

With free choice of the base system, both approaches are completely equivalent. However, the Theory of Special Relativity has the disadvantage that it excludes the existence of a system of absolute rest in principle, but this can be integrated without problems into the extended Lorentz concept by a simple choice of the base system. From today's point of view, it seems reasonable to use for it the system which is the basis for the uniform cosmic background radiation in the universe. However, since up to now no experimental proof has succeeded, a decision cannot be made at present. In the context of this elaboration a proposal was made, how an experiment could be arranged, which makes a clear decision possible concerning the different approaches (chapter 13.1).

## 14.1 Principles of SRT and their presentation in the literature

It is quite surprising that until today there is no uniform formulation of the two central conditions "principle of relativity" and "constancy of the speed of light". Every author of a publication about the SRT chooses his own approach for this (only in individual cases, no presentation is made at all and without comment the Lorentz equations are used [89]). In order to represent the occurring bandwidth, the possible formulations were divided into "objective observation criterion" and "axiom" (Tab. 14.1). In more recent publications, the axiomatic approach is rather (but not exclusively) chosen.

| Objective observation criterion | Axiom of Special Relativity |
|---|---|
| 1. The execution of any physical experiment leads to the same result in all inertial systems. | 1. Principle of Relativity:<br>All inertial systems are equivalent. |
| 2. Measurements of the speed of light in different spatial directions lead to the same result in all inertial systems. | 2. Constant speed of light:<br>The speed of light in different spatial directions are the same in all inertial systems. |

Tab. 14.1: Currently common representations of the basics of Special Relativity

To show the differences, individual examples are presented in the following. The principle of relativity is defined in its original form by Einstein as follows [12]:

> “Principle of Relativity: The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems in uniform translatory motion relative to each other.”

This is therefore a formulation that can be assigned to an objective observation criterion. Some other authors also use the reference to measurements, although the representation can be completely different [27]:

> “Postulate I: It is impossible to measure, or detect, the unaccelerated translatory motion of a system through free space or through any ether-like medium which might be assumed to pervade it.”

This is different with the constancy of the speed of light. For this exist only few cases with the reference to measurements, e.g. M. Born with the following formulation [26a]:

> “The principle of the constancy of the speed of light: In all inertial systems, the speed of light, measured with physically identical rods and clocks, has the same value.”

In almost all other cases, the reference to measurement methods is not mentioned and the form as an axiom is used. Einstein himself used a more complicated form of representation which describes a measuring method but makes a clear assignment difficult:

> “Principle of constancy of the speed of light: Every light ray moves in the "resting" coordinate system with a certain speed $V$, independent of whether this light ray is emitted by a resting or a moving body. Here is
>
> $$\text{velocity} = \frac{\text{lightpath}}{\text{time period}}$$
>
> where "time period" is to be understood in the sense of the definition of § 1.”

The overall situation can be simplified as follows:

- Objective observation: No difference can be determined. The facts are verified by experiments.
- Axiom: In principle, there is no difference.

The interpretations associated with these representations are significant in the following and will therefore be evaluated in detail. The discussion starts with the constancy of the speed of light.

## 14.2 Constant Speed of light in every inertial system

First the possibilities to measure the speed of light shall be presented and discussed on a principal basis. The options for measurements can be characterized first by direct and indirect procedures (Tab. 14.2). Whereas direct measurements create quantifiable values, the indirect approach only allows the comparison between values measured in different spatial directions.

| 1. <u>Direct</u><br>Use of time measurements | 2. <u>Indirect</u><br>Comparison of oscillations |
|---|---|
| **1a) Measurements using light pulses**<br>Measurement of time differences at sender/receiver between emitting and receiving a signal after reflection at a mirror. | **2a) Measurement of frequency**<br>Comparison of frequency at sender/receiver between emitting and receiving a signal after reflection at a mirror. |
| **1b) Measurements using moved clocks**<br>Two or more identical clocks shall be synchronized. After the transport to reference objects light signals are exchanged and time is measured. | **2b) Oscillation measurements**<br>Analysis of light signals between sender/receiver and mirror as reference (Number of oscillations referring to travelling distance going and coming after reflection). |

Tab. 14.2: Possibilities for measurements of the speed of light

In case when direct measurements are chosen, it is essential that the distance between emitter and reference object must be known exactly. It makes no difference, whether the reference object is at rest relative to the sender or moving. First the possibility exists that the time difference between emitting and receiving a light signal after reflection at a mirror is measured (1a). In addition, identical clocks can be synchronized and transported to defined reference points, then signals can be exchanged followed by time measurements (1b). The disadvantage of this procedure is, however, that for test evaluation it must be recognized that moving clocks are subject to time dilatation and that this effect must be considered during test evaluation.

With the indirect methods, only possibly existing differences between the light velocities in different spatial directions can be determined. The distance to a reference object might be unknown but must remain constant during the measurement. First, the comparison of frequencies between outgoing and incoming signals is possible (2a). Furthermore, oscillation measurements have often been performed in the past, comparing the number of oscillations on the way to and from a mirror (2b). Here, the use of measurement providing interference patterns is particularly suitable, such as it is the case in the Michelson-Morley experiment.

The methods were all examined in the context of this elaboration, namely 1a) in chap. 2, then 1b) in chap. 5 as well as 2a) and 2b) in chap. 8. It is important for the interpretation of experiments of the type 2b) that here the phase velocity of light must be used for the evaluation. In the past, this was not done in a sufficient way, so that new and consistent results became visible in a new interpretation of the Michelson-Morley and Kennedy Thorndike experiments, taking this effect into account. If this effect is not respected, false conclusions are drawn.

In the following, another important aspect about the speed of light will be dealt with. The statement: "The speed of light is the same in all inertial systems" must be considered and interpreted carefully. Equal speed of light means:

In every inertial system the speed of light can be chosen in such a way that the own system serves as basis. All conditions of the theory of special relativity are then valid without restrictions. The following relation was defined by Einstein for a base system called "resting" by him, related to another arbitrarily moved system [12a]:

$$\frac{1}{2}(\tau_0 + \tau_2) = \tau_1 \tag{3.60}$$

This condition, today also called “Einstein synchronization”, means that the times for a signal exchange between two points are divided exactly in half for the way there and back (for details see chapter 3.4 and 12.2). This statement is independent of whether the reference object is at rest with respect to the origin or is in motion. Together with the statement that the speed of light is constant in all directions, the distances must also be the same.

The situation is different when the system emitting the signal itself is moving. Let's consider the simple case that the origin of the signal and the reference object have the same velocity. Also, here it is possible that the light velocity of the origin is taken as resting and the same conditions apply as already derived. The same procedure is possible for a signal exchange likewise for any other system from its subjective view.

However, if several test participants from different inertial systems moving against each other observe the *same* event, e.g. the signal exchange between different spatially separated points, different observations must occur. If the speed of light of the own system is taken as a basis for measurements and if the times and distances necessary for the signal exchange are determined for the way there and back, different results appear. Distance and time are *not* divided symmetrically. This effect is caused by the "relativity of simultaneity".

This fact has already been presented in detail in chapter 12.2. At this point it shall be shown additionally that the data taken from this diagram correspond exactly to the results of the Lorentz transformation. For this purpose, first in Fig. 14.1 the left side of Fig. 12.3 is shown again, which represents the correct signal course from the point of view of the moving system S.

The determination of the Einstein synchronization for the outgoing and returning path for the signal exchange between two points (e.g. the ends of a laboratory A and E), which means time and path are in each case divided to the half, is valid only subjectively for the system L which is at rest to the laboratory. If from another moving inertial system S this determination would also apply and the times $t_1 = t_2$ would be equal, the situation would arise as shown in the right part of the diagram 12.3 with signal velocities larger or smaller than $c$ as well as measurable synchronization differences. Moreover, according to these considerations, a situation where the path is constant in both directions cannot even theoretically occur because the lab end moves away from the original point immediately after the signal is emitted and is at a different location on the return path. Instead, the situation as shown in the left part of the diagram applies. This means that the determination of a reference system can always only be subjective.

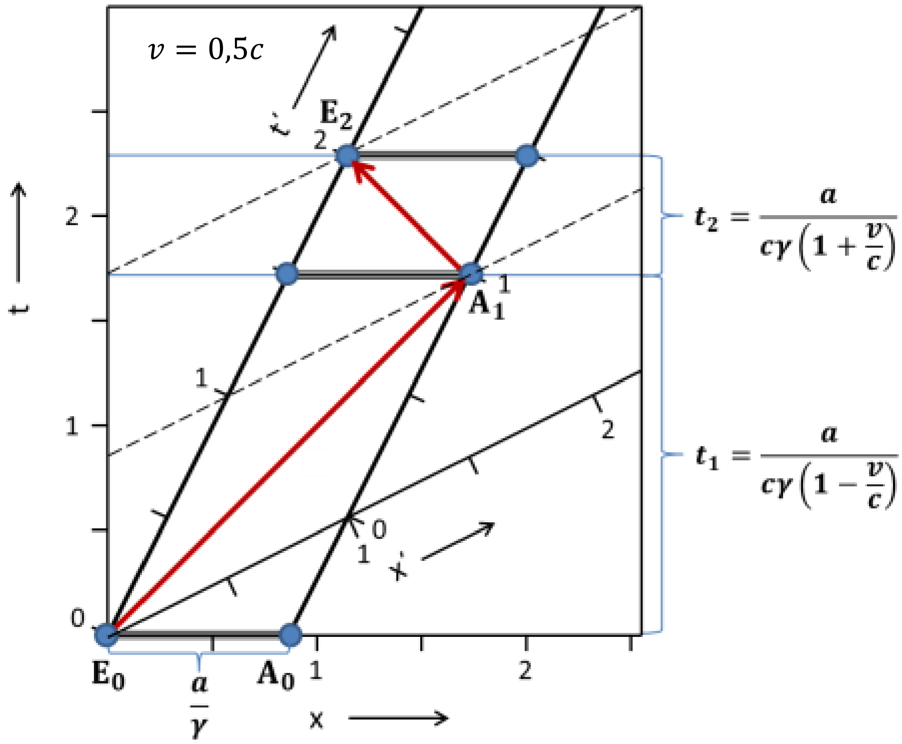



Fig. 14.1: Schematic presentation of a signal in a laboratory L between E and A from the point of view of an inertial system S moving relative to it ($v = 0{,}5c$).

Table 14.3 shows the coordinates for displacement and time taken from Fig. 14.1. The values subjectively valid for the moving system were calculated by using the Lorentz equations. It is immediately recognizable that in this normalized representation the value of the speed of light is $c$ in all cases; for the reference system this results immediately from the position of the signal course in the diagram (45° to $x$ and $t$), for $x'$ and $t'$ from the relations between path and time.

| | $x$ | $t$ | $x'$ | $t'$ |
|---|---|---|---|---|
| $\mathrm{E}_0$ | 0 | 0 | 0 | 0 |
| $\mathrm{A}_1$ | 1,73205081 | 1,73205081 | 1 | 1 |
| $\mathrm{E}_2$ | 1,15470054 | 2,30940108 | 0 | 2 |

Tab. 14.3: Determination of the coordinates of $\mathrm{E}_0$, $\mathrm{A}_1$ and $\mathrm{E}_2$ from Fig. 13.1
The values of $x'$ and $t'$ were calculated using the Lorentz-Transformation.

In summary, the following is valid: If the same event is considered from different inertial systems, this leads subjectively to the situation that in all cases the definition of the own speed of light is possible as a basis. The connection between the systems is given by the Lorentz equations, furthermore the principle of the relativity of simultaneity is valid.

## 14.3 Principle of relativity

For a better understanding of the specifics of this point, it is useful to consider the historical development first. As a main issue to mention here is the conviction, which lasted until the 20th century, that light, because of the wave properties attributed to it, requires a carrier medium for propagation, which was called "ether". This was a general consensus for centuries, although there were great differences in the understanding of the structure of this ether.

Until the Michelson-Morley experiment was carried out in 1887, the idea existed that this ether penetrates everything and shows similarities in its properties with air and sound waves transported in it. Derived from various experimental results, however, there were different opinions about whether ether is influenced by matter and is carried with it completely, partially, or not at all. (Further details of these experiments and subsequent discussions are presented in chapter 1.3).

However, there was a general understanding that when passing through ether, there must be an effect caused by an occurring "ether wind". On the basis of these considerations, the Michelson-Morley experiment was carried out, which, however, gave a null result. This result led to a multiplicity of considerations, which brought however over nearly two decades no breakthrough. It is reported that Lord Kelvin spoke on the subject of "ether" during the international physics congress in Paris in 1900. He said at that time: "The only cloud in the clear sky of the theory was the null result of the Michelson-Morley experiment" [49h]. He as well as many other physicists of his time shared the opinion that the experiment should be repeated with higher accuracy and then would bring the expected positive result; however, none of these attempts were successful.

A first solution appeared when Hendrik A. Lorentz developed the equations later named after him, which allowed a contradiction-free calculation of the correlations. The key point was the introduction of different local times and an effect which was later called "relativity of simultaneity" by Einstein. It was essential in the development that these relations had a similar structure as the previously developed Maxwell equations for electromagnetism. Lorentz was convinced that the ether, which he still considered necessary, must have these properties.

Einstein revolutionized the view on this problem. In 1905, he first showed that light propagation does not need a medium but can be understood as emission of "discontinuous energy quanta" [48]. Until then, the idea of their existence had not existed, but only the nature of light as a wave and the existence of a transport medium connected with it was in the focus. With this approach, Einstein was able to reduce the fundamentals of the theory he presented to the two principles already discussed. The dualism between corpuscle and wave, which is evident for physics today, was not yet known at that time; it was formulated for the first time in 1924 by Louis de Broglie.

The principle of relativity formulated by Einstein also requires a precise interpretation. First, this can be divided into the following detailed statements:

a) If identical experiments are carried out by different observers in reference systems moving uniformly relative to each other, the results will be the same.
b) An observer can describe results of any experiment in another inertial system that shows a constant relative movement using only the Lorentz transformation equations and the relativistic increase of mass. In particular, the observation of the time sequence of events is the same in all cases.
c) All systems moving uniformly relative to each other are equivalent and there is no absolute "system at rest".

The statement a) will now be defined as "principle of identity", b) as "principle of equivalent observations" and c) as "principle of complete equivalence of all inertial systems". While points a) and b) are today backed up by multiple test results, this must be considered in a differentiated manner for point c). This will be done in the following. From the literature, several argumentations are known to support the statement of point c), namely:

1. The results of the Michelson-Morley experiment show that there can be no system of absolute rest.

This becomes clear e.g. in the formulation of Kneubühl [46c] with the evaluation of the Michelson-Morley experiment:

> "The Galilei transformation is not valid for the light! The concept of a "resting" universe is not tenable."

While the first sentence is correct without doubt (the Lorentz transformations are valid as known) the conclusion in the second sentence cannot be derived from it. If the principle of constancy of the phase velocity of light is taken as a basis, the integration of a system of absolute rest is possible without contradictions, which has already been presented in detail in chapter 8. Therefore, contrary to the author's opinion, the Michelson-Morley experiment does not provide evidence for this thesis.

Furthermore, there exists another argument:

2. What is not measurable does not exist.

This view is held, for example, by Born [26b]. The formulation he uses is:

> "If two observers moving relative to each other have the same right to say that they are resting in the ether, there can be no ether."

The term "ether" is to be understood here as a synonym for a system of absolute rest, whose existence is completely rejected based on the available knowledge. Einstein himself has said the following about the topic ether and Theory of Relativity in his inaugural speech as visiting professor in Leiden in 1920 (for explanation: the systems K and K1 are inertial systems moving relatively to each other) [86]:

> "Now the anxious question arises: Why should I distinguish the system K, to which the systems K1 are physically completely equivalent, in the theory in favor to the latter by

the assumption that ether rests relative to it? Such an asymmetry of the theoretical building, to which no asymmetry of the system of experiences corresponds, is unbearable for the theoretical physicist. It seems to me that the physical equivalence of K and K1 with the assumption that ether is resting relative to K, but is moving relative to K1, is not exactly incorrect from the logical point of view, but nevertheless unacceptable."

The ether concept was not completely rejected by him. In the following explanations he even pointed out that it is necessary for General Relativity; however, he contradicts the idea that it is a system of absolute rest and was of the opinion that ether must exist for every inertial system.

From this representation another argument becomes recognizable, which can be formulated as follows:

3. The Theory of Relativity is preferable to the ether theory according to "Ockham's principle".

"Ockham's principle" is the basic approach to a problem and is named after William of Ockham (1287-1347) and concerns the "law of parsimony". In short, it describes a problem-solving procedure according to which, when several possible explanations are available, the simplest theory is always to be preferred to all others. The simplest theory has the fewest variables and hypotheses. The application of this principle is also called "Occam's razor" because it cuts off everything superfluous and allows only one sufficient explanation.

If the theories on this fundamental basis are compared with each other, then the Theory of Relativity contains 2 basic assumptions, the ether theory on the other hand needs, with the condition of a state of absolute rest (which cannot be proved experimentally at present) a further one. According to the general concept that a theory should be based on as few assumptions as possible, the Theory of Special Relativity is therefore preferable.

The topic ether versus relativity principle was subject of long and controversial discussions at the beginning of the 20th century. Especially because of the considerations presented here, the controversy was clearly decided in favor of Special Relativity and there were no serious objections against it for many decades.

This did not change before the beginning of the second half of the 20th century with the discovery of the uniform cosmic background radiation. Latest measurements with extreme precision showed, that our sun is moving relative to it with a velocity of 369.1 km/s. The maximum deviation of the measurements is actually 0.9 km/s, i.e. 0.25% [23]. Various approaches have been developed to reconcile this measurement result with SRT. However, these were all connected with the consideration to cancel the "relativity of simultaneity" and to introduce a state of absolute rest on this basis. None of these theories were able to show results without severe discrepancies to experimental findings. Major characteristic for all formulated theories was that the simple concept of the invariant phase velocity of light had found no entrance into the considerations and consequently the following interpretations could not be useful.

The Theory of Special Relativity says so far nothing about the cosmic background radiation. But if this phenomenon is also considered, the fact would have to be added that an unspecified coincidence has led to the uniform alignment of this radiation. Here the view of a coincidental givenness is possible; this theory is represented e.g. by Johann Rafelski in "Relativity Matters" (2017). Thereby the cosmic background radiation is ascribed the status of an appearing "beacon" to which one can refer [93].

If now the two competing theories are compared again, it becomes clear that Ockham's principle cannot be effective here because of the same number of fundamental assumptions, since Special Relativity needs an additional hypothesis by the appearance of the cosmic background radiation. This is not necessary for the ether theory. So, based on these considerations it is not possible to decide which of the theories is preferable. Only an unambiguous experiment could provide clarity.

As already mentioned, in this compilation the basic approach was applied that all phenomena are considered from the point of view of a stationary and a moving observer. However, none of the calculations performed showed any difference. These are the following topics, for which the relevant chapter is given in this elaboration:

→ Exchange of signals between point-shaped observers (2.1)
→ Exchange of signals inside moving bodies (2.2)
→ Exchange of signals and correlation of angles (2.3)
→ Signal exchange in any spatial direction (2.4)
→ Experiments with transparent media in motion (4.2)
→ Triggering of engines after synchronization (4.3)
→ Exchange of signals between observers with spatial geometry (4.4)
→ Clock transport t (5.1)
→ Twin paradox (5.2)
→ Relativistic mass increase and energy (6.1)
→ Spring paradox (6.2)
→ Relativistic elastic collision (6.3)
→ Exchange of signals in systems with constant acceleration (6.4.1)
→ Relativistic rocket equation (6.4.2)
→ Relativistic non-elastic collisions (7.1)
→ Analysis of disintegration into 2 particles (7.2.1)
→ Disintegration into 2 photons (7.2.2)
→ Invariance of phase velocity during transition between different inertial systems (8.)

In summary, there is only one reason to prefer the Theory of Special Relativity to the approach of Lorentz. This is the fact that SRT generally covers all conceivable physical experiments, while the Lorentz transformation only describes the signal exchange between different inertial systems. To guarantee a general validity, therefore, an addition must be made, which is given by the solution of the Einstein equation regarding kinetic energy. This will be shown in the following chapter.

## 14.4 Alternative presentation: Extended Lorentz-Theory

As already explained in chapter 1.6, Einstein had chosen a top-down approach for the Theory of Special Relativity. For this purpose, the principle of relativity and the constant speed of light were defined as basics and the Lorentz transformation and later also the relativistic mass increase were derived from them. For the formulation of the principle of relativity, a similar variant must be chosen as by Einstein himself, namely the representation as objective observation criterion. Also, the statement about the velocity of light can be made in this way, but here it is better to use the constancy of the phase velocity of light. The proposal for a contradiction-free and unambiguous formulation of the principles of the SRT reads accordingly:

1. The execution of any physical experiments leads to the same results in all inertial systems.
2. The phase velocity of the light is invariant in all inertial systems and its speed is equal to the value of the velocity of light measurable in every inertial frame.

However, the investigations presented here have also shown that a bottom-up approach with an extended Lorentz theory is also possible. In this case, the necessary physical basic laws are defined, and the relativity principle can then be derived from them. This approach reads as follows:

1. From the unlimited number of existing inertial systems, one is selected as base system and marked with index 0.
2. In this basic system, measurements of the speed of light show the same value $c$ in all directions.
3. The properties of all other inertial systems are defined by their relative velocity $v$ to the base system, and the following relations are valid for time $t$, displacement $x$ and mass $m$

   a) $t = \gamma\left(t_0 - \frac{v}{c^2}x_0\right), \qquad x = \gamma(x_0 - vt_0)$

   b) $m = \gamma m_0$

   with: $\qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

First some formal remarks: The equations under a) are the Lorentz transformation (related to the basic system with index 0). In order to unify the formulas, the traditional representation with $t'$ and $x'$ was not used here (see. chapter 1.6). Equation b) describes the relativistic mass increase and contains the Einstein equation for the kinetic energy (see also chapters 1.6 and 6.1)

$$E_{kin} = m_0 c^2(\gamma - 1) \qquad (6.14)$$

In this representation, special relativity and the extended Lorentz approach are mathematically completely equivalent. However, the Theory of Special Relativity excludes with usual interpretation the existence of a system of absolute rest, which can be integrated in the extended Lorentz approach by simple choice of the basic system without further assumptions or restrictions. From today's point of view, it seems to be reasonable to use for this the system which is the basis for the uniform cosmic background radiation. However, since up to now no experimental proof has succeeded, a decision cannot be made at present.

From today's point of view, the only possibility for an experimental proof of a system of absolute rest is the realization of experiments with superluminal velocities. Today there are investigations within quantum mechanics, e.g. in tunneling experiments, where superluminal effects have been detected. Regarding the interpretation of the results, however, there are still big differences. On the one hand it is assumed that despite superluminal effects were detected, no information is transmitted faster than light and therefore the validity of Special Relativity need not be questioned, on the other hand it is assumed that a simple signal transmission, e.g. by a pulse, can indeed be faster than light. In the context of this elaboration a proposal was made, how an experiment could be arranged, which allows a clear decision concerning the different approaches (chapter 13.1).

Further experiments were also presented, which should experimentally confirm other interesting aspects such as the "relativity of simultaneity" and "mass increase after a non-elastic impact".

If these experiments would be carried out, important fundamental questions of physics could be investigated and possibly finally decided. There is certainly some effort involved, but compared to today's costs for experiments, this should be bearable. It is hoped that teams of researchers will be found to undertake the experiments.

In conclusion, it is remarkable that even more than a century after the formulation of the Theory of Special Relativity, new aspects still become apparent when intensively examined.

# Annex

The attachments presented in the following were utilized in those cases when calculations could not be provided in a closed analytical way, and it was necessary to use numerical calculations.

For every calculation first the mathematical foundations are presented and based on this the used formula for the program. For the execution Microsoft Excel© was used. In every case in addition the original codes are provided to allow a simple confirmation when requested.

To every evaluation, examples with selected reasonable basic conditions are added.

# Annex A: Relativistic elastic collision

In this attachment the necessary calculations for the elastic collision are presented (see also chapter 6.3). For this purpose, the equations

$$p = m_1\gamma_1 v_1 + m_2\gamma_2 v_2 = m_1\gamma_3 v_3 + m_2\gamma_4 v_4 \qquad (A.01)$$

$$\frac{E_0}{c^2} = (\gamma_1 - 1)m_1 + (\gamma_2 - 1)m_2 = (\gamma_3 - 1)m_1 + (\gamma_4 - 1)m_2 \qquad (A.02)$$

are used. Eq. (A.02) is transformed to

$$\gamma_4 = \frac{\frac{E_0}{c^2} - (\gamma_3 - 1)m_1}{m_2} + 1 \qquad (A.03)$$

with

$$v_4 = \pm c \cdot \sqrt{1 - \frac{1}{\gamma_4^2}} \qquad (A.04)$$

Further Eq. (A.03) and Eq. (A.04) are inserted in Eq. (A.01)

$$f(v_3) = m_1\gamma_3 v_3 \pm c\left(\frac{E_0}{c^2} - (\gamma_3 - 1)m_1 + m_2\right)\left[1 - \left(\frac{\frac{E_0}{c^2} - (\gamma_3 - 1)m_1}{m_2} + 1\right)^{-2}\right]^{1/2} \qquad (A.05)$$

This relation is depending solely on the defined values for $v_1$ and $v_2$. Using the principle of bisection, the values for $v_3$ and in a second step also $v_4$ can now be determined (for comparisons of different calculation methods see annex D). First the appropriate starting values $(v_{3+})_0$ and $(v_{3-})_0$ must be identified for which the following conditions apply:

$$f(v_{3+})_0 > p \qquad (A.06)$$

$$f(v_{3-})_0 < p \qquad (A.07)$$

In the interval $[(v_{3-})_0; (v_{3+})_0]$ the function $f(v_3)$ must be continuous and differentiable and further $f'(v_3) \neq 0$ is required. This means, that in the chosen interval minima and maxima are not allowed, because otherwise no exact solution exists. Now the mean value is determined using

$$(v_3)_1 = \frac{(v_{3+})_0 + (v_{3-})_0}{2} \qquad (A.08)$$

and $f(v_3)_1$ is calculated according to Eq. (A.05). The following equations apply:

$$f(v_3)_1 > p \quad \Rightarrow \quad \begin{cases} (v_{3+})_1 = (v_3)_1 \\ (v_{3-})_1 = (v_{3-})_0 \end{cases} \qquad (A.09)$$

$$f(v_3)_1 \leq p \quad \Rightarrow \quad \begin{cases} (v_{3+})_1 = (v_{3+})_0 \\ (v_{3-})_1 = (v_3)_1 \end{cases} \tag{A.10}$$

The calculation is repeated with increasing index 1 to $K$ until the required accuracy is achieved. Caused by the appearance of the indication $\pm$ in the relations Eq. (A.04) and Eq. (A.05), which is caused by the determination of the square root, the calculation of $v_4$ provides 2 different results, which must be interpreted using plausibility considerations according to the applicable situation.

If a simple spreadsheet is used for the calculation (cf. Chap. A.2), the input parameters are limited due to the previously discussed boundary conditions. For the calculations, the starting conditions must be chosen so that the values for $v_1$ are positive in all cases. It is also assumed that, through appropriate index selection, the values of $v_1$ are always greater than $v_2$ and the values for the calculated momentum in Eq. (A.01) are $p > 0$. If the actual default values deviate from these prerequisites, adjustments are necessary whose definition is shown below.

## A.1 Program flow of the calculation process

In the following it is described which process steps a program must execute in order to carry out the necessary calculations (cf. Fig. A.1). To ensure an unrestricted selection of the output parameters, their determination is first carried out via the subprogram "Parameter Input" and after completion of the calculations the reconversion is carried out by means of the subprogram "Parameter Output" (Fig. A.2).

The specification of the input-parameters is determined by the following criteria:

1. For the consideration of the velocities of objects with mass $m_1$ and $m_2$ the precondition $v_1 > v_2$ is necessary. The reason for this is, that the calculation starts with the determination of $v_3$ (of the object with mass $m_1$ after collision); values with $v_1 < v_2$ would represent a situation that object $m_2$ is moving faster than $m_1$ and this would mean that the incident could not take place.
2. Further the general conditions $v_1 > 0$ and $p > 0$ must apply. These preconditions are necessary to guarantee an undisturbed execution of the program because the presence of the square root in the formula would otherwise lead to interpretation problems. In the case discussed here, only positive values must be obeyed instead of plus and minus as possible results.

The definition of these preconditions for the execution are severe restrictions at first sight, but they are representing no limit for the calculations. This is the case because several possibilities exist to modify the starting conditions as

1. The algebraic sign for velocities $v_1$ and $v_2$ can be determined as desired, under the condition that they are changed simultaneously.
2. The index between $v_1; m_1$ and $v_2; m_2$ can be changed.

When an appropriate combination of these conditions is used, this will cover all possible situations. To show this, first the case $v_1 > 0$ shall be discussed. Instead of the theoretically possible $2^3 = 8$ combinations defined by the 3 starting conditions $v_2 > 0$, $v_1 > v_2$ and $p > 0$ only 4 alternatives are remaining. This can be explained by discussing the following situations:

- For case $v_2 > 0$ in combination with $v_1 > 0$ the resulting total momentum is always positive and so it is not necessary to consider it further. A negative momentum can only occur when the velocities show different algebraic signs (or are both negative).
- The discussed case $v_1 > 0$ in combination with $v_2 < 0$ is obviously always leading to the result $v_1 > v_2$.

These cases can be excluded from further considerations. The remaining variants can be summarized as follows:

| Condition 1 | Condition 2 | Action | Code |
|---|---|---|---|
| $v_2 > 0$ | $v_1 > v_2$ | No action necessary | F1 |
| $v_2 > 0$ | $v_1 < v_2$ | Change of index | F2 |
| $v_2 < 0$ | $p > 0$ | No action necessary | F1 |
| $v_2 < 0$ | $p < 0$ | Change of index and algebraic sign | F4 |

Tab. A.1: Input-parameter depending on starting conditions for $v_1 > 0$

For the situation $v_1 < 0$ the determination follows the same procedure with the only difference, that first a general change of the algebraic sign is necessary. It must be obeyed that in this case the algebraic sign of the momentum is changing also. Finally, the following cases apply:

| Condition 1 | Condition 2 | Action | Code |
|---|---|---|---|
| $v_2 > 0$ | $p > 0$ | Change of index and algebraic sign | F4 |
| $v_2 > 0$ | $p < 0$ | Change algebraic sign | F3 |
| $v_2 < 0$ | $v_1 < v_2$ | Change of index | F2 |
| $v_2 < 0$ | $v_1 < v_2$ | Change algebraic sign | F3 |

Tab. A.2: Input-parameter depending on starting conditions for $v_1 < 0$

The values obtained in this way shall be named $V_1, V_2, M_1, M_2$ and can be used for further calculations. Following this procedure all possible combinations of appearing masses and velocities can be addressed. After finishing the calculations, the results for $V_3, V_4, M_1, M_2$ must be converted into the needed values using a reverse scheme reapplying the code defined during the Input-process.

It shall be mentioned that according to the transformation described above the results for $V_4$ always show positive values and only after a transformation, which may be necessary according to the preconditions, shifting to a negative result is possible. This is important because the values are calculated according to Eq. (A.04) and, thus, concerning the square root with

$$V_4 = +\sqrt{1 - \frac{1}{\gamma_4^2}} \tag{A.11}$$

only the positive result must be used.

The values determined in this way allow the initial values for $(V_{3+})_0$ and $(V_{3-})_0$ required for the calculations to be established in a straightforward manner. It can be easily shown that for all cases the conditions $(V_{3+})_0 = V_1$ as well as $(V_{3-})_0 = -V_1$ fulfill the requirements and always lead to usable results.

For the further calculations here (as in the other cases) the method of bisection was chosen. For the definition of the parameter for the termination of the calculations the possibility is given here that the values of $(v_3)_{K-1}$ and $(v_3)_K$ or $(v_4)_{K-1}$and $(v_4)_K$ are compared with each other and with equality the calculation process is terminated. However, if one of these queries is chosen, the situation may arise that - if the values are close to zero - the other has not yet been calculated exactly. To avoid this problem the fact was used that from a number of approx. 60 iteration steps with the available accuracy of 15 digits the possible limit accuracy is reached (see discussion in appendix D). To avoid any problem a fixed number of 80 iteration steps to stop the process was defined.

All necessary process steps are represented with the help of program flow charts, namely in Fig. A.1 for the general flow and in Fig. A.2 for the described subprograms. Subsequently, a VBA program code created for the calculations (Fig. A.3) as well as the assignment of the formula characters used (Tab. A.3) is reproduced.

In the following, a simple spreadsheet calculation program is shown in chapter A.2, which can be used to perform the same calculations. However, the already mentioned boundary conditions $v_1 > v_2$, $v_1 > 0$ and $p > 0$ must be observed or, if necessary, manually adjusted.

As already mentioned in chapter 6.3, the results from VBA program and spreadsheet are not completely identical, although they follow exactly the same calculation scheme. While this does not matter for large values, deviations are noticeable for very small values of $v_1$. These are caused by rounding errors during the calculation, which have different effects on the different procedures. However, this does not affect the general statement that in elastic relativistic impact no effects can occur which allow measurements to identify a system at absolute rest.

If cases with very low velocities shall be investigated numerically in more detail, computer systems with higher accuracy must be used to get reliable results.

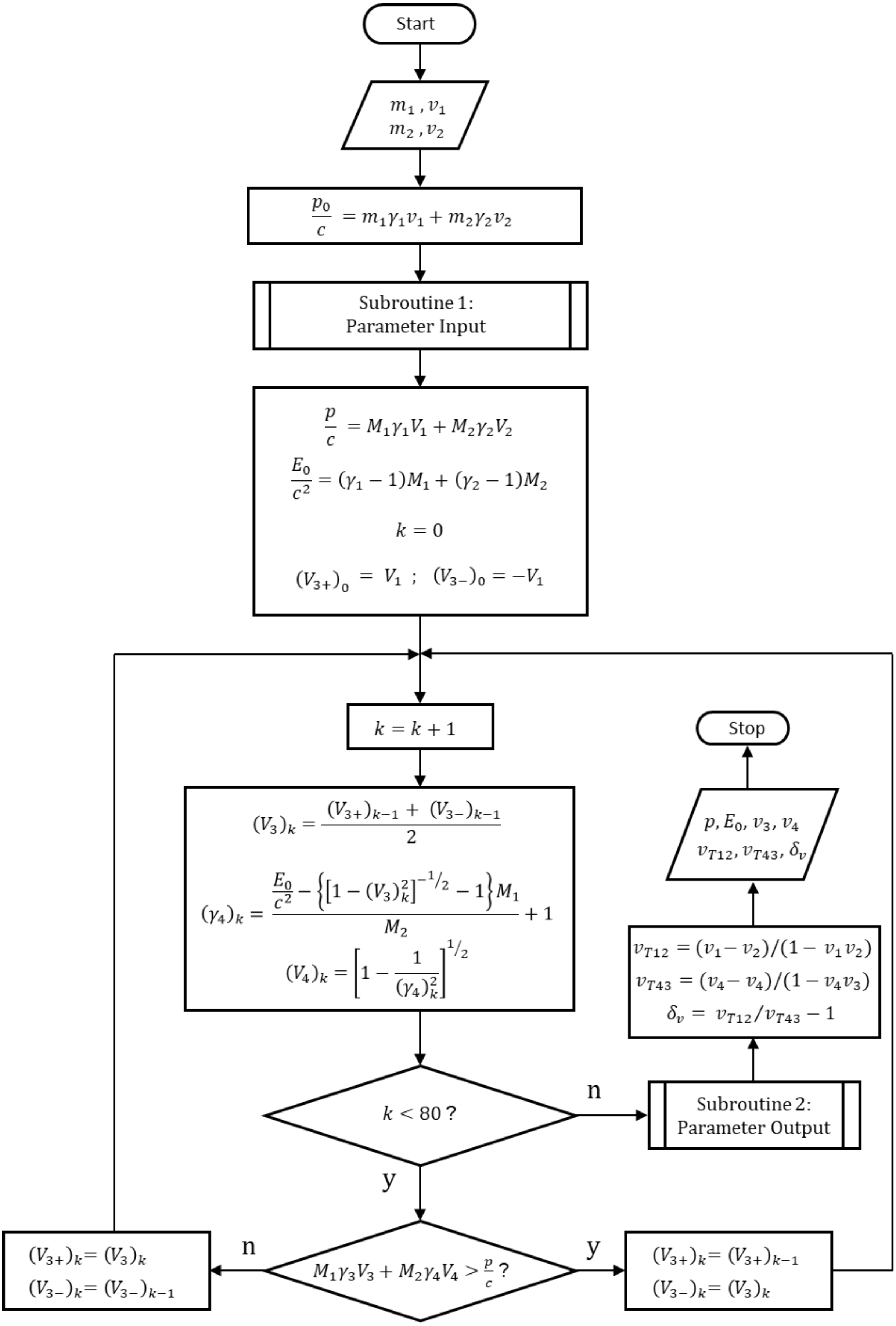



Fig. A.1: Flowchart of the calculation process

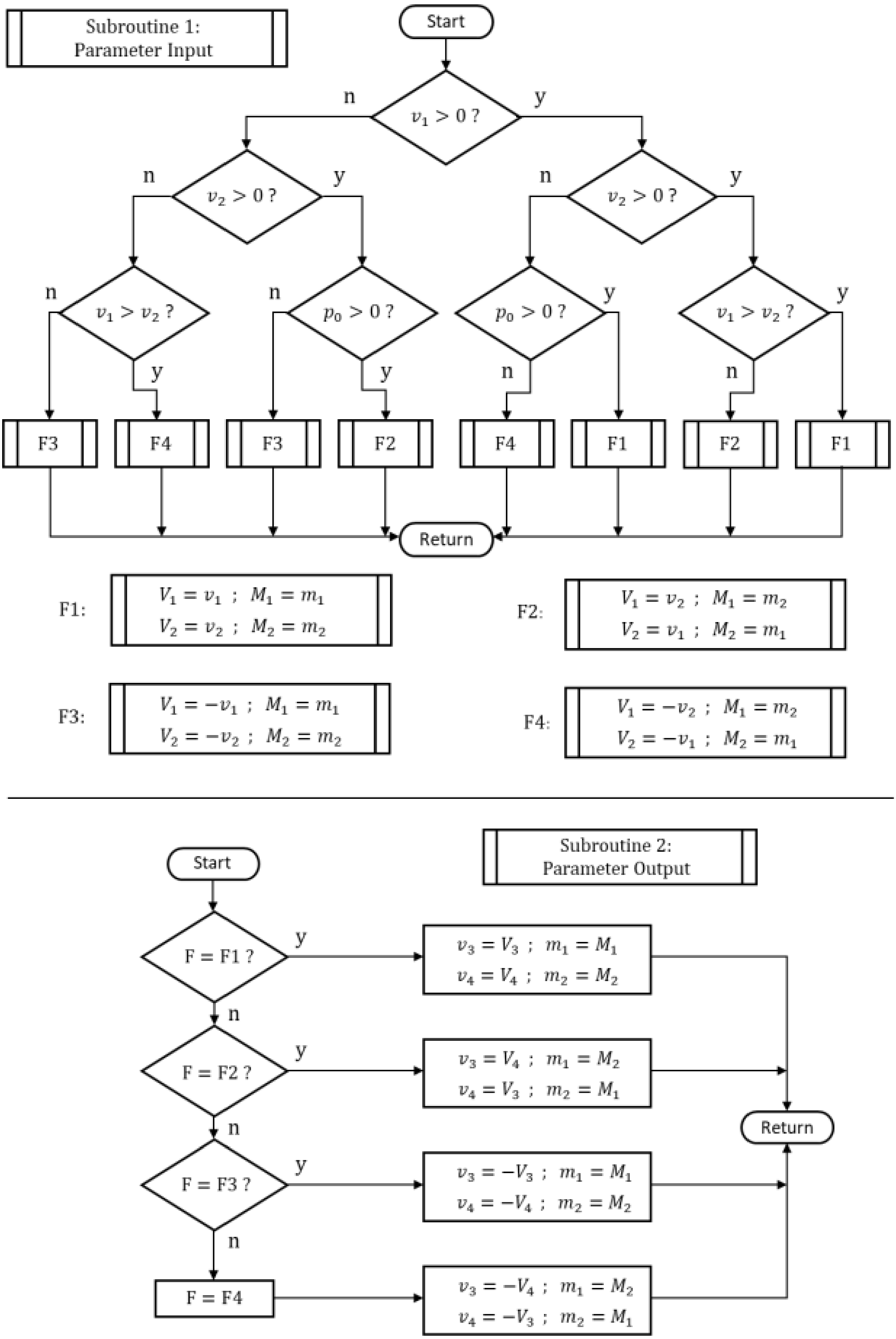



Fig. A.2: Subroutines for process in Fig. A.1

| Symbol | VBA-Code | Symbol | VBA-Code | Symbol | VBA-Code |
|---|---|---|---|---|---|
| $v_1$ | v1 | $v_2$ | v2 | $vc_1$ | vc1 |
| $vc_1$ | vc2 | $v_3$ | v3 | $vc_3$ | vc3 |
| $vc_{3-}$ | vc3m | $vc_{3+}$ | vc3p | $v_4$ | v4 |
| $vc_4$ | vc4 | $m_1$ | m1 | $mc_1$ | mc1 |
| $v_T(v_1, v_2)$ | vt12 | $v_T(v_4, v_3)$ | vt43 | $\delta_v$ | Dv |
| $m_2$ | m2 | $mc_2$ | mc2 | $p_0$ | p0 |
| $p$ | pc0 | $E_0$ | E0 | $\gamma_4$ | Ga4 |

Tab. A.3: Formula symbols and referring VBA-Codes

```
Sub A()
Dim v1, v2, vc1, vc2, v3, vc3, vc3m, vc3p, v4, vc4, vt12, vt43, Dv, m1,
mc1, m2, mc2, p0, pc0, E0, Ga4, Gav, K As Double
Dim F, F1, F2, F3, F4 As String
'Input
    v1 = 0.3
    v2 = -0.1
    m1 = 1
    m2 = 3
'Start calculation
    If v1 = v2 Then
        Debug.Print "Calculation not possible: v1 = v2"
        GoTo Out1:
        End If
    p0 = v1 * m1 / (1 - v1 ^ 2) ^ 0.5 + v2 * m2 / (1 - v2 ^ 2) ^ 0.5
'Subroutine 1
    If v1 > 0 Then
        GoTo P1:
    End If
    If v2 > 0 Then
        GoTo P2:
    End If
    If v1 > v2 Then
        F = "F4"
        Else
        F = "F3"
    End If
    GoTo Def1:
P2:
    If p0 > 0 Then
        F = "F2"
        Else
        F = "F3"
    End If
    GoTo Def1:
P1:
    If v2 > 0 Then
        GoTo P3:
```

```
    End If
    If p0 > 0 Then
        F = "F1"
        Else
        F = "F4"
    End If
    GoTo Def1:
P3:
    If v1 > v2 Then
        F = "F1"
        Else
        F = "F2"
    End If
    GoTo Def1:
Def1:
    If F = "F1" Then
        vc1 = v1
        vc2 = v2
        mc1 = m1
        mc2 = m2
    End If
    If F = "F2" Then
        vc1 = v2
        vc2 = v1
        mc1 = m2
        mc2 = m1
     End If
     If F = "F3" Then
        vc1 = -v1
        vc2 = -v2
        mc1 = m1
        mc2 = m2
    End If
    If F = "F4" Then
        vc1 = -v2
        vc2 = -v1
        mc1 = m2
        mc2 = m1
    End If
'End Subroutine 1
'Calculation
   pc0 = vc1 * mc1 / (1 - vc1 ^ 2) ^ 0.5 + vc2 * mc2 / (1 - vc2 ^ 2) ^ 0.5
   E0 = mc1 * ((1 - vc1 ^ 2) ^ -0.5 - 1) + mc2 * ((1 - vc2 ^ 2) ^ -0.5 - 1)
    vc3m = -vc1            'Values for start
    vc3p = vc1
    K = 0
Do
        K = K + 1
        vc3 = (vc3m + vc3p) / 2
        Ga4 = (E0 - ((1 - vc3 ^ 2) ^ -0.5 - 1) * mc1) / mc2 + 1
        vc4 = (1 - 1 / Ga4 ^ 2) ^ 0.5
        If (vc3 * mc1 / (1 - vc3 ^ 2) ^ 0.5 + vc4 * mc2 / (1 - vc4 ^ 2) ^
0.5) > pc0 Then
        vc3p = vc3
        Else
        vc3m = vc3
        End If
Loop Until K = 80
'Subroutine 2
    If F = "F1" Then
        v3 = vc3
        v4 = vc4
        m1 = mc1
```

```
        m2 = mc2
    End If
    If F = "F2" Then
        v3 = vc4
        v4 = vc3
        m1 = mc2
        m2 = mc1
     End If
     If F = "F3" Then
        v3 = -vc3
        v4 = -vc4
        m1 = mc1
        m2 = mc2
    End If
    If F = "F4" Then
        v3 = -vc4
        v4 = -vc3
        m1 = mc2
        m2 = mc1
    End If
'End Subroutine 2
    vt12 = (v1 - v2) / (1 - v1 * v2)
    vt43 = (v4 - v3) / (1 - v4 * v3)
    Dv = (vt12 / vt43) - 1
'Presentation of results: Calculated values in view of observer at rest
    Debug.Print "F =", F
    Debug.Print "v3 =", v3
    Debug.Print "v4 =", v4
    Debug.Print "vt12 =", vt12
    Debug.Print "vt43 =", vt43
    Debug.Print "Dv =", Dv
Out1:
End Sub
```

Fig. A.3: VBA Program-Code for the calculation process presented in Fig. A.1 and A.2

## A.2 Spreadsheet calculation

The following equations are used for calculation:

$$p_0 = \frac{p}{c} = \frac{m_1\gamma_1 v_1 + m_2\gamma_2 v_2}{c} \qquad \frac{E_0}{c^2} = (\gamma_1 - 1)m_1 + (\gamma_2 - 1)m_2$$

$$\frac{(v_3)_k}{c} = \frac{(v_{3+})_{k-1} + (v_{3-})_{k-1}}{2 \cdot c} \qquad \gamma_4 = \frac{\frac{E_0}{c^2} - (\gamma_3 - 1)m_1}{m_2} + 1$$

$$\frac{v_4}{c} = \sqrt{1 - \frac{1}{\gamma_4^2}}$$

(Remark: Because of appropriate selection of basic conditions, only positive results of the square root must be considered.

Determination: $f(v_3)_1 > p\colon \Rightarrow \frac{(v_{3+})_k}{c} = \frac{(v_3)_k}{c}$ and $\frac{(v_{3-})_k}{c} = \frac{(v_{3-})_{k-1}}{c}$

Determination: $f(v_3)_1 < p : \Rightarrow \frac{(v_{3+})_k}{c} = \frac{(v_{3+})_{k-1}}{c}$ and $\frac{(v_{3-})_k}{c} = \frac{(v_3)_k}{c}$

Useful starting values: For $\frac{(v_{3-})_0}{c} = -\frac{v_1}{c}$ and for $\frac{(v_{3+})_0}{c} = \frac{v_1}{c}$

Values in the fields for results (blue color):

$$\frac{v_3}{c} = \frac{(v_3)_{k=80}}{c} \qquad \frac{v_4}{c} = \frac{(v_4)_{k=80}}{c}$$

$$\frac{v_T(v_1,\ v_2)}{c} = \frac{v_1 - v_2}{1 - \frac{v_1 v_2}{c^2}} \qquad \frac{v_T(v_4,\ v_3)}{c} = \frac{v_4 - v_3}{1 - \frac{v_4 v_3}{c^2}}$$

$$\delta_v = \frac{v_T(v_1,\ v_2)}{v_T(v_4,\ v_3)} - 1$$

As examples for $m_1 = 2;\ m_2 = 1$ the cases $v_1 = 0{,}5c$ and $v_2 = -0{,}5c$ as well as $v_1 = 0{,}00001c$ and $v_2 = 0$ are shown.

### Codes for calculation:

| Coordinate | | Code |
|---|---|---|
| B3 | = | B1*D1*(1-B1^2)^-0,5+B2*D2*(1-B2^2)^-0,5 |
| D3 | = | D1*((1-B1^2)^-0,5-1)+D2*((1-B2^2)^-0,5-1) |
| B8 | = | (E7+F7)/2 |
| C8 | = | (D$3-((1-B8^2)^-0,5-1)*D$1)/D$2+1 |
| D8 | = | (1-1/C8^2)^0,5 |
| E8 | = | IF((B8*D$1*(1-B8^2)^-0,5+D8*D$2*(1-D8^2)^-0,5)>B$3;E7;B8) |
| F8 | = | IF((B8*D$1*(1-B8^2)^-0,5+D8*D$2*(1-D8^2)^-0,5)>B$3;B8;F7) |
| G9 | = | IF(B9=B8;"x";"") |
| H9 | = | IF(D9=D8;"x";"") |
| F1 | = | B87 |
| F2 | = | D87 |
| F3 | = | (B1-B2)/(1-B1*B2) |
| F4 | = | (F2-F1)/(1-F2*F1) |
| F5 | = | F3/F4-1 |

Codes B8 to G8 to be copied as far as B87 to G87.

The status queries in columns G and H are used to determine whether the values for $v_3$ and $v_4$ still differ. For $v_3$ there are only slight deviations (Fig. A.4: step 51, Fig. A.5: step 52), $v_4$ shows strongly different behavior depending on the initial values; in these examples there are no further changes from step 49 (with interruptions), resp. already from step 19.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | $v_1$/c= | 0,5 | $m_1$= | 2 | $v_3$/c= | -0,209677419354839 | | |
| 2 | $v_2$/c= | -0,5 | $m_2$= | 1 | $v_4$/c= | 0,709302325581396 | | |
| 3 | $p_0$/c= | 0,5773502692 | $E_0$/c²= | 0,4641016151 | $v_T$/c ($v_1$, $v_2$)= | 0,800000000000000 | | |
| 4 | | | | | $v_T$/c ($v_4$, $v_3$)= | 0,800000000000000 | | |
| 5 | | | | | $\delta_V$= | 0,0E+00 | | |
| 6 | k | $v_3$/c | $\gamma_4$ | $v_4$/c | $v_{3-}$/c | $v_{3+}$/c | St. | |
| 7 | 0 | | | | -0,5 | 0,5 | 3 | 4 |
| 8 | 1 | 0,000000000000000 | 1,46410161513776 | 0,730406495763757 | -0,500000000000000 | 0,000000000000000 | | |
| 9 | 2 | -0,250000000000000 | 1,39851049716047 | 0,699076919847366 | -0,250000000000000 | 0,000000000000000 | | |
| 10 | 3 | -0,125000000000000 | 1,44829109242188 | 0,723362051517259 | -0,250000000000000 | -0,125000000000000 | | |
| 11 | 4 | -0,187500000000000 | 1,42799037361527 | 0,713863539464603 | -0,250000000000000 | -0,187500000000000 | | |
| 12 | 5 | -0,218750000000000 | 1,41446124643264 | 0,707230579852351 | -0,218750000000000 | -0,187500000000000 | | |
| 13 | 6 | -0,203125000000000 | 1,42151952770374 | 0,710722406262171 | -0,218750000000000 | -0,203125000000000 | | |
| 14 | 7 | -0,210937500000000 | 1,41806486850481 | 0,709022002845605 | -0,210937500000000 | -0,203125000000000 | | |
| 15 | 8 | -0,207031250000000 | 1,41981068269019 | 0,709883363160283 | -0,210937500000000 | -0,207031250000000 | | |
| 16 | 9 | -0,208984375000000 | 1,41894241344356 | 0,709455499378042 | -0,210937500000000 | -0,208984375000000 | | |
| 17 | 10 | -0,209960937500000 | 1,41850480254676 | 0,709239458600344 | -0,209960937500000 | -0,208984375000000 | | |
| 18 | 11 | -0,209472656250000 | 1,41872389812336 | 0,709347655434525 | -0,209960937500000 | -0,209472656250000 | | |
| 19 | 12 | -0,209716796875000 | 1,41861442290016 | 0,709293601181963 | -0,209716796875000 | -0,209472656250000 | | |
| 20 | 13 | -0,209594726562500 | 1,41866917864890 | 0,709320639342717 | -0,209716796875000 | -0,209594726562500 | | |
| 21 | 14 | -0,209655761718750 | 1,41864180530934 | 0,709307123021790 | -0,209716796875000 | -0,209655761718750 | | |
| 22 | 15 | -0,209686279296875 | 1,41862811523852 | 0,709300362791843 | -0,209686279296875 | -0,209655761718750 | | |
| 23 | 16 | -0,209671020507812 | 1,41863496055736 | 0,709303743079295 | -0,209686279296875 | -0,209671020507812 | | |
| 24 | 17 | -0,209678649902344 | 1,41863153796880 | 0,709302052978691 | -0,209678649902344 | -0,209671020507812 | | |
| 25 | 18 | -0,209674835205078 | 1,41863324928079 | 0,709302898039773 | -0,209678649902344 | -0,209674835205078 | | |
| 26 | 19 | -0,209676742553711 | 1,41863239362922 | 0,709302475511927 | -0,209678649902344 | -0,209676742553711 | | |
| 27 | 20 | -0,209677696228027 | 1,41863196580012 | 0,709302264245983 | -0,209677696228027 | -0,209676742553711 | | |
| 47 | 40 | -0,209677419355103 | 1,41863209000868 | 0,709302325581337 | -0,209677419355103 | -0,209677419354193 | | |
| 48 | 41 | -0,209677419354648 | 1,41863209000888 | 0,709302325581438 | -0,209677419355103 | -0,209677419354648 | | |
| 49 | 42 | -0,209677419354875 | 1,41863209000878 | 0,709302325581387 | -0,209677419354875 | -0,209677419354648 | | |
| 50 | 43 | -0,209677419354762 | 1,41863209000883 | 0,709302325581413 | -0,209677419354875 | -0,209677419354762 | | |
| 51 | 44 | -0,209677419354819 | 1,41863209000880 | 0,709302325581400 | -0,209677419354875 | -0,209677419354819 | | |
| 52 | 45 | -0,209677419354847 | 1,41863209000879 | 0,709302325581394 | -0,209677419354847 | -0,209677419354819 | | |
| 53 | 46 | -0,209677419354833 | 1,41863209000880 | 0,709302325581397 | -0,209677419354847 | -0,209677419354833 | | |
| 54 | 47 | -0,209677419354840 | 1,41863209000879 | 0,709302325581395 | -0,209677419354840 | -0,209677419354833 | | |
| 55 | 48 | -0,209677419354836 | 1,41863209000880 | 0,709302325581396 | -0,209677419354840 | -0,209677419354836 | | |
| 56 | 49 | -0,209677419354838 | 1,41863209000880 | 0,709302325581396 | -0,209677419354840 | -0,209677419354838 | | X |
| 57 | 50 | -0,209677419354839 | 1,41863209000880 | 0,709302325581396 | -0,209677419354840 | -0,209677419354839 | | X |
| 58 | 51 | -0,209677419354839 | 1,41863209000879 | 0,709302325581395 | -0,209677419354839 | -0,209677419354839 | X | |
| 59 | 52 | -0,209677419354839 | 1,41863209000879 | 0,709302325581395 | -0,209677419354839 | -0,209677419354839 | X | X |
| 60 | 53 | -0,209677419354839 | 1,41863209000879 | 0,709302325581395 | -0,209677419354839 | -0,209677419354839 | X | X |
| 61 | 54 | -0,209677419354839 | 1,41863209000879 | 0,709302325581395 | -0,209677419354839 | -0,209677419354839 | X | X |
| 62 | 55 | -0,209677419354839 | 1,41863209000880 | 0,709302325581396 | -0,209677419354839 | -0,209677419354839 | X | |

Fig. A.4: Results when using the spreadsheet calculation. $v_1 = 0{,}5c$, $v_2 = -0{,}5c$
Green fields: Input values. Steps between 20 and 40 hidden

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | $v_1$/c= | 0,00001 | $m_1$= | 2 | $v_3$/c= | 3,33333305742102E-06 | | |
| 2 | $v_2$/c= | 0 | $m_2$= | 1 | $v_4$/c= | 1,33333338849358E-05 | | |
| 3 | $p_0$/c= | 2,00000000E-05 | $E_0$/c²= | 1,00000008E-10 | $v_T$/c ($v_1$, $v_2$)= | 0,000010000000000 | | |
| 4 | | | | | $v_T$/c ($v_4$, $v_3$)= | 0,000010000000828 | | |
| 5 | | | | | $\delta_V$= | -8,3E-08 | | |
| 6 | k | $v_3$/c | $\gamma_4$ | $v_4$/c | $v_{3-}$/c | $v_{3+}$/c | St. | |
| 7 | 0 | | | | -0,5 | 0,5 | 3 | 4 |
| 8 | 1 | 0,00000000000000E+00 | 1,00000000010000 | 1,41421362087937E-05 | 0,00000000000000E+00 | 1,00000000000000E-05 | | |
| 9 | 2 | 5,00000000000000E-06 | 1,00000000007500 | 1,22474492205951E-05 | 0,00000000000000E+00 | 5,00000000000000E-06 | | |
| 10 | 3 | 2,50000000000000E-06 | 1,00000000009375 | 1,36930563961891E-05 | 2,50000000000000E-06 | 5,00000000000000E-06 | | |
| 11 | 4 | 3,75000000000000E-06 | 1,00000000008594 | 1,31101090273823E-05 | 2,50000000000000E-06 | 3,75000000000000E-06 | | |
| 12 | 5 | 3,12500000000000E-06 | 1,00000000009023 | 1,34338740912243E-05 | 3,12500000000000E-06 | 3,75000000000000E-06 | | |
| 13 | 6 | 3,43750000000000E-06 | 1,00000000008818 | 1,32803375392606E-05 | 3,12500000000000E-06 | 3,43750000000000E-06 | | |
| 14 | 7 | 3,28125000000000E-06 | 1,00000000008923 | 1,33591548737510E-05 | 3,28125000000000E-06 | 3,43750000000000E-06 | | |
| 15 | 8 | 3,35937500000000E-06 | 1,00000000008871 | 1,33202712640041E-05 | 3,28125000000000E-06 | 3,35937500000000E-06 | | |
| 16 | 9 | 3,32031250000000E-06 | 1,00000000008898 | 1,33398271082881E-05 | 3,32031250000000E-06 | 3,35937500000000E-06 | | |
| 17 | 10 | 3,33984375000000E-06 | 1,00000000008885 | 1,33300694297534E-05 | 3,32031250000000E-06 | 3,33984375000000E-06 | | |
| 18 | 11 | 3,33007812500000E-06 | 1,00000000008891 | 1,33349658128449E-05 | 3,33007812500000E-06 | 3,33984375000000E-06 | | |
| 19 | 12 | 3,33496093750000E-06 | 1,00000000008888 | 1,33325345004281E-05 | 3,33007812500000E-06 | 3,33496093750000E-06 | | |
| 20 | 13 | 3,33251953125000E-06 | 1,00000000008889 | 1,33337335592179E-05 | 3,33251953125000E-06 | 3,33496093750000E-06 | | |
| 21 | 14 | 3,33374023437500E-06 | 1,00000000008889 | 1,33331340433020E-05 | 3,33251953125000E-06 | 3,33374023437500E-06 | | |
| 22 | 15 | 3,33312988281250E-06 | 1,00000000008889 | 1,33334338046295E-05 | 3,33312988281250E-06 | 3,33374023437500E-06 | | |
| 23 | 16 | 3,33343505859375E-06 | 1,00000000008889 | 1,33332672713907E-05 | 3,33312988281250E-06 | 3,33343505859375E-06 | | |
| 24 | 17 | 3,33328247070313E-06 | 1,00000000008889 | 1,33333671915836E-05 | 3,33328247070313E-06 | 3,33343505859375E-06 | | |
| 25 | 18 | 3,33335876464844E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33328247070313E-06 | 3,33335876464844E-06 | | |
| 26 | 19 | 3,33332061767578E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33332061767578E-06 | 3,33335876464844E-06 | | X |
| 27 | 20 | 3,33333969116211E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33332061767578E-06 | 3,33333969116211E-06 | | X |
| | | | | | | | | |
| 47 | 40 | 3,33333305743509E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305741690E-06 | 3,33333305743509E-06 | | X |
| 48 | 41 | 3,33333305742599E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305741690E-06 | 3,33333305742599E-06 | | X |
| 49 | 42 | 3,33333305742144E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305741690E-06 | 3,33333305742144E-06 | | X |
| 50 | 43 | 3,33333305741917E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305741917E-06 | 3,33333305742144E-06 | | X |
| 51 | 44 | 3,33333305742031E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742031E-06 | 3,33333305742144E-06 | | X |
| 52 | 45 | 3,33333305742087E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742087E-06 | 3,33333305742144E-06 | | X |
| 53 | 46 | 3,33333305742116E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742087E-06 | 3,33333305742116E-06 | | X |
| 54 | 47 | 3,33333305742102E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742087E-06 | 3,33333305742102E-06 | | X |
| 55 | 48 | 3,33333305742095E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742095E-06 | 3,33333305742102E-06 | | X |
| 56 | 49 | 3,33333305742098E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742098E-06 | 3,33333305742102E-06 | | X |
| 57 | 50 | 3,33333305742100E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742100E-06 | 3,33333305742102E-06 | | X |
| 58 | 51 | 3,33333305742101E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742101E-06 | 3,33333305742102E-06 | | X |
| 59 | 52 | 3,33333305742101E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742101E-06 | 3,33333305742102E-06 | X | X |
| 60 | 53 | 3,33333305742101E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742101E-06 | 3,33333305742102E-06 | X | X |
| 61 | 54 | 3,33333305742102E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742102E-06 | 3,33333305742102E-06 | | X |
| 62 | 55 | 3,33333305742102E-06 | 1,00000000008889 | 1,33333338849358E-05 | 3,33333305742102E-06 | 3,33333305742102E-06 | X | X |

Fig. A.5: Representation as in Fig. A.4. $v_1 = 0{,}00001c$, $v_2 = 0$
Values for $v_4$ already unchanged as of iteration step 19

# Annex B: Exchange of signals during and after acceleration

In this annex it is shown that the reception of signals from an accelerated system by an observer at rest at the beginning of the acceleration phase and by an observer in uniform motion leads to the same results. The analytic relations valid here were already derived in chapter 6.4.1 in the equations (6.60) to (6.80). However, there is also a numerical method for the solution of this problem, which will be presented in the following. There are advantages and disadvantages between the analytical and the numerical method, which become visible in a comparison, also with comparable results of the numerical method from Annex C.

## B.1 Numerical solution

The following general correlation between velocity and acceleration within the moving system S apply

$$\Delta v = a(v) \cdot \Delta t(v) = a_S \cdot \Delta t_S \tag{B.01}$$

The values of $a_S$ and $\Delta t_S$ are constant by definition. A numerical solution requires the multiple calculation of different steps; for this, first the relativistic velocity addition is used, then the determination of the increase of time and distance follows for each case.

1st step:

$$v_1 = \frac{v_0 + \Delta v}{1 + \frac{v_0 \Delta v}{c^2}} = \frac{v_0 + a_S \Delta t_S}{1 + \frac{v_0 a_S \Delta t_S}{c^2}} \tag{B.02}$$

2nd step:

$$\Delta t_1 = \Delta t_S \cdot \frac{\gamma(v_1) + \gamma(v_0)}{2} \tag{B.03}$$

3rd step:

$$\Delta x_1 = \Delta t_1 v_0 + \frac{v_1 + v_0}{2} \Delta t_1 \tag{B.04}$$

It should be noted that the functions for $\gamma(v)$ and $v(\Delta v)$ are not linear and thus the formation of a mean value is only an approximation, and the error must be compensated by choosing suitably small intervals for $\Delta t_S$. These steps are now to be repeated $N$ times and the single results added. In general, it applies

$$t_N = \Delta t_S \sum_{K=1}^{N} \frac{\gamma(v_K) + \gamma(v_{K-1})}{2} \tag{B.05}$$

$$x_N = \sum_{K=1}^{N} \Delta t_K \frac{v_K + v_{K-1}}{2} \tag{B.06}$$

with

$$v_{K+1} = \frac{v_K + \Delta v}{1 + \frac{v_K \Delta v}{c^2}} - v_0 = \frac{v_K + a_S \Delta t_S}{1 + \frac{v_K a_S \Delta t_S}{c^2}} - v_0 \tag{B.07}$$

At any arbitrary time $t_K$ a signal from the accelerated system S shall be transmitted back to observer A. In case of $v_0 \neq 0$ observer A is moving during signal propagation in view of B either in direction to S or in the opposite way. Because $a_S$ and $v_0$ can be both positive and/or negative, for the calculation different regulations are necessary (see also the comprehensive presentations in chapter 2.1). If first the situation is discussed that $a_S$ and $v_0$ are both positive, then observer B will find the situation according to type "b" referring to Fig. 2.2 as

$$\Delta t = \Delta t_S \left(1 + \frac{v_0}{c}\right) \tag{B.08}$$

When $a_S$ and $v_0$ show in different directions, however, the algebraic sign is changing in equation Eq. (B.08) according to situation of type "d" from Fig. 2.2.

In summary, the following combinations arise for the time between two pulses $t_{K,R}$ perceived by observer B due to the increasing distance, into which any positive or negative values for the velocity $v_0$ can be inserted:

$a_S > 0$:
$$t_{K,R} = \frac{x_K - v_0 t_K}{c\left(1 + \frac{v_0}{c}\right)} \tag{B.09}$$

$a_S < 0$:
$$t_{K,R} = \frac{|(x_K - v_0 t_K)|}{c\left(1 - \frac{v_0}{c}\right)} \tag{B.10}$$

For $v_0 = 0$ both equations for any value of $a_S$ simplify to

$$t_{K,R} = \frac{|x_K|}{c} \tag{B.11}$$

The total time from the start of the acceleration to the transmission and subsequent reception of the signal is then in all cases

$$t_{K,T} = t_K + t_{K,R} \tag{B.12}$$

Further, the signals received by observer A must be adjusted in view of B according to equation

$$t_{K,T}(v_0) = \frac{t_K + t_{K,R}}{\gamma(v_0)} \tag{B.13}$$

to cover the effect, that for A in view of B the time is running slower by the factor $\gamma(v_0)$ according to the Lorentz-equations.

With the relations presented here it is possible to determine the values for the reception times of observers moving relative to each other. For this purpose, first the time intervals are calculated, with which the accelerated system S transmits the signals. While these are subjectively $\Delta t_S$ within the system S, from a non-accelerated observer the values for the time interval can be determined using the equations presented before. The calculation scheme can also be used to define the distance of S when transmitting the signals. Thus, the total times for the arrival of the signals can be determined for any arbitrarily moving observer.

Fig. B.1 shows the program flow chart for the numerical calculation of $v_N$, $t_N$, $t_T$ and $x_N$ according to the equations mentioned (the values for $t_K$ and $x_K$ are calculated throughout; since only the last results are considered, these correspond to $t_N$ and $x_N$). In addition, the acceleration $a_N$ is determined for an observer moving relative to the system S; the value deviates from the acceleration $a_S$, which can be measured subjectively in S. As already shown in chapter 6.4.1, the subjectively adjusted acceleration in S and the acceleration measured by an external observer moving relative to it with the velocity $v$ must differ by the factor $\gamma^3(v)$. Therefore, to verify this theoretically expected effect, the value $\gamma^3 a_N$ was also calculated from the data. The results show a very good agreement between $a_S$ and $\gamma^3 a_N$.

The used VBA program (Visual Basic) code is shown in Fig. B.1. In Tab. B.1 the formula symbols taken for the calculation program are assigned to those used in the text. The program was designed in such a way that the initial velocity $v_0$, as well as the subjectively valid acceleration $a_S$ and total duration of the experiment $t_S$ are to be specified. In addition, the number of intended iteration steps N can be freely selected, which provides an important influencing variable. With the VBA program, values up to $\mathrm{N} = 10^7$ were investigated. These calculations only make sense with such programs, since with a conventional spreadsheet each iteration step requires separate program fields, and this would lead to enormous file sizes.

Tab. B.2 shows in the parts a) to c) the results from calculations with the boundary conditions $a_S = 10\,\mathrm{m/s^2}$ and $t_S = 1000\mathrm{s}$. Values of $v_0 = 0$, $v_0 = 369\,\mathrm{km/s}$ and $v_0 = 0.5c$ were chosen as initial velocities. For all results, $\delta$-values were calculated according to the scheme

$$\delta v_T = \frac{v_T(\mathrm{K})}{v_T(\mathrm{K}-1)} - 1 \qquad \text{(B.14)}$$

and compared, where K in this case represents a potency of 10 according to the specifications in the table.

The calculations performed show that within a range of about $10^2$ to $10^4$ the differences between the results reach a minimum. This suggests that these zones have the largest confidence range. This is primarily dependent on the chosen calculation system; Microsoft Excel© was used as the method here, which has an accuracy of 15 digits. If computer systems with higher accuracy would be used, other results are to be expected. However, the overall quality of the calculations can only be verified in the comparison between the analytical and numerical methods, which will be carried out subsequently.

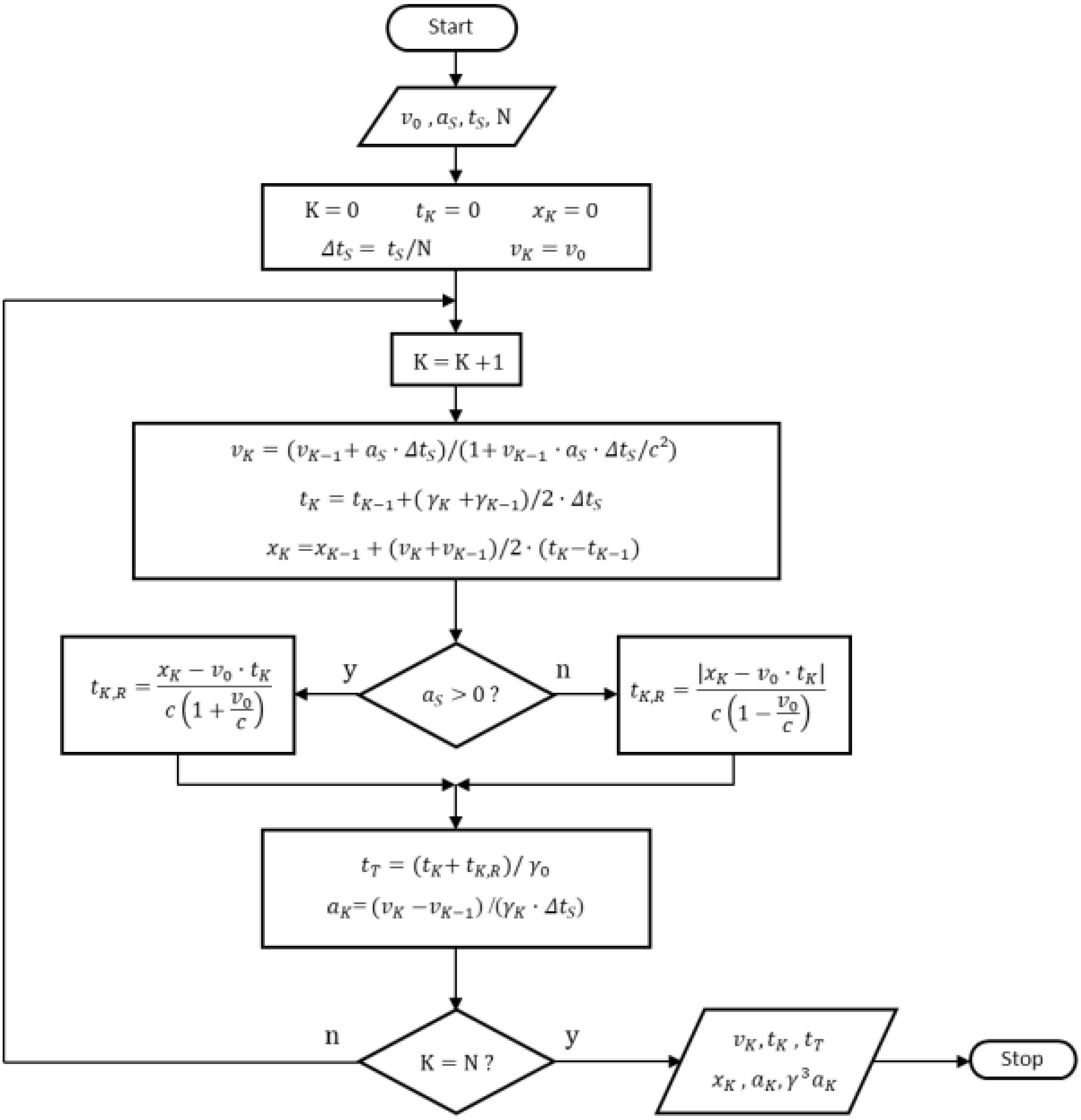



Fig. B.1: Flowchart of the calculation process

| Symbol | VBA-Code | Symbol | VBA-Code | Symbol | VBA-Code |
|---|---|---|---|---|---|
| $v_0$ | v0 | $a_S$ | a0 | $t_S$ | tS |
| $\Delta t_S$ | dtS | $t_K$ | tK | $t_{K-1}$ | tKm1 |
| $x_K$ | xK | $v_K$ | vK | $v_{K-1}$ | vKm1 |
| $\gamma_K$ | GaK | $\gamma_{K-1}$ | GaKm1 | $t_{K,R}$ | tKR |
| $t_T$ | tT | $a_K$ | aK | $\gamma^3 a_K$ | aKGa3 |

Tab. B.1: Formula symbols and referring VBA-Codes

```
Sub B()
Dim c, v0, a0, aK, tS, dtS, tK, tKm1, xK, vK, vKm1, GaK, GaKm1 As Double
Dim aKGa3, tKR, tT, vT, K, N As Double
'Input
    v0 = 299792.458 / 2 'Initial velocity in km/s
    a0 = 10             'Acceleration in m/s²
    N = 1000            'Number of iteration steps
    tS = 1000           'Time for S between transmission of signals in s
'Start Calculation
    c = 299792.458       'Speed of light in km/s
    a0 = a0 / 1000       'Acceleration in km/s²
    dtS = tS / N
    tK = 0
    xK = 0
    vK = v0
    For K = 1 To N
        vKm1 = vK
        tKm1 = tK
        GaKm1 = 1 / (1 - (vKm1 / c) ^ 2) ^ 0.5
        vK = (vK + a0 * dtS) / (1 + vK * a0 * dtS / c ^ 2)
        GaK = 1 / (1 - (vK / c) ^ 2) ^ 0.5
        tK = tK + (GaKm1 + GaK) / 2 * dtS
        xK = xK + (vK + vKm1) / 2 * (tK - tKm1)
            If a0 > 0 Then
            tKR = (xK - tK * v0) / c / (1 + v0 / c)
            Else
            tKR = Abs((xK - tK * v0) / c / (1 - v0 / c))
            End If
        tT = (tK + tKR) * (1 - (v0 / c) ^ 2) ^ 0.5
    aK = (vK - vKm1) / (GaK * dtS) * 1000
    aKGa3 = aK * GaK ^ 3
    vT = vK - v0
    Next K
'Results in view of an observer moving with v0 at beginning of trial
    Debug.Print "vT", "vK", "tN", "xN", "aN", "aNGa3"
    Debug.Print vT, vK, tT, xK, aK, aKGa3
End Sub
```

Fig. B.2: VBA Program-Code for the calculation process presented in Fig. B1

Basically, it can be stated that all $\delta$-values are very low at $v_0 = 0$ and then increase slightly at higher numbers. In particular, the values for $t_T$, which would be well suited for experimental verification, hardly differ between the individual values of $v_0$ within a range with constant acceleration $a_S$. Also, between the different acceleration values the differences are so small that a systematic influence cannot be assumed, but the effects are due to influences of the numerical calculation.

The deviations between the results for the selected iteration steps between 1 and $10^7$ show that there are no systematic deviations. In the range of $10^3$ the results show a high stability and the smallest differences; therefore, they are particularly suitable for comparative considerations.

The additional value of $v_0 = 369$ km/s was chosen because it corresponds to the velocity of the sun with respect to the cosmic background radiation and therefore, if an effect would show up in the calculations, it could be an appropriate basis for further considerations (see also chapter 1.7).

It is to be noted, however, that in none of these evaluations a noticeable difference becomes recognizable and thus the subjectively determined observations between differently moving observers agree. This is also true for the high velocity of $v_0 = 0{,}5c$.

In addition, it should be mentioned that the values of $t_N$, $x_N$ etc. used here were named in this way exclusively because of the numerical calculation method and correspond to the analytically determined data for $t_A$ and $x_A$, respectively. Accordingly, these values also refer to the measurement results of the observer A moving with the same speed as S at the beginning of an experiment.

| K | N | $v_T$ | $t_N$ | $t_T$ | $x_N$ | $\gamma^3 a_N$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 10,0000000000000 | 1000,0000027816 | 1000,01667848293 | 5000,0000139081 | 10,0000000111265 |
| 2 | 10 | 9,99999999632825 | 1000,0000018637 | 1000,01667839113 | 5000,0000047288 | 10,000000011126 |
| 3 | $10^2$ | 9,99999999629152 | 1000,0000018545 | 1000,01667839021 | 5000,0000046369 | 10,0000000001113 |
| 4 | $10^3$ | 9,99999999629152 | 1000,0000018545 | 1000,01667839020 | 5000,0000046369 | 10,0000000000098 |
| 5 | $10^4$ | 9,99999999629107 | 1000,0000018544 | 1000,01667839020 | 5000,0000046378 | 10,000000000010 |
| 6 | $10^5$ | 9,99999999628186 | 1000,0000018545 | 1000,01667839021 | 5000,0000046244 | 9,99999999991214 |
| 7 | $10^6$ | 9,99999999612586 | 1000,0000018732 | 1000,01667839208 | 5000,0000044603 | 10,0000000000898 |
| 8 | $10^7$ | 9,99999999923743 | 1000,0000024389 | 1000,01667844866 | 5000,00000204189 | 9,99999998587893 |
| | | $\delta v_T$ | $\delta t_N$ | $\delta t_T$ | $\delta x_N$ | $\delta\gamma^3 a_N$ |
| 1/2 | | $3{,}67 \cdot 10^{-10}$ | $9{,}18 \cdot 10^{-11}$ | $9{,}18 \cdot 10^{-11}$ | $1{,}84 \cdot 10^{-10}$ | $1{,}00 \cdot 10^{-9}$ |
| 2/3 | | $3{,}67 \cdot 10^{-12}$ | $9{,}20 \cdot 10^{-13}$ | $9{,}20 \cdot 10^{-13}$ | $1{,}84 \cdot 10^{-12}$ | $1{,}00 \cdot 10^{-10}$ |
| 3/4 | | 0 | 0 | $9{,}99 \cdot 10^{-15}$ | 0 | $1{,}01 \cdot 10^{-11}$ |
| 4/5 | | $4{,}49 \cdot 10^{-14}$ | $9{,}99 \cdot 10^{-15}$ | 0 | $-1{,}80 \cdot 10^{-14}$ | $8{,}88 \cdot 10^{-13}$ |
| 5/6 | | $9{,}21 \cdot 10^{-13}$ | $-9{,}99 \cdot 10^{-15}$ | $-9{,}99 \cdot 10^{-15}$ | $2{,}68 \cdot 10^{-13}$ | $8{,}88 \cdot 10^{-12}$ |
| 6/7 | | $1{,}56 \cdot 10^{-11}$ | $-1{,}87 \cdot 10^{-12}$ | $-1{,}87 \cdot 10^{-12}$ | $3{,}28 \cdot 10^{-12}$ | $-1{,}78 \cdot 10^{-11}$ |
| 7/8 | | $-3{,}11 \cdot 10^{-10}$ | $-5{,}66 \cdot 10^{-11}$ | $-5{,}66 \cdot 10^{-11}$ | $-3{,}19 \cdot 10^{-10}$ | $1{,}42 \cdot 10^{-9}$ |

a) $v_0 = 0$, $a_S = 10\text{m/s}^2$, $t_S = 1000\text{s}$

| K | N | $v_T$ | $t_N$ | $t_T$ | $x_N$ | $\gamma^3 a_N$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 378,999984439478 | 1000,00077830515 | 1000,01667848259 | 374000,283305861 | 10,0000004216944 |
| 2 | 10 | 378,999984435807 | 1000,00077821336 | 1000,01667839113 | 374000,283372686 | 10,0000000421697 |
| 3 | $10^2$ | 378,999984435772 | 1000,00077821244 | 1000,01667839021 | 374000,283373356 | 10,0000000042204 |
| 4 | $10^3$ | 378,999984435760 | 1000,00077821243 | 1000,01667839020 | 374000,283373357 | 10,0000000004517 |
| 5 | $10^4$ | 378,999984435533 | 1000,00077821243 | 1000,01667839020 | 374000,283373242 | 9,99999999988323 |
| 6 | $10^5$ | 378,999984433259 | 1000,00077821243 | 1000,01667839020 | 374000,283372125 | 10,0000000010201 |
| 7 | $10^6$ | 378,999984411821 | 1000,00077821244 | 1000,01667839018 | 374000,283362434 | 9,99999998396706 |
| 8 | $10^7$ | 378,999984156412 | 1000,00077821289 | 1000,01667839010 | 374000,283206699 | 10,0000000408106 |
| | | $\delta v_T$ | $\delta t_N$ | $\delta t_T$ | $\delta x_N$ | $\delta\gamma^3 a_N$ |
| 1/2 | | $9{,}69 \cdot 10^{-12}$ | $9{,}18 \cdot 10^{-11}$ | $9{,}15 \cdot 10^{-11}$ | $-1{,}79 \cdot 10^{-10}$ | $3{,}80 \cdot 10^{-8}$ |
| 2/3 | | $9{,}10 \cdot 10^{-14}$ | $9{,}20 \cdot 10^{-13}$ | $9{,}20 \cdot 10^{-13}$ | $-1{,}79 \cdot 10^{-12}$ | $3{,}79 \cdot 10^{-9}$ |
| 3/4 | | $3{,}09 \cdot 10^{-14}$ | $9{,}99 \cdot 10^{-15}$ | $9{,}99 \cdot 10^{-15}$ | $-2{,}66 \cdot 10^{-15}$ | $3{,}77 \cdot 10^{-10}$ |
| 4/5 | | $6{,}01 \cdot 10^{-13}$ | 0 | 0 | $3{,}07 \cdot 10^{-13}$ | $5{,}68 \cdot 10^{-11}$ |
| 5/6 | | $6{,}00 \cdot 10^{-12}$ | 0 | 0 | $2{,}99 \cdot 10^{-12}$ | $-1{,}14 \cdot 10^{-10}$ |
| 6/7 | | $5{,}66 \cdot 10^{-11}$ | $-9{,}99 \cdot 10^{-15}$ | $2{,}00 \cdot 10^{-14}$ | $2{,}59 \cdot 10^{-11}$ | $1{,}71 \cdot 10^{-9}$ |
| 7/8 | | $6{,}74 \cdot 10^{-10}$ | $-4{,}50 \cdot 10^{-13}$ | $7{,}99 \cdot 10^{-14}$ | $4{,}16 \cdot 10^{-10}$ | $-5{,}68 \cdot 10^{-9}$ |

b) $v_0 = 369\ \text{km/s}$, $a_S = 10\text{m/s}^2$, $t_S = 1000\text{s}$

| K | N | $v_T$ | $t_N$ | $t_T$ | $x_N$ | $\gamma^3 a_N$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 149903,728874916 | 1154,71016786646 | 1000,01667841339 | 173091029,842051 | 10,0001667931727 |
| 2 | 10 | 149903,728874913 | 1154,71016776046 | 1000,01667839043 | 173091029,861909 | 10,0000166791061 |
| 3 | $10^2$ | 149903,728874913 | 1154,71016775940 | 1000,01667839021 | 173091029,862108 | 10,0000016684019 |
| 4 | $10^3$ | 149903,728874922 | 1154,71016775940 | 1000,01667839022 | 173091029,862117 | 10,0000002015203 |
| 5 | $10^4$ | 149903,728874803 | 1154,71016775925 | 1000,01667838996 | 173091029,862026 | 9,99999992987018 |
| 6 | $10^5$ | 149903,728875378 | 1154,71016775999 | 1000,01667839124 | 173091029,862469 | 10,0000022582798 |
| 7 | $10^6$ | 149903,728863738 | 1154,71016774494 | 1000,01667836518 | 173091029,853446 | 9,99998285456124 |
| 8 | $10^7$ | 149903,729001552 | 1154,71016792619 | 1000,01667867908 | 173091029,962107 | 10,0000216670928 |
| | | $\delta v_T$ | $\delta t_N$ | $\delta t_T$ | $\delta x_N$ | $\delta \gamma^3 a_N$ |
| 1/2 | | $1{,}84 \cdot 10^{-14}$ | $9{,}18 \cdot 10^{-11}$ | $2{,}30 \cdot 10^{-11}$ | $-1{,}15 \cdot 10^{-10}$ | $1{,}50 \cdot 10^{-5}$ |
| 2/3 | | 0 | $9{,}18 \cdot 10^{-13}$ | $2{,}20 \cdot 10^{-13}$ | $-1{,}15 \cdot 10^{-12}$ | $1{,}50 \cdot 10^{-6}$ |
| 3/4 | | $-5{,}84 \cdot 10^{-14}$ | 0 | $-9{,}99 \cdot 10^{-15}$ | $-5{,}20 \cdot 10^{-14}$ | $1{,}47 \cdot 10^{-7}$ |
| 4/5 | | $7{,}93 \cdot 10^{-13}$ | $1{,}30 \cdot 10^{-13}$ | $2{,}60 \cdot 10^{-13}$ | $5{,}26 \cdot 10^{-13}$ | $2{,}72 \cdot 10^{-8}$ |
| 5/6 | | $-3{,}84 \cdot 10^{-12}$ | $-6{,}41 \cdot 10^{-13}$ | $-1{,}28 \cdot 10^{-12}$ | $-2{,}56 \cdot 10^{-12}$ | $-2{,}33 \cdot 10^{-7}$ |
| 6/7 | | $7{,}77 \cdot 10^{-11}$ | $1{,}30 \cdot 10^{-11}$ | $2{,}61 \cdot 10^{-11}$ | $5{,}21 \cdot 10^{-11}$ | $1{,}94 \cdot 10^{-6}$ |
| 7/8 | | $-9{,}19 \cdot 10^{-10}$ | $-1{,}57 \cdot 10^{-10}$ | $-3{,}14 \cdot 10^{-10}$ | $-6{,}28 \cdot 10^{-10}$ | $-3{,}88 \cdot 10^{-6}$ |

c) $v_0 = 0.5c,\ a_S = 10\mathrm{m/s^2},\ t_S = 1000\mathrm{s}$

Tab. B.2: Results for $v_T$, $t_N$, $t_T$, $x_N$ and $\gamma^3 a_N$ acc. to calculations of Program B presented in Fig. B.2 as a function of the number of iteration steps N. Values for $v_T$ in $\mathrm{km/s}$, $t_T$ in s, $x_N$ in km and $a_N$ in $\mathrm{m/s^2}$.

## B.3 Improved accuracy by using a Taylor expansion

If the analytical calculations shown are to be carried out for very small values for time or speed, larger differences result depending on the calculation accuracy. This concerns in particular equation (6.74) for the distance covered during an experiment

$$x_A = \frac{c^2}{a_S}\left\{\left(1 - \frac{v_A^2}{c^2}\right)^{-1/2} - 1\right\} = \frac{c^2}{a_S}(\gamma - 1) \tag{6.74}$$

For small values for $v_A$, the effect arises that the value for $\gamma$ deviates only slightly from 1 and the final result becomes inaccurate because of the difference formation to 1. In the present case, the spreadsheet program Microsoft Excel© was used which provides an accuracy of 15 digits, and thus for values for $v_A$ below about 400 km/s, deviations occur which can become very high for small values. In this case, instead of using Eq. (6.74), it is recommended to use a Taylor expansion for $\gamma$ that contains "1" as the first value. This is:

$$\gamma = \left(1 - \frac{v_A^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2}\frac{v_A^2}{c^2} + \frac{3}{8}\frac{v_A^4}{c^4} + \frac{15}{48}\frac{v_A^6}{c^6} + + \cdots \tag{B.15}$$

$$1. \quad 2. \quad 3. \quad 4. \quad \text{Taylor} - \text{elements}$$

The following table B.3 shows the effect on the results for different test times $t_S$ or velocities of $v_A$ using different approaches.

| $t_S$ | $v_A$ | $t_A$ | $x_A(1)$ | $x_A(2)$ | $x_A(3)$ |
|---|---|---|---|---|---|
| 1 | 0,0100000000000000 | 1,0000000000000 | 0,00399127477173885 | 0,0050000000000000 | 0,0050000000000000 |
| 10 | 0,0999999999999963 | 10,0000000000002 | 0,498909346467356 | 0,500000000000004 | 0,500000000000004 |
| 100 | 0,999999999996291 | 100,000000000185 | 50,0006947029584 | 50,0000000000463 | 50,0000000000463 |
| 1.000 | 9,99999999629117 | 1000,00000018544 | 5000,00161862472 | 5000,0000046360 | 5000,0000046360 |
| 10.000 | 99,9999962911666 | 10000,0001854417 | 500000,004207119 | 500000,004636038 | 500000,004636042 |
| 20.000 | 199,999970329337 | 20000,0014835334 | 2000000,07470196 | 2000000,07417642 | 2000000,07417667 |
| 40.000 | 399,999762634824 | 40000,0118682683 | 8000001,18686648 | 8000001,18681095 | 8000001,18682679 |
| 60.000 | 599,999198893243 | 60000,0400554100 | 18000006,0086520 | 18000006,0081306 | 18000006,0083111 |
| 80.000 | 799,998101082647 | 80000,0949461719 | 32000018,9903299 | 32000018,9882180 | 32000018,9892321 |
| 100.000 | 999,996291182986 | 100000,185441779 | 50000046,3602845 | 50000046,3565675 | 50000046,3604362 |
| 200.000 | 1999,97032986004 | 200001,483536709 | 200000741,768963 | 200000741,520219 | 200000741,767795 |
| 1.000.000 | 9996,29281639030 | 1000185,45199287 | 5000463621,38578 | 5000459757,50365 | 5000463617,62560 |

Tab. B.3: Values of $v_A$, $t_A$ and $x_A$ depending on $t_S$ acc. to different procedures
$x_A(1)$: Eq. (6.74)
$x_A(2)$: Eq. (B.15) Taylor elements 1–3
$x_A(3)$: Eq. (B.15) Taylor elements 1–4
Optimal values for $x_A$ marked in green. Results in km and s.

For $t_S$ values up to 20,000s, the calculation according to $x_A(3)$ using the first 4 Taylor elements has the highest accuracy, up to 1,000s the solution with $x_A(2)$ is also sufficiently accurate. For values from approx. 40,000s, Eq. (6.74) is preferable (or further Taylor elements would have to be added).

## B.4 Comparison of results of the different methods

Finally, the results calculated from the different methods will be compared. In addition to the numerical and analytical methods presented here, the numerically obtained results from Annex C based on the relativistic rocket equation have been added. While in the first two calculations a constant acceleration is made a prerequisite, the same situation arises in the relativistic rocket equation for the special case that the ejection of the propellant mass is kept constant in relation to the remaining mass of the rocket.

Tab. B.4 shows the values determined according to the different methods $v_T = v_N - v_0$, $t_A$, $t_T$ and $x_N$ for the initial velocities $v_0 = 0$ as well as 369 km/s and 0.5c. The values listed in A were calculated analytically using the equations Eq. (6.60) to (6.74). For the velocities $v_0 = 0$ and 369 km/s the Taylor expansion was used as described in Tab. B3, details are presented in the table. The values for B are the numerical results corresponding to Annex B, and C are from Annex C, type "B1". The comparison shows that the velocities $v_T$ for A and B agree very well, but this deviates somewhat for variant C, especially for higher initial values. Moreover, for A, slightly higher values for $x_N$ result in the range of small velocities. In general, however, it can be said that the agreement of the results is good despite the completely different approaches.

Furthermore, for a comprehensive overlook the values for $\gamma^3 a_N$ were added. It is shown in all cases that they correspond very exactly to the value of $a_S$ subjectively valid for the accelerated observer.

| a) | St. | $v_A$ | $t_A$ | $t_T$ | $x_A$ | $a_S$ |
|---|---|---|---|---|---|---|
| A | (1) | 0 | 0 | | 0 | 10 |
| A | (2) | 9,99999999629117 | 1000,00000018544 | | 5000,0000046361 | 10 |
| A | Diff. | 9,99999999629117 | 1000,00000018544 | 1000,01667839020 | 5000,0000046361 | |
| | N | $v_T$ | $t_N$ | $t_T$ | $x_N$ | $\gamma^3 a_N$ |
| B | $10^2$ | 9,99999999629152 | 1000,00000018545 | 1000,01667839021 | 5000,0000046369 | 10,0000000001113 |
| B | $10^3$ | 9,99999999629152 | 1000,00000018545 | 1000,01667839020 | 5000,0000046369 | 10,0000000000098 |
| B | $10^4$ | 9,99999999629107 | 1000,00000018544 | 1000,01667839020 | 5000,0000046378 | 10,0000000000010 |
| B | $10^5$ | 9,99999999628186 | 1000,00000018545 | 1000,01667839021 | 5000,0000046244 | 9,99999999991214 |
| | N | $\delta v_T$ | $\delta t_N$ | $\delta t_T$ | $\delta x_N$ | $\delta\gamma^3 a_N$ |
| B | $10^2$ | $-3{,}52 \cdot 10^{-14}$ | $-8{,}22 \cdot 10^{-15}$ | $-6{,}78 \cdot 10^{-15}$ | $-1{,}69 \cdot 10^{-14}$ | $-1{,}11 \cdot 10^{-11}$ |
| B | $10^3$ | $-3{,}52 \cdot 10^{-14}$ | $-8{,}22 \cdot 10^{-15}$ | 0 | $-1{,}69 \cdot 10^{-14}$ | $-9{,}84 \cdot 10^{-13}$ |
| B | $10^4$ | $9{,}77 \cdot 10^{-15}$ | 0 | 0 | $-3{,}50 \cdot 10^{-14}$ | $-9{,}65 \cdot 10^{-14}$ |
| B | $10^5$ | $9{,}31 \cdot 10^{-13}$ | $-8{,}22 \cdot 10^{-15}$ | $-6{,}78 \cdot 10^{-15}$ | $2{,}34 \cdot 10^{-13}$ | $8{,}79 \cdot 10^{-12}$ |
| | N | $v_T$ | $t_N$ | $t_T$ | $x_N$ | $\gamma^3 a_N$ |
| C | $10^2$ | 9,99999999608546 | 1000,00000018545 | 1000,01667839021 | 5000,0000054144 | 9,99999999883114 |
| C | $10^3$ | 9,99999999607074 | 1000,00000018544 | 1000,01667839020 | 5000,0000053831 | 9,99999999868366 |
| C | $10^4$ | 9,99999999607296 | 1000,00000018544 | 1000,01667839020 | 5000,0000053928 | 9,99999999869584 |
| C | $10^5$ | 9,99999999610921 | 1000,00000018545 | 1000,01667839021 | 5000,0000055217 | 9,99999999845206 |
| | N | $\delta v_T$ | $\delta t_N$ | $\delta t_T$ | $\delta x_N$ | $\delta\gamma^3 a_N$ |
| C | $10^2$ | $2{,}06 \cdot 10^{-11}$ | $-8{,}22 \cdot 10^{-15}$ | $-6{,}78 \cdot 10^{-15}$ | $3{,}24 \cdot 10^{-7}$ | $1{,}17 \cdot 10^{-10}$ |
| C | $10^3$ | $2{,}20 \cdot 10^{-11}$ | 0 | 0 | $3{,}24 \cdot 10^{-7}$ | $1{,}32 \cdot 10^{-10}$ |
| C | $10^4$ | $2{,}19 \cdot 10^{-11}$ | 0 | 0 | $3{,}24 \cdot 10^{-7}$ | $1{,}30 \cdot 10^{-10}$ |
| C | $10^5$ | $1{,}82 \cdot 10^{-11}$ | $-8{,}22 \cdot 10^{-15}$ | $-6{,}78 \cdot 10^{-15}$ | $3{,}24 \cdot 10^{-7}$ | $1{,}55 \cdot 10^{-10}$ |

| b) | St. | $v_A$ | $t_A$ | $t_T$ | $x_A$ | $a_S$ |
|---|---|---|---|---|---|---|
| A | (1) | 369 | 36900,0279516977 | | 6808057,73563331 | 10 |
| A | (2) | 378,99998443578 | 37900,0287299112 | | 7182058,01900707 | 10 |
| A | Diff. | 9,99998443578 | 1000,0007782135 | 1000,01667839124 | 374000,28337376 | |
| | N | $v_T$ | $t_N$ | $t_T$ | $x_N$ | $\gamma^3 a_N$ |
| B | $10^2$ | 9,99998443577 | 1000,0007782124 | 1000,01667839021 | 374000,28337336 | 10,0000000042204 |
| B | $10^3$ | 9,99998443576 | 1000,0007782124 | 1000,01667839020 | 374000,28337336 | 10,0000000004517 |
| B | $10^4$ | 9,99998443553 | 1000,0007782124 | 1000,01667839020 | 374000,28337324 | 9,99999999988323 |
| B | $10^5$ | 9,99998443326 | 1000,0007782124 | 1000,01667839020 | 374000,28337213 | 10,0000000010201 |
| | N | $\delta v_T$ | $\delta t_N$ | $\delta t_T$ | $\delta x_N$ | $\delta\gamma^3 a_N$ |
| B | $10^2$ | $2{,}09 \cdot 10^{-14}$ | $1{,}04 \cdot 10^{-12}$ | $1{,}03 \cdot 10^{-12}$ | $2{,}87 \cdot 10^{-12}$ | $-4{,}22 \cdot 10^{-10}$ |
| B | $10^3$ | $5{,}17 \cdot 10^{-14}$ | $1{,}05 \cdot 10^{-12}$ | $1{,}04 \cdot 10^{-12}$ | $1{,}08 \cdot 10^{-12}$ | $-4{,}52 \cdot 10^{-11}$ |
| B | $10^4$ | $6{,}52 \cdot 10^{-13}$ | $1{,}05 \cdot 10^{-12}$ | $1{,}04 \cdot 10^{-12}$ | $1{,}08 \cdot 10^{-12}$ | $1{,}17 \cdot 10^{-11}$ |
| B | $10^5$ | $6{,}65 \cdot 10^{-12}$ | $1{,}05 \cdot 10^{-12}$ | $1{,}04 \cdot 10^{-12}$ | $1{,}38 \cdot 10^{-13}$ | $-1{,}02 \cdot 10^{-10}$ |
| | N | $v_T$ | $t_N$ | $t_T$ | $x_N$ | $\gamma^3 a_N$ |
| C | $10^2$ | 9,99998435551 | 1000,0007782124 | 1000,01667839008 | 374000,28333340 | 9,99999992287606 |
| C | $10^3$ | 9,99998435367 | 1000,0007782124 | 1000,01667839007 | 374000,28333240 | 9,99999991716842 |
| C | $10^4$ | 9,99998435376 | 1000,0007782124 | 1000,01667839006 | 374000,28333254 | 9,99999991716842 |
| C | $10^5$ | 9,99998435645 | 1000,0007782124 | 1000,01667839007 | 374000,28333389 | 9,99999992496930 |
| | N | $\delta v_T$ | $\delta t_N$ | $\delta t_T$ | $\delta x_N$ | $\delta\gamma^3 a_N$ |
| C | $10^2$ | $2{,}12 \cdot 10^{-10}$ | $1{,}04 \cdot 10^{-12}$ | $1{,}17 \cdot 10^{-12}$ | $-5{,}74 \cdot 10^{-10}$ | $7{,}71 \cdot 10^{-9}$ |
| C | $10^3$ | $2{,}17 \cdot 10^{-10}$ | $1{,}05 \cdot 10^{-12}$ | $1{,}18 \cdot 10^{-12}$ | $1{,}10 \cdot 10^{-10}$ | $8{,}28 \cdot 10^{-9}$ |
| C | $10^4$ | $2{,}16 \cdot 10^{-10}$ | $1{,}05 \cdot 10^{-12}$ | $1{,}19 \cdot 10^{-12}$ | $1{,}10 \cdot 10^{-10}$ | $8{,}28 \cdot 10^{-9}$ |
| C | $10^5$ | $2{,}09 \cdot 10^{-10}$ | $1{,}05 \cdot 10^{-12}$ | $1{,}18 \cdot 10^{-12}$ | $1{,}07 \cdot 10^{-10}$ | $7{,}50 \cdot 10^{-9}$ |

| c) | St. | $v_A$ | $t_A$ | $t_T$ | $x_A$ | $a_S$ |
|---|---|---|---|---|---|---|
| A | (1) | 149896,229000000 | 17308525,6327320 | | 1390379100217,26 | 10 |
| A | (2) | 149903,728874913 | 17309680,3428997 | | 1390552191247,12 | 10 |
| A | Diff. | 7,499874913 | 1154,7101678 | 1000,01667838490 | 173091029,86 | |
| | N | $v_T$ | $t_N$ | $t_T$ | $x_N$ | $\gamma^3 a_N$ |
| B | $10^2$ | 7,499874913 | 1154,7101678 | 1000,01667839021 | 173091029,86 | 10,0000016684019 |
| B | $10^3$ | 7,499874922 | 1154,7101678 | 1000,01667839022 | 173091029,86 | 10,0000002015203 |
| B | $10^4$ | 7,499874803 | 1154,7101678 | 1000,01667838996 | 173091029,86 | 9,99999992987018 |
| B | $10^5$ | 7,499875378 | 1154,7101678 | 1000,01667839124 | 173091029,86 | 10,0000022582798 |
| | N | $\delta v_T$ | $\delta t_N$ | $\delta t_T$ | $\delta x_N$ | $\delta \gamma^3 a_N$ |
| B | $10^2$ | $-4{,}22 \cdot 10^{-15}$ | $-4{,}29 \cdot 10^{-12}$ | $-5{,}31 \cdot 10^{-12}$ | $-7{,}33 \cdot 10^{-12}$ | $-1{,}67 \cdot 10^{-7}$ |
| B | $10^3$ | $-6{,}27 \cdot 10^{-14}$ | $-4{,}29 \cdot 10^{-12}$ | $-5{,}32 \cdot 10^{-12}$ | $-7{,}38 \cdot 10^{-12}$ | $-2{,}02 \cdot 10^{-8}$ |
| B | $10^4$ | $7{,}30 \cdot 10^{-13}$ | $-4{,}16 \cdot 10^{-12}$ | $-5{,}06 \cdot 10^{-12}$ | $-6{,}85 \cdot 10^{-12}$ | $7{,}01 \cdot 10^{-9}$ |
| B | $10^5$ | $-3{,}10 \cdot 10^{-12}$ | $-4{,}81 \cdot 10^{-12}$ | $-6{,}34 \cdot 10^{-12}$ | $-9{,}41 \cdot 10^{-12}$ | $-2{,}26 \cdot 10^{-7}$ |
| | N | $v_T$ | $t_N$ | $t_T$ | $x_N$ | $\gamma^3 a_N$ |
| C | $10^2$ | 7,499850523 | 1154,7101677 | 1000,01667833597 | 173091029,84 | 9,99996914760263 |
| C | $10^3$ | 7,499849967 | 1154,7101677 | 1000,01667833473 | 173091029,84 | 9,99996690196617 |
| C | $10^4$ | 7,499849989 | 1154,7101677 | 1000,01667833478 | 173091029,84 | 9,99996743452330 |
| C | $10^5$ | 7,499850786 | 1154,7101677 | 1000,01667833657 | 173091029,84 | 9,99996654696513 |
| | N | $\delta v_T$ | $\delta t_N$ | $\delta t_T$ | $\delta x_N$ | $\delta \gamma^3 a_N$ |
| C | $10^2$ | $3{,}25 \cdot 10^{-6}$ | $2{,}28 \cdot 10^{-11}$ | $4{,}89 \cdot 10^{-11}$ | $1{,}01 \cdot 10^{-10}$ | $3{,}09 \cdot 10^{-6}$ |
| C | $10^3$ | $3{,}33 \cdot 10^{-6}$ | $2{,}35 \cdot 10^{-11}$ | $5{,}02 \cdot 10^{-11}$ | $1{,}04 \cdot 10^{-10}$ | $3{,}31 \cdot 10^{-6}$ |
| C | $10^4$ | $3{,}32 \cdot 10^{-6}$ | $2{,}34 \cdot 10^{-11}$ | $5{,}01 \cdot 10^{-11}$ | $1{,}04 \cdot 10^{-10}$ | $3{,}26 \cdot 10^{-6}$ |
| C | $10^5$ | $3{,}22 \cdot 10^{-6}$ | $2{,}25 \cdot 10^{-11}$ | $4{,}83 \cdot 10^{-11}$ | $9{,}99 \cdot 10^{-11}$ | $3{,}35 \cdot 10^{-6}$ |

Tab. B.4: Calculated values for $v_T$, $t_A$, $t_T$, $x_N$ and $\gamma^3 a_N$ using different procedures
A: Analytically acc. to calculation using Eq. $(6.60)$ to $(6.74)$
B: Numerically acc. to VBA-Code from Fig. B.2
C: Numerically acc. to VBA-Code from Fig. C.2, Type “B1”
$a_S = 10\text{m/s}^2$. $\Delta t_S = 1.000\text{s}$. Results in km and s.
a) $v_0 = 0$, results for $x_A$ calculated using Eq. $(\text{B}.15)$, $x_A(3)$ and $x_A(2)$
b) $v_0 = 369\ \text{km/s}$, results for $x_A$ calculated using Eq. $(\text{B}.15)$, $x_A(3)$
c) $v_0 = 0.5c$, results for $x_A$ calculated using Eq. $(6.74)$

# Annex C: Relativistic rocket equation

For numerical calculation, the equations derived in chapter 6.4.2

$$p_K + p'_K = (m_{K-1} - \Delta m_{K-1}) v_K \gamma_k + \Delta m_{K-1} v'_K \gamma'_K = m_{K-1} v_{K-1} \gamma_{K-1} \tag{6.84}$$

and

$$v'_K = \frac{v_K + v'_0}{1 + \frac{v_K v'_0}{c^2}} \tag{6.85}$$

are used. For the determination of $v_K$, as already presented in other chapters, the method of bisection was chosen (see also the comparison of different numerical calculation methods in annex D). The basis is the momentum calculation of the total system, consisting of the momentum of the rocket $p_K$ as well as that of the propulsion gas $p'_K$ with mass $\Delta m_{K-1}$ moving in the opposite direction, and the determination of the corresponding rocket velocity $v_K$. Due to the law of conservation of momentum, the total value must be constant before and after the velocity increase of the rocket including the consideration of mass ejection.

First, suitable starting values for $(v_+)_0$ and $(v_-)_0$ must be defined; it makes sense that these values should be far apart since it must be ensured that the final result $v_K$ lies within these limits. Thereupon a new index $L$ is defined. Now the mean value

$$(v_K)_{L=1} = \frac{(v_+)_0 + (v_-)_0}{2} \tag{C.01}$$

is formed and for the velocity calculated here the momentum is determined according to equation (6.84). Then the following definitions must be used:

$$(p_K + p'_K)_{L=1} > m_{K-1} v_{K-1} \gamma_{K-1} \quad \Rightarrow \quad \begin{cases} (v_+)_1 = (v)_1 \\ (v_-)_1 = (v_-)_0 \end{cases} \tag{C.02}$$

$$(p_K + p'_K)_{L=1} \leq m_{K-1} v_{K-1} \gamma_{K-1} \quad \Rightarrow \quad \begin{cases} (v_+)_1 = (v_+)_0 \\ (v_-)_1 = (v)_1 \end{cases} \tag{C.03}$$

This calculation is repeated with increasing index $L$ until the results for $v_+$ and $v_-$ are equal. Thus, the velocity of the rocket, whose mass is now reduced by $\Delta m_{K-1}$, is determined for this partial step. Subsequently, the next step is performed for $K = 2$ and so on.

The time that subjectively elapses inside the rocket between the emission of 2 signals is by definition $\Delta t_0$. For an external observer the view is different, and the value must be supplemented according to

$$\Delta t_K = \Delta t_0 \gamma_K \tag{C.04}$$

and the distance covered is

$$\Delta x_K = \Delta t_K v_K \tag{C.05}$$

After adding all $N$ single values, the final result is

$$t_N = \sum_{K=1}^{N} \Delta t_0 \gamma_K \tag{C.06}$$

$$x_N = \sum_{K=1}^{N} \Delta t_0 v_K \tag{C.07}$$

At any arbitrary time $t_K$, a signal is sent back from the accelerated system S to the observers A and B. Observer A has moved with the same velocity as the rocket at the beginning of the experiment and continues its path without acceleration, while B measures a velocity $v_0$ with respect to A. From B's point of view, A is either moving in direction to S or in the opposite way during signal propagation. In case of $v_0 \neq 0$ the values for acceleration $a_K$ and velocity $v_0$ can each be positive or negative, so different arrangements must be made for performing the calculations. This was already done in a similar form in Chap. 6.4.1 with the equations Eq. (6.60) to (6.74), but there the acceleration of the rocket was kept constant over the entire course of the experiment. In contrast, here the exit direction of the propulsion gas $v'$ represents the effect of precondition. If $v' > 0$ then the acceleration is negative, at $v' < 0$ it is positive. The equations used in section 6.4.1 must therefore be modified with respect to the boundary conditions and read as follows here

$v' < 0 \quad (a_S > 0)$:
$$t_{K,R} = \frac{x_K - v_0 t_K}{c\left(1 + \frac{v_0}{c}\right)} \tag{C.08}$$

$v' > 0 \quad (a_S < 0)$:
$$t_{K,R} = \frac{|x_K - v_0 t_K|}{c\left(1 - \frac{v_0}{c}\right)} \tag{C.09}$$

Thus, for the limiting case applies

$v_0 = 0$:
$$t_{K,R} = \frac{|x_K|}{c} \tag{C.10}$$

Generally follows

$$t_T(K) = \frac{t_K + t_{K,R}}{\gamma(v_0)} \tag{C.11}$$

In addition, for the determined final velocity $v_N$, the following is specified for different system velocities $v_0$ for better comparability of the calculations

$$v_T = v_N - v_0 \tag{C.12}$$

## C.2 Specific specifications for the calculation

When defining the boundary conditions for the calculation, the ratio of outflowing mass per time interval is relevant. In order to simplify the representation, here the outflow mass of the rocket is normalized to 1 and the standard time interval, valid subjectively inside the rocket, is set to $\Delta t_0 = 1\mathrm{s}$. From this it follows, for example, for the case when 0.5% of the rocket mass flows out per second for propulsion, that when 50% of the mass is ejected, a total of 100 iteration steps have been performed. This case can be defined for the calculations using the form

$$\Delta m_0 = \Delta t_0 \cdot 0{,}5\% \qquad \mathrm{N}/\Delta t_0 = 100 \tag{C.13}$$

If, for example, the number of iteration steps is then increased by a factor of 10, the time interval and the outflowing supporting mass are reduced by the same factor for the subsequent calculations.

The initial values of the velocities $(v_+)_{L=0}$ and $(v_-)_{L=0}$ for the bisection should be chosen far apart, but the mean value must be non-zero, otherwise there will be disturbances during the calculation; $(v_+)_{L=0} = 0{,}9c$ and $(v_-)_{L=0} = -0{,}8c$ were chosen in this case.

## C.3 Flowchart and VBA program code of the process

A flow chart (Fig. C.1) shows how the running program is designed. It is a process with two nested iteration loops; the running indices have been labeled K and L. The representation of the VBA program code (Fig. C.2) follows the flowchart representation. The VBA codes used for the formula characters are shown in the following listing.

| Symbol | VBA-Code | Symbol | VBA-Code | Symbol | VBA-Code |
|---|---|---|---|---|---|
| $v_0$ | `v0` | $v_0'$ | `v0g` | $\Delta t_0$ | `dt0` |
| $(v_+)_L$ | `vmax` | $(v_-)_L$ | `vmin` | $(v_+)_{L=0}$ | `vmax0` |
| $(v_-)_{L=0}$ | `vmin0` | $t_K$ | `tK` | $t_{K-1}$ | `tKm1` |
| $t_T$ | `tT` | $x_K$ | `xK` | $t_{K,R}$ | `tKR` |
| $(v_K)_L$ | `vL` | $v_{K-1}$ | `vKm1` | $v_K$ | `vK` |
| $m_K$ | `mK` | $\Delta m_0$ | `dm0` | $\Delta m_K$ | `dmK` |
| $p_{K-1}$ | `pKm1` | $(p_K + p_K')_L$ | `pL` | $v_K'$ | `vKg` |
| $(v_K)_{L-1}$ | `vLm1` | $(v_K')_L$ | `vLg` | $v_T$ | `vT` |
| $a_K$ | `aK` | $\gamma^3$ | `Ga3` | $\gamma^3 a_K$ | `aKGa3` |

Tab. C.1: Formula symbols and referring VBA-Codes

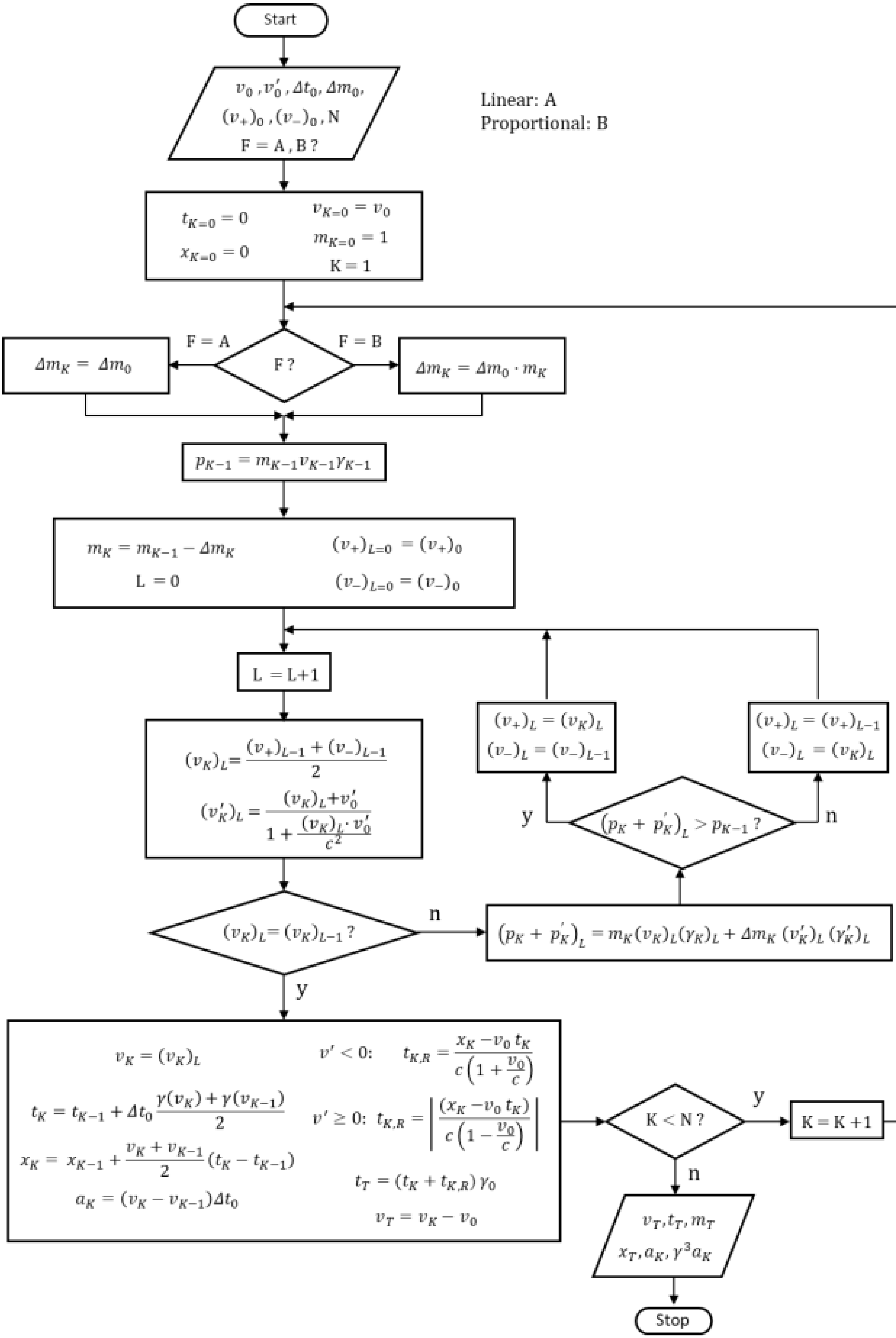



Fig. C.1: Flowchart of the calculation process

```
Sub C()
Dim v0, v0g, tS, dtS, dm0, mF, vmax0, vmin0, vmax, vmin, mK, tK As Double
Dim tKm1, tKR, tT, xK, vK, vKm1, dmK, pKm1, pL, vL, vLm1 As Double
Dim N, K, L, vKg, vT, vLg, c, aK, Ga3, aKGa3 As Double
Dim F, A1, A2, B1, B2 As String
'Input
    F = "B1"            'Define A1, A2, B1 or B2
                        'A: Linear mass reduction, B: Prop. mass reduction
                        '1: Def. number of iteration steps, 2: Def. end mass
    v0 = 0              'Initial velocity in km/s
    v0g = -4            'Initial velocity gas in km/s
    dm0 = 0.25 / 100    'Initial output mass in %/s
'Specific input Def. 1
    tS = 400            'Time until a signal is emitted
    N = 1000            'Number of iteration steps
'Specific input Def. 2
    dtS = 1             'Iteration time in s
    mF = 10 / 100       'Mass at end of trial in %
'Start Calculation
    If F = "A1" Or F = "A2" Or F = "B1" Or F = "B2" Then
        GoTo Calc:
        Else
        Debug.Print "Input error: Chose A1, A2, B1, or B2"
        GoTo Out1:
        End If
Calc:
    If F = "A1" Or F = "B1" Then
        dtS = tS / N
        End If
    mK = 1               'Initial value mass
    vmax0 = 0.9          'Initial value max. for calculation (in rel. to c)
    vmin0 = -0.8         'Initial value min. for calculation (in rel. to c)
    c = 299792.458       'speed of light in km/s
    tK = 0
    xK = 0
    vK = v0 / c
    v0g = v0g / c
Mainloop:
        K = K + 1
        If F = "A1" Or F = "A2" Then
            dmK = dm0 * dtS
            Else
            dmK = dm0 * dtS * mK
            End If
        pKm1 = mK * vK / (1 - vK ^ 2) ^ 0.5     'Momentum rocket for K - 1
        mK = mK - dmK                           'Rest rocket mass for K
        If mK <= 0 Then
            K = K - 1
            mK = mK + dmK
            Debug.Print "Rocket mass zero"
            GoTo Out2:
            End If
        vmax = vmax0
        vmin = vmin0                            'Req.: vmin0 unequal -vmax0
        L = 0
Do
        L = L + 1
        vLm1 = vL
        vL = (vmax + vmin) / 2
        vLg = (vL + v0g) / (1 + vL * v0g)
        pL = mK * vL / (1 - vL ^ 2) ^ 0.5 + dmK * vLg / (1 - vLg ^ 2) ^ 0.5
```

```
If pL > pKm1 Then
            vmax = vL
            Else: vmin = vL
            End If
Loop Until vLm1 = vL
    vKm1 = vK
    vK = vL
    vKg = vLg
    tKm1 = tK
    tK = tK + dtS * (1 / (1 - vK ^ 2) ^ 0.5 + 1 / (1 - vKm1 ^ 2) ^ 0.5) / 2
    xK = xK + (vK + vKm1) / 2 * (tK - tKm1) * c
    aK = (vK - vKm1) / (dtS / (1 - ((vK + vKm1) / 2) ^ 2) ^ 0.5) * c * 1000
    Ga3 = (1 / (1 - ((vK + vKm1) / 2) ^ 2)) ^ 1.5
    If v0g > 0 Then
            tKR = Abs(xK - v0 * tK) / c / (1 - v0 / c)
            Else: tKR = (xK - v0 * tK) / c / (1 + v0 / c)
            End If
        tT = (tK + tKR) * (1 - (v0 / c) ^ 2) ^ 0.5
        vT = (vK * c - v0)
        aKGa3 = aK * Ga3
    If F = "A1" Or F = "B1" Then
        If K < N Then
            GoTo Mainloop:
            End If
        End If
    If F = "A2" Or F = "B2" Then
        If mK > mF Then
            GoTo Mainloop:
            End If
        End If
Out2:
Results in view of an observer moving with v0 at beginning of trial
Debug.Print "vT =", vT 'velocity when signal is emitted in km/s
Debug.Print "tN =", tK 'Total time until a signal is emitted in s
Debug.Print "tT =", tT 'Total time for transmission of signal in s
Debug.Print "mN =", mK 'Rocket mass at emission in relation to 1
Debug.Print "xN =", xK 'Distance covered at emission of signal in km
Debug.Print "aN =", aK 'Acceleration in m/s²
Debug.Print "aNGa3 =", aKGa3     'Acceleration * Gamma ^ 3 in m/s²
Out1:
End Sub
```

Fig. C2: VBA Program-Code for the calculation process presented in Fig. C1

In the following tables Tab. C.2, C.3 and C.4 supplementary calculations are shown according to Tab. 6.4 from Chap. 6.4.2. Instead of using the program "A1", the variant "A2" could also have been selected. In this case, the desired final value of the rocket mass and the iteration time are specified, and the number of iteration steps results from the calculation. Example from Tab C2: Parameters "A1" $t_S = 100s$, $N = 1000$ correspond to "A2" $m_F = 50\%$ and $\Delta t_S = 0{,}1s$. The calculated value for $K$ is then $N = 1001$. The results are very similar, but not completely identical. Since in this case the influence of the number of iteration steps was in the foreground, calculation "A1" was chosen.

The values of $t_T$ are of particular interest for comparisons, since they would be accessible for experimental testing due to the simple use of precision clocks. The results of $t_T$ obtained here are shown separately in Tab. 6.6, Tab. 6.7 and Fig. 6.4, but do not show any systematic differences, so that the principle of relativity is also observed here as in all other cases.

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 2,67508561278727 | 100,000397329364 | 0,500000000000000 | 119,116010675216 |
| $10^2$ | 2,76261372200990 | 100,000408141269 | 0,500000000000000 | 122,357320955608 |
| $10^3$ | 2,77158897232187 | 100,000409292747 | 0,500000000000055 | 122,702523336750 |
| $10^4$ | 2,77248872482278 | 100,000409408634 | 0,500000000000055 | 122,737265091767 |
| $10^5$ | 2,77257872237194 | 100,000409420246 | 0,499999999996724 | 122,740741494222 |
| $10^6$ | 2,77258772224753 | 100,000409422400 | 0,500000000041133 | 122,741089155569 |
| $10^7$ | 2,77258862465211 | 100,000409440862 | 0,499999999708066 | 122,741124020357 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | 8,9931 | $1,4064 \cdot 10^{-3}$ | | $3,4698 \cdot 10^2$ |
| $10^2$ | $8,7528 \cdot 10^{-2}$ | $1,0812 \cdot 10^{-5}$ | 0 | 3,2413 |
| $10^3$ | $8,9753 \cdot 10^{-3}$ | $1,1515 \cdot 10^{-6}$ | $5,4956 \cdot 10^{-14}$ | $3,4520 \cdot 10^{-1}$ |
| $10^4$ | $8,9975 \cdot 10^{-4}$ | $1,1589 \cdot 10^{-7}$ | 0 | $3,4742 \cdot 10^{-2}$ |
| $10^5$ | $8,9998 \cdot 10^{-5}$ | $1,1612 \cdot 10^{-8}$ | $-3,3309 \cdot 10^{-12}$ | $3,4764 \cdot 10^{-3}$ |
| $10^6$ | $8,9998 \cdot 10^{-6}$ | $2,1540 \cdot 10^{-9}$ | $4,4409 \cdot 10^{-11}$ | $3,4766 \cdot 10^{-4}$ |
| $10^7$ | $9,0240 \cdot 10^{-7}$ | $1,8462 \cdot 10^{-8}$ | $-3,3307 \cdot 10^{-10}$ | $3,4865 \cdot 10^{-5}$ |
| $10^8$ | $8,9931 \cdot 10^{-8}$ | $1,4069 \cdot 10^{-11}$ | | $3,4698 \cdot 10^{-6}$ |
| $10^9$ | $8,9931 \cdot 10^{-9}$ | $1,4069 \cdot 10^{-12}$ | | $3,4698 \cdot 10^{-7}$ |
| $10^{10}$ | $8,9931 \cdot 10^{-10}$ | $1,4211 \cdot 10^{-13}$ | | $3,4698 \cdot 10^{-8}$ |
| $10^{11}$ | $8,9931 \cdot 10^{-11}$ | 0 | | $3,4698 \cdot 10^{-9}$ |
| $10^{12}$ | $8,9933 \cdot 10^{-12}$ | 0 | | $3,4699 \cdot 10^{-10}$ |
| $10^{13}$ | $8,9928 \cdot 10^{-13}$ | 0 | | $3,4703 \cdot 10^{-11}$ |
| $10^{14}$ | $9,0150 \cdot 10^{-14}$ | 0 | | $3,4674 \cdot 10^{-12}$ |
| $10^{15}$ | $8,8818 \cdot 10^{-15}$ | 0 | | $3,4106 \cdot 10^{-13}$ |
| $10^{16}$ | 0 | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 2,77258862155768 | 100,000409422541 | | 122,741123853607 |
| $10^8$ | 2,77258871148869 | 100,000409422555 | | 122,741127323411 |
| $10^9$ | 2,77258872048179 | 100,000409422556 | | 122,741127670391 |
| $10^{10}$ | 2,77258872138110 | 100,000409422556 | | 122,741127705089 |
| $10^{11}$ | 2,77258872147103 | 100,000409422556 | | 122,741127708559 |
| $10^{12}$ | 2,77258872148003 | 100,000409422556 | | 122,741127708906 |
| $10^{13}$ | 2,77258872148093 | 100,000409422556 | | 122,741127708941 |
| $10^{14}$ | 2,77258872148102 | 100,000409422556 | | 122,741127708944 |
| $10^{15}$ | 2,77258872148102 | 100,000409422556 | | 122,741127708945 |
| $10^{16}$ | 2,77258872148102 | 100,000409422556 | | 122,741127708945 |

*a)*

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 2,67508151022224 | 100,000397329361 | 0,500000000000000 | 37019,1440520908 |
| $10^2$ | 2,76260948280941 | 100,000408141266 | 0,500000000000000 | 37022,3853647753 |
| $10^3$ | 2,77158471909860 | 100,000409292744 | 0,500000000000055 | 37022,7305674122 |
| $10^4$ | 2,77248447045912 | 100,000409408630 | 0,500000000000055 | 37022,7653092072 |
| $10^5$ | 2,77257447227356 | 100,000409420227 | 0,499999999996724 | 37022,7687858303 |
| $10^6$ | 2,77258347644647 | 100,000409421377 | 0,500000000041133 | 37022,7691339111 |
| $10^7$ | 2,77258481950150 | 100,000409421716 | 0,499999999708066 | 37022,7691906506 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | 8,9943 | $1,1550 \cdot 10^{-3}$ | | $3,4709 \cdot 10^2$ |
| $10^2$ | $8,7528 \cdot 10^{-2}$ | $1,0812 \cdot 10^{-5}$ | 0 | 3,2413 |
| $10^3$ | $8,9752 \cdot 10^{-3}$ | $1,1515 \cdot 10^{-6}$ | $5,4956 \cdot 10^{-14}$ | $3,4520 \cdot 10^{-1}$ |
| $10^4$ | $8,9975 \cdot 10^{-4}$ | $1,1589 \cdot 10^{-7}$ | 0 | $3,4742 \cdot 10^{-2}$ |
| $10^5$ | $9,0002 \cdot 10^{-5}$ | $1,1597 \cdot 10^{-8}$ | $-3,3309 \cdot 10^{-12}$ | $3,4766 \cdot 10^{-3}$ |
| $10^6$ | $9,0042 \cdot 10^{-6}$ | $1,1500 \cdot 10^{-9}$ | $4,4409 \cdot 10^{-11}$ | $3,4808 \cdot 10^{-4}$ |
| $10^7$ | $1,3431 \cdot 10^{-6}$ | $3,3900 \cdot 10^{-10}$ | $-3,3307 \cdot 10^{-10}$ | $5,6740 \cdot 10^{-5}$ |
| $10^8$ | $8,9943 \cdot 10^{-8}$ | $1,1553 \cdot 10^{-11}$ | | $3,4709 \cdot 10^{-6}$ |
| $10^9$ | $8,9943 \cdot 10^{-9}$ | $1,1511 \cdot 10^{-12}$ | | $3,4709 \cdot 10^{-7}$ |
| $10^{10}$ | $8,9943 \cdot 10^{-10}$ | $1,1369 \cdot 10^{-13}$ | | $3,4706 \cdot 10^{-8}$ |
| $10^{11}$ | $8,9943 \cdot 10^{-11}$ | 0 | | $3,4706 \cdot 10^{-9}$ |
| $10^{12}$ | $8,9941 \cdot 10^{-12}$ | 0 | | $3,4925 \cdot 10^{-10}$ |
| $10^{13}$ | $8,9928 \cdot 10^{-13}$ | 0 | | 0 |
| $10^{14}$ | $9,0150 \cdot 10^{-14}$ | 0 | | 0 |
| $10^{15}$ | $8,8818 \cdot 10^{-15}$ | 0 | | 0 |
| $10^{16}$ | 0 | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 2,77258437587408 | 100,000409421493 | | 37022,7691686202 |
| $10^8$ | 2,77258446581684 | 100,000409421504 | | 37022,7691720911 |
| $10^9$ | 2,77258447481111 | 100,000409421505 | | 37022,7691724381 |
| $10^{10}$ | 2,77258447571054 | 100,000409421505 | | 37022,7691724729 |
| $10^{11}$ | 2,77258447580048 | 100,000409421505 | | 37022,7691724764 |
| $10^{12}$ | 2,77258447580948 | 100,000409421505 | | 37022,7691724767 |
| $10^{13}$ | 2,77258447581038 | 100,000409421505 | | 37022,7691724768 |
| $10^{14}$ | 2,77258447581047 | 100,000409421505 | | 37022,7691724768 |
| $10^{15}$ | 2,77258447581048 | 100,000409421505 | | 37022,7691724768 |
| $10^{16}$ | 2,77258447581048 | 100,000409421505 | | 37022,7691724768 |

*b)*

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 2,67496628539311 | 100,000397329348 | 0,500000000000000 | 200123,569407464 |
| $10^2$ | 2,76249047724150 | 100,000408141251 | 0,500000000000000 | 200126,810789598 |
| $10^3$ | 2,77146532579354 | 100,000409292729 | 0,500000000000055 | 200127,155999630 |
| $10^4$ | 2,77236503944528 | 100,000409408616 | 0,500000000000055 | 200127,190742224 |
| $10^5$ | 2,77245505647284 | 100,000409420216 | 0,499999999996724 | 200127,194219809 |
| $10^6$ | 2,77246407688722 | 100,000409421380 | 0,500000000041133 | 200127,194569079 |
| $10^7$ | 2,77246734040796 | 100,000409422016 | 0,499999999708066 | 200127,194715347 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | 8,9985 | $1,1586 \cdot 10^{-3}$ | | $3,4742 \cdot 10^2$ |
| $10^2$ | $8,7524 \cdot 10^{-2}$ | $1,0812 \cdot 10^{-5}$ | 0 | 3,2414 |
| $10^3$ | $8,9748 \cdot 10^{-3}$ | $1,1515 \cdot 10^{-6}$ | $5,4956 \cdot 10^{-14}$ | $3,4521 \cdot 10^{-1}$ |
| $10^4$ | $8,9971 \cdot 10^{-4}$ | $1,1589 \cdot 10^{-7}$ | 0 | $3,4743 \cdot 10^{-2}$ |
| $10^5$ | $9,0017 \cdot 10^{-5}$ | $1,1600 \cdot 10^{-8}$ | $-3,3309 \cdot 10^{-12}$ | $3,4776 \cdot 10^{-3}$ |
| $10^6$ | $9,0204 \cdot 10^{-6}$ | $1,1640 \cdot 10^{-9}$ | $4,4409 \cdot 10^{-11}$ | $3,4927 \cdot 10^{-4}$ |
| $10^7$ | $3,2635 \cdot 10^{-6}$ | $6,3601 \cdot 10^{-10}$ | $-3,3307 \cdot 10^{-10}$ | $1,4627 \cdot 10^{-4}$ |
| $10^8$ | $8,9985 \cdot 10^{-8}$ | $1,1582 \cdot 10^{-11}$ | | $3,4742 \cdot 10^{-6}$ |
| $10^9$ | $8,9985 \cdot 10^{-9}$ | $1,1653 \cdot 10^{-12}$ | | $3,4741 \cdot 10^{-7}$ |
| $10^{10}$ | $8,9985 \cdot 10^{-10}$ | $1,1369 \cdot 10^{-13}$ | | $3,4750 \cdot 10^{-8}$ |
| $10^{11}$ | $8,9985 \cdot 10^{-11}$ | 0 | | $3,4634 \cdot 10^{-9}$ |
| $10^{12}$ | $8,9986 \cdot 10^{-12}$ | 0 | | $3,4925 \cdot 10^{-10}$ |
| $10^{13}$ | $8,9972 \cdot 10^{-13}$ | 0 | | 0 |
| $10^{14}$ | $9,0150 \cdot 10^{-14}$ | 0 | | 0 |
| $10^{15}$ | $8,8818 \cdot 10^{-15}$ | 0 | | 0 |
| $10^{16}$ | 0 | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 2,77246497673978 | 100,000409421496 | | 200127,194603821 |
| $10^8$ | 2,77246506672503 | 100,000409421507 | | 200127,194607295 |
| $10^9$ | 2,77246507572356 | 100,000409421509 | | 200127,194607642 |
| $10^{10}$ | 2,77246507662341 | 100,000409421509 | | 200127,194607677 |
| $10^{11}$ | 2,77246507671339 | 100,000409421509 | | 200127,194607680 |
| $10^{12}$ | 2,77246507672239 | 100,000409421509 | | 200127,194607681 |
| $10^{13}$ | 2,77246507672329 | 100,000409421509 | | 200127,194607681 |
| $10^{14}$ | 2,77246507672338 | 100,000409421509 | | 200127,194607681 |
| $10^{15}$ | 2,77246507672339 | 100,000409421509 | | 200127,194607681 |
| $10^{16}$ | 2,77246507672339 | 100,000409421509 | | 200127,194607681 |

*c)*

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 2,67210782993243 | 100,000397329281 | 0,500000000000000 | 1000675,97202456 |
| $10^2$ | 2,75953844190371 | 100,000408141179 | 0,500000000000000 | 1000679,21513818 |
| $10^3$ | 2,76850369436397 | 100,000409292656 | 0,500000000000055 | 1000679,56053261 |
| $10^4$ | 2,76940245165497 | 100,000409408544 | 0,500000000000055 | 1000679,59529404 |
| $10^5$ | 2,76949246576442 | 100,000409420158 | 0,499999999996724 | 1000679,59877786 |
| $10^6$ | 2,76950155149825 | 100,000409421342 | 0,500000000041133 | 1000679,59913297 |
| $10^7$ | 2,76951415107033 | 100,000409423363 | 0,499999999708066 | 1000679,59971782 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | 9,0100 | $1,1639 \cdot 10^{-3}$ | | $3,4913 \cdot 10^2$ |
| $10^2$ | $8,7431 \cdot 10^{-2}$ | $1,0812 \cdot 10^{-5}$ | 0 | 3,2431 |
| $10^3$ | $8,9653 \cdot 10^{-3}$ | $1,1515 \cdot 10^{-6}$ | $5,4956E \cdot 10^{-14}$ | $3,4539 \cdot 10^{-1}$ |
| $10^4$ | $8,9876 \cdot 10^{-4}$ | $1,1589 \cdot 10^{-7}$ | 0 | $3,4761 \cdot 10^{-2}$ |
| $10^5$ | $9,0014 \cdot 10^{-5}$ | $1,1614 \cdot 10^{-8}$ | $-3,3309 \cdot 10^{-12}$ | $3,4838 \cdot 10^{-3}$ |
| $10^6$ | $9,0857 \cdot 10^{-6}$ | $1,1840 \cdot 10^{-9}$ | $4,4409 \cdot 10^{-11}$ | $3,5511 \cdot 10^{-4}$ |
| $10^7$ | $1,2600 \cdot 10^{-5}$ | $2,0210 \cdot 10^{-9}$ | $-3,3307 \cdot 10^{-10}$ | $5,8485 \cdot 10^{-4}$ |
| $10^8$ | $9,0100 \cdot 10^{-8}$ | $1,1639 \cdot 10^{-11}$ | | $3,4913 \cdot 10^{-6}$ |
| $10^9$ | $9,0100 \cdot 10^{-9}$ | $1,1653 \cdot 10^{-12}$ | | $3,4913 \cdot 10^{-7}$ |
| $10^{10}$ | $9,0100 \cdot 10^{-10}$ | $1,1369 \cdot 10^{-13}$ | | $3,4925 \cdot 10^{-8}$ |
| $10^{11}$ | $9,0100 \cdot 10^{-11}$ | 0 | | $3,4925 \cdot 10^{-9}$ |
| $10^{12}$ | $9,0101 \cdot 10^{-12}$ | 0 | | 0 |
| $10^{13}$ | $9,0106 \cdot 10^{-13}$ | 0 | | 0 |
| $10^{14}$ | $9,0150 \cdot 10^{-14}$ | 0 | | 0 |
| $10^{15}$ | $8,8818 \cdot 10^{-15}$ | 0 | | 0 |
| $10^{16}$ | 0 | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 2,76950245249750 | 100,000409421458 | | 1000679,59916788 |
| $10^8$ | 2,76950254259743 | 100,000409421470 | | 1000679,59917137 |
| $10^9$ | 2,76950255160742 | 100,000409421471 | | 1000679,59917172 |
| $10^{10}$ | 2,76950255250842 | 100,000409421471 | | 1000679,59917176 |
| $10^{11}$ | 2,76950255259852 | 100,000409421471 | | 1000679,59917176 |
| $10^{12}$ | 2,76950255260753 | 100,000409421471 | | 1000679,59917176 |
| $10^{13}$ | 2,76950255260843 | 100,000409421471 | | 1000679,59917176 |
| $10^{14}$ | 2,76950255260852 | 100,000409421471 | | 1000679,59917176 |
| $10^{15}$ | 2,76950255260853 | 100,000409421471 | | 1000679,59917176 |
| $10^{16}$ | 2,76950255260853 | 100,000409421471 | | 1000679,59917176 |

*d)*

Tab: C.2: Calculation of relativistic rocket velocity according to program
Type: “A1”, $v_0' = -4\ \mathrm{km/s}$, $\Delta m_0 = 0.5\%$, $t_0 = 100s$
a) $v_0 = 0$, b) $v_0 = 369\ \mathrm{km/s}$, c) $v_0 = 2000\ \mathrm{km/s}$, d) $v_0 = 10000\ \mathrm{km/s}$

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 7,84085772014717 | 1000,00912888441 | 0,100000000000000 | 2736,750700612000 |
| $10^2$ | 9,05101119073233 | 1000,00983714046 | 0,099999999999999 | 2949,077033897790 |
| $10^3$ | 9,19416709875657 | 1000,00991970498 | 0,099999999999989 | 2973,828794295530 |
| $10^4$ | 9,20872063626419 | 1000,00992810957 | 0,099999999999856 | 2976,348382091860 |
| $10^5$ | 9,21017837171215 | 1000,00992895150 | 0,099999999998368 | 2976,600796650620 |
| $10^6$ | 9,21032416890996 | 1000,00992904517 | 0,100000000009585 | 2976,626042686920 |
| $10^7$ | 9,21033875458106 | 1000,00992885138 | 0,099999999922464 | 2976,628567836020 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | $1,4507 \cdot 10^{2}$ | $8,6118 \cdot 10^{-2}$ | | $2,5109 \cdot 10^{4}$ |
| $10^2$ | 1,2102 | $7,0826 \cdot 10^{-4}$ | $-8,0491 \cdot 10^{-16}$ | $2,1233 \cdot 10^{2}$ |
| $10^3$ | $1,4316 \cdot 10^{-1}$ | $8,2565 \cdot 10^{-5}$ | $-1,0700 \cdot 10^{-14}$ | $2,4752 \cdot 10^{1}$ |
| $10^4$ | $1,4554 \cdot 10^{-2}$ | $8,4046 \cdot 10^{-6}$ | $-1,3300 \cdot 10^{-13}$ | 2,5196 |
| $10^5$ | $1,4577 \cdot 10^{-3}$ | $8,4193 \cdot 10^{-7}$ | $-1,4880 \cdot 10^{-12}$ | $2,5241 \cdot 10^{-1}$ |
| $10^6$ | $1,4580 \cdot 10^{-4}$ | $9,3670 \cdot 10^{-8}$ | $1,1217 \cdot 10^{-11}$ | $2,5246 \cdot 10^{-2}$ |
| $10^7$ | $1,4586 \cdot 10^{-5}$ | $-1,9379 \cdot 10^{-7}$ | $-8,7121 \cdot 10^{-11}$ | $2,5251 \cdot 10^{-3}$ |
| $10^8$ | $1,4507 \cdot 10^{-6}$ | $8,6118 \cdot 10^{-10}$ | | $2,5109 \cdot 10^{-4}$ |
| $10^9$ | $1,4507 \cdot 10^{-7}$ | $8,6175 \cdot 10^{-11}$ | | $2,5109 \cdot 10^{-5}$ |
| $10^{10}$ | $1,4507 \cdot 10^{-8}$ | $8,6402 \cdot 10^{-12}$ | | $2,5109 \cdot 10^{-6}$ |
| $10^{11}$ | $1,4507 \cdot 10^{-9}$ | $9,0949 \cdot 10^{-13}$ | | $2,5109 \cdot 10^{-7}$ |
| $10^{12}$ | $1,4507 \cdot 10^{-10}$ | 0 | | $2,5109 \cdot 10^{-8}$ |
| $10^{13}$ | $1,4506 \cdot 10^{-11}$ | 0 | | $2,5107 \cdot 10^{-9}$ |
| $10^{14}$ | $1,4513 \cdot 10^{-12}$ | 0 | | $2,5102 \cdot 10^{-10}$ |
| $10^{15}$ | $1,4566 \cdot 10^{-13}$ | 0 | | $2,5011 \cdot 10^{-11}$ |
| $10^{16}$ | $1,4211 \cdot 10^{-14}$ | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 9,21033867546060 | 1000,00992905378 | | 2976,628553565180 |
| $10^8$ | 9,21034012611567 | 1000,00992905464 | | 2976,628804653010 |
| $10^9$ | 9,21034027118117 | 1000,00992905473 | | 2976,628829761790 |
| $10^{10}$ | 9,21034028568772 | 1000,00992905474 | | 2976,628832272670 |
| $10^{11}$ | 9,21034028713838 | 1000,00992905474 | | 2976,628832523760 |
| $10^{12}$ | 9,21034028728345 | 1000,00992905474 | | 2976,628832548870 |
| $10^{13}$ | 9,21034028729795 | 1000,00992905474 | | 2976,628832551380 |
| $10^{14}$ | 9,21034028729940 | 1000,00992905474 | | 2976,628832551630 |
| $10^{15}$ | 9,21034028729955 | 1000,00992905474 | | 2976,628832551650 |
| $10^{16}$ | 9,21034028729956 | 1000,00992905474 | *a)* | 2976,628832551660 |

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 7,84084554146182 | 1000,00912888434 | 0,100000000000000 | 371737,0322956670 |
| $10^2$ | 9,05099707036669 | 1000,00983714038 | 0,099999999999999 | 371949,3587899240 |
| $10^3$ | 9,19415274738003 | 1000,00991970490 | 0,099999999999989 | 371974,1105690590 |
| $10^4$ | 9,20870626170580 | 1000,00992810949 | 0,099999999999856 | 371976,6301589470 |
| $10^5$ | 9,21016399816938 | 1000,00992895148 | 0,099999999998368 | 371976,8825756170 |
| $10^6$ | 9,21030981978799 | 1000,00992903570 | 0,100000000009585 | 371976,9078311300 |
| $10^7$ | 9,21032474295856 | 1000,00992904906 | 0,099999999922464 | 371976,9105254130 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | $1,4507 \cdot 10^{2}$ | $8,3757 \cdot 10^{-2}$ | | $2,5111 \cdot 10^{4}$ |
| $10^2$ | 1,2102 | $7,0826 \cdot 10^{-4}$ | $-8,0491 \cdot 10^{-16}$ | $2,1233 \cdot 10^{2}$ |
| $10^3$ | $1,4316 \cdot 10^{-1}$ | $8,2565 \cdot 10^{-5}$ | $-1,0700 \cdot 10^{-14}$ | $2,4752 \cdot 10^{1}$ |
| $10^4$ | $1,4554 \cdot 10^{-2}$ | $8,4046 \cdot 10^{-6}$ | $-1,3300 \cdot 10^{-13}$ | 2,5196 |
| $10^5$ | $1,4577 \cdot 10^{-3}$ | $8,4199 \cdot 10^{-7}$ | $-1,4880 \cdot 10^{-12}$ | $2,5242 \cdot 10^{-1}$ |
| $10^6$ | $1,4582 \cdot 10^{-4}$ | $8,4220 \cdot 10^{-8}$ | $1,1217 \cdot 10^{-11}$ | $2,5256 \cdot 10^{-2}$ |
| $10^7$ | $1,4923 \cdot 10^{-5}$ | $1,3360 \cdot 10^{-8}$ | $-8,7121 \cdot 10^{-11}$ | $2,6943 \cdot 10^{-3}$ |
| $10^8$ | $1,4507 \cdot 10^{-6}$ | $8,3753 \cdot 10^{-10}$ | | $2,5111 \cdot 10^{-4}$ |
| $10^9$ | $1,4507 \cdot 10^{-7}$ | $8,3787 \cdot 10^{-11}$ | | $2,5111 \cdot 10^{-5}$ |
| $10^{10}$ | $1,4507 \cdot 10^{-8}$ | $8,4128 \cdot 10^{-12}$ | | $2,5111 \cdot 10^{-6}$ |
| $10^{11}$ | $1,4507 \cdot 10^{-9}$ | 0 | | $2,5111 \cdot 10^{-7}$ |
| $10^{12}$ | $1,4507 \cdot 10^{-10}$ | 0 | | $2,5088 \cdot 10^{-8}$ |
| $10^{13}$ | $1,4508 \cdot 10^{-11}$ | 0 | | $2,5029 \cdot 10^{-9}$ |
| $10^{14}$ | $1,4513 \cdot 10^{-12}$ | 0 | | 0 |
| $10^{15}$ | $1,4566 \cdot 10^{-13}$ | 0 | | 0 |
| $10^{16}$ | $1,4211 \cdot 10^{-14}$ | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 9,21032432694012 | 1000,00992904408 | | 371976,9103422510 |
| $10^8$ | 9,21032577765533 | 1000,00992904491 | | 371976,9105933640 |
| $10^9$ | 9,21032592272685 | 1000,00992904500 | | 371976,9106184750 |
| $10^{10}$ | 9,21032593723401 | 1000,00992904501 | | 371976,9106209860 |
| $10^{11}$ | 9,21032593868472 | 1000,00992904501 | | 371976,9106212370 |
| $10^{12}$ | 9,21032593882979 | 1000,00992904501 | | 371976,9106212620 |
| $10^{13}$ | 9,21032593884430 | 1000,00992904501 | | 371976,9106212650 |
| $10^{14}$ | 9,21032593884575 | 1000,00992904501 | | 371976,9106212650 |
| $10^{15}$ | 9,21032593884590 | 1000,00992904501 | | 371976,9106212650 |
| $10^{16}$ | 9,21032593884591 | 1000,00992904501 | *b)* | 371976,9106212650 |

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 7,84050712993712 | 1000,00912888406 | 0,100000000000000 | 2002781,31911852 |
| $10^2$ | 9,05060615444427 | 1000,00983714003 | 0,099999999999999 | 2002993,65017758 |
| $10^3$ | 9,19375561457377 | 1000,00991970454 | 0,099999999999989 | 2003018,40248872 |
| $10^4$ | 9,20830849818162 | 1000,00992810913 | 0,099999999999856 | 2003020,92213354 |
| $10^5$ | 9,20976618582245 | 1000,00992895115 | 0,099999999998368 | 2003021,17456373 |
| $10^6$ | 9,20991210652369 | 1000,00992903552 | 0,100000000009585 | 2003021,19985815 |
| $10^7$ | 9,20992848275819 | 1000,00992904651 | 0,099999999922464 | 2003021,20324950 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | $1,4509 \cdot 10^{2}$ | $8,3796 \cdot 10^{-2}$ | | $2,5122 \cdot 10^{4}$ |
| $10^2$ | 1,2101 | $7,0826 \cdot 10^{-4}$ | $-8,0491 \cdot 10^{-16}$ | $2,1233 \cdot 10^{2}$ |
| $10^3$ | $1,4315 \cdot 10^{-1}$ | $8,2565 \cdot 10^{-5}$ | $-1,0700 \cdot 10^{-14}$ | $2,4752 \cdot 10^{1}$ |
| $10^4$ | $1,4553 \cdot 10^{-2}$ | $8,4046 \cdot 10^{-6}$ | $-1,3300 \cdot 10^{-13}$ | 2,5196 |
| $10^5$ | $1,4577 \cdot 10^{-3}$ | $8,4202 \cdot 10^{-7}$ | $-1,4880 \cdot 10^{-12}$ | $2,5243 \cdot 10^{-1}$ |
| $10^6$ | $1,4592 \cdot 10^{-4}$ | $8,4370 \cdot 10^{-8}$ | $1,1217 \cdot 10^{-11}$ | $2,5294 \cdot 10^{-2}$ |
| $10^7$ | $1,6376 \cdot 10^{-5}$ | $1,0990 \cdot 10^{-8}$ | $-8,7121 \cdot 10^{-11}$ | $3,3913 \cdot 10^{-3}$ |
| $10^8$ | $1,4509 \cdot 10^{-6}$ | $8,3799 \cdot 10^{-10}$ | | $2,5122 \cdot 10^{-4}$ |
| $10^9$ | $1,4509 \cdot 10^{-7}$ | $8,3787 \cdot 10^{-11}$ | | $2,5121 \cdot 10^{-5}$ |
| $10^{10}$ | $1,4509 \cdot 10^{-8}$ | $8,4128 \cdot 10^{-12}$ | | $2,5122 \cdot 10^{-6}$ |
| $10^{11}$ | $1,4509 \cdot 10^{-9}$ | 0 | | $2,5122 \cdot 10^{-7}$ |
| $10^{12}$ | $1,4509 \cdot 10^{-10}$ | 0 | | $2,5146 \cdot 10^{-8}$ |
| $10^{13}$ | $1,4509 \cdot 10^{-11}$ | 0 | | $2,5611 \cdot 10^{-9}$ |
| $10^{14}$ | $1,4513 \cdot 10^{-12}$ | 0 | | 0 |
| $10^{15}$ | $1,4566 \cdot 10^{-13}$ | 0 | | 0 |
| $10^{16}$ | $1,4211 \cdot 10^{-14}$ | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 9,20992661571773 | 1000,00992904390 | | 2003021,20237030 |
| $10^8$ | 9,20992806663713 | 1000,00992904474 | | 2003021,20262152 |
| $10^9$ | 9,20992821172907 | 1000,00992904482 | | 2003021,20264664 |
| $10^{10}$ | 9,20992822623827 | 1000,00992904483 | | 2003021,20264915 |
| $10^{11}$ | 9,20992822768919 | 1000,00992904483 | | 2003021,20264941 |
| $10^{12}$ | 9,20992822783428 | 1000,00992904483 | | 2003021,20264943 |
| $10^{13}$ | 9,20992822784879 | 1000,00992904483 | | 2003021,20264943 |
| $10^{14}$ | 9,20992822785024 | 1000,00992904483 | | 2003021,20264943 |
| $10^{15}$ | 9,20992822785039 | 1000,00992904483 | | 2003021,20264943 |
| $10^{16}$ | 9,20992822785040 | 1000,00992904483 | *c)* | 2003021,20264943 |

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 7,83212547315452 | 1000,00912888264 | 0,100000000000000 | 10008306,1716827 |
| $10^2$ | 9,04092953464169 | 1000,00983713831 | 0,099999999999999 | 10008518,6162407 |
| $10^3$ | 9,18392577772829 | 1000,00991970277 | 0,099999999999989 | 10008543,3817823 |
| $10^4$ | 9,19846309125933 | 1000,00992810738 | 0,099999999999856 | 10008545,9027777 |
| $10^5$ | 9,19991928923446 | 1000,00992894953 | 0,099999999998368 | 10008546,1553823 |
| $10^6$ | 9,20006556392400 | 1000,00992903452 | 0,100000000009585 | 10008546,1808763 |
| $10^7$ | 9,20008899787535 | 1000,00992905726 | 0,099999999922464 | 10008546,1876819 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | $1,4507 \cdot 10^{2}$ | $8,3954 \cdot 10^{-2}$ | | $2,5182 \cdot 10^{2}$ |
| $10^2$ | 1,2088 | $7,0826 \cdot 10^{-4}$ | $-8,0491 \cdot 10^{-16}$ | $2,1244 \cdot 10^{2}$ |
| $10^3$ | $1,4300 \cdot 10^{-1}$ | $8,2564 \cdot 10^{-5}$ | $-1,0700 \cdot 10^{-14}$ | $2,4766 \cdot 10^{1}$ |
| $10^4$ | $1,4537 \cdot 10^{-2}$ | $8,4046 \cdot 10^{-6}$ | $-1,3300 \cdot 10^{-13}$ | 2,5210 |
| $10^5$ | $1,4562 \cdot 10^{-3}$ | $8,4215 \cdot 10^{-7}$ | $-1,4880 \cdot 10^{-12}$ | $2,5260 \cdot 10^{-1}$ |
| $10^6$ | $1,4627 \cdot 10^{-4}$ | $8,4990 \cdot 10^{-8}$ | $1,1217 \cdot 10^{-11}$ | $2,5494 \cdot 10^{-2}$ |
| $10^7$ | $2,3434 \cdot 10^{-5}$ | $2,2740 \cdot 10^{-8}$ | $-8,7121 \cdot 10^{-11}$ | $6,8056 \cdot 10^{-3}$ |
| $10^8$ | $1,4507 \cdot 10^{-6}$ | $8,3958 \cdot 10^{-10}$ | | $2,5182 \cdot 10^{-4}$ |
| $10^9$ | $1,4507 \cdot 10^{-7}$ | $8,3901 \cdot 10^{-11}$ | | $2,5183 \cdot 10^{-5}$ |
| $10^{10}$ | $1,4507 \cdot 10^{-8}$ | $8,4128 \cdot 10^{-12}$ | | $2,5183 \cdot 10^{-6}$ |
| $10^{11}$ | $1,4507 \cdot 10^{-9}$ | 0 | | $2,5146 \cdot 10^{-7}$ |
| $10^{12}$ | $1,4507 \cdot 10^{-10}$ | 0 | | $2,6077 \cdot 10^{-8}$ |
| $10^{13}$ | $1,4506 \cdot 10^{-11}$ | 0 | | 0 |
| $10^{14}$ | $1,4513 \cdot 10^{-12}$ | 0 | | 0 |
| $10^{15}$ | $1,4566 \cdot 10^{-13}$ | 0 | | 0 |
| $10^{16}$ | $1,4211 \cdot 10^{-14}$ | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 9,20008007052064 | 1000,00992904292 | | 10008546,1833945 |
| $10^8$ | 9,20008152118030 | 1000,00992904376 | | 10008546,1836464 |
| $10^9$ | 9,20008166624627 | 1000,00992904384 | | 10008546,1836716 |
| $10^{10}$ | 9,20008168075286 | 1000,00992904385 | | 10008546,1836741 |
| $10^{11}$ | 9,20008168220352 | 1000,00992904385 | | 10008546,1836743 |
| $10^{12}$ | 9,20008168234859 | 1000,00992904385 | | 10008546,1836744 |
| $10^{13}$ | 9,20008168236309 | 1000,00992904385 | | 10008546,1836744 |
| $10^{14}$ | 9,20008168236455 | 1000,00992904385 | | 10008546,1836744 |
| $10^{15}$ | 9,20008168236469 | 1000,00992904385 | | 10008546,1836744 |
| $10^{16}$ | 9,20008168236471 | 1000,00992904385 | *d)* | 10008546,1836744 |

Tab. C.3: Calculation of relativistic rocket velocity according to program
Type: "A1", $v'_0 = -4\ \mathrm{km/s}$, $\Delta m_0 = 0.09\%$, $t_0 = 1000s$
a) $v_0 = 0$, b) $v_0 = 369\ \mathrm{km/s}$, c) $v_0 = 2000\ \mathrm{km/s}$, d) $v_0 = 10000\ \mathrm{km/s}$

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 196,021417688228 | 10002,2826214110 | 0,100000000000000 | 684187,711109235 |
| $10^2$ | 226,275235660056 | 10002,4597552575 | 0,099999999999999 | 737269,299206788 |
| $10^3$ | 229,854130642931 | 10002,4804055720 | 0,099999999999989 | 743457,239721413 |
| $10^4$ | 230,217968797944 | 10002,4825076705 | 0,099999999999856 | 744087,136710344 |
| $10^5$ | 230,254412155672 | 10002,4827182607 | 0,099999999998368 | 744150,240353761 |
| $10^6$ | 230,258057081830 | 10002,4827393235 | 0,100000000009585 | 744156,551855839 |
| $10^7$ | 230,258421713536 | 10002,4827414384 | 0,099999999922464 | 744157,183283767 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | $3,6266 \cdot 10^2$ | $2,0948 \cdot 10^1$ | | $6,2771 \cdot 10^6$ |
| $10^2$ | $3,0254 \cdot 10^1$ | $1,7713 \cdot 10^{-1}$ | $-8,0491 \cdot 10^{-16}$ | $5,3082 \cdot 10^4$ |
| $10^3$ | 3,5789 | $2,0650 \cdot 10^{-2}$ | $-1,0700 \cdot 10^{-14}$ | $6,1879 \cdot 10^3$ |
| $10^4$ | $3,6384 \cdot 10^{-1}$ | $2,1021 \cdot 10^{-3}$ | $-1,3300 \cdot 10^{-13}$ | $6,2990 \cdot 10^2$ |
| $10^5$ | $3,6443 \cdot 10^{-2}$ | $2,1059 \cdot 10^{-4}$ | $-1,4880 \cdot 10^{-12}$ | $6,3104 \cdot 10^1$ |
| $10^6$ | $3,6449 \cdot 10^{-3}$ | $2,1063 \cdot 10^{-5}$ | $1,1217 \cdot 10^{-11}$ | 6,3115 |
| $10^7$ | $3,6463 \cdot 10^{-4}$ | $2,1149 \cdot 10^{-6}$ | $-8,7121 \cdot 10^{-11}$ | $6,3143 \cdot 10^{-1}$ |
| $10^8$ | $3,6266 \cdot 10^{-5}$ | $2,0948 \cdot 10^{-7}$ | | $6,2772 \cdot 10^{-2}$ |
| $10^9$ | $3,6266 \cdot 10^{-6}$ | $2,0947 \cdot 10^{-8}$ | | $6,2772 \cdot 10^{-3}$ |
| $10^{10}$ | $3,6266 \cdot 10^{-7}$ | $2,0955 \cdot 10^{-9}$ | | $6,2772 \cdot 10^{-4}$ |
| $10^{11}$ | $3,6266 \cdot 10^{-8}$ | $2,0918 \cdot 10^{-10}$ | | $6,2772 \cdot 10^{-5}$ |
| $10^{12}$ | $3,6266 \cdot 10^{-9}$ | $2,1828 \cdot 10^{-11}$ | | $6,2772 \cdot 10^{-6}$ |
| $10^{13}$ | $3,6266 \cdot 10^{-10}$ | 0 | | $6,2771 \cdot 10^{-7}$ |
| $10^{14}$ | $3,6266 \cdot 10^{-11}$ | 0 | | $6,2748 \cdot 10^{-8}$ |
| $10^{15}$ | $3,6380 \cdot 10^{-12}$ | 0 | | $6,2864 \cdot 10^{-9}$ |
| $10^{16}$ | $3,6948 \cdot 10^{-13}$ | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 230,258419745292 | 10002,4827414183 | | 744157,179575260 |
| $10^8$ | 230,258456011638 | 10002,4827416278 | | 744157,242347202 |
| $10^9$ | 230,258459638272 | 10002,4827416488 | | 744157,248624396 |
| $10^{10}$ | 230,258460000936 | 10002,4827416509 | | 744157,249252115 |
| $10^{11}$ | 230,258460037202 | 10002,4827416511 | | 744157,249314887 |
| $10^{12}$ | 230,258460040829 | 10002,4827416511 | | 744157,249321164 |
| $10^{13}$ | 230,258460041192 | 10002,4827416511 | | 744157,249321792 |
| $10^{14}$ | 230,258460041228 | 10002,4827416511 | | 744157,249321855 |
| $10^{15}$ | 230,258460041231 | 10002,4827416511 | *a)* | 744157,249321861 |
| $10^{16}$ | 230,258460041232 | 10002,4827416511 | | 744157,249321862 |

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 196,020933328363 | 10002,2826210019 | 0,100000000000000 | 4374191,05576218 |
| $10^2$ | 226,274637774396 | 10002,4597547615 | 0,099999999999999 | 4427272,68489423 |
| $10^3$ | 229,853518480855 | 10002,4804050636 | 0,099999999999989 | 4433460,62991467 |
| $10^4$ | 230,217355174200 | 10002,4825071609 | 0,099999999999856 | 4434090,52736042 |
| $10^5$ | 230,253798388818 | 10002,4827177511 | 0,099999999998368 | 4434153,63106899 |
| $10^6$ | 230,257443325741 | 10002,4827388142 | 0,100000000009585 | 4434159,94267449 |
| $10^7$ | 230,257808300206 | 10002,4827409268 | 0,099999999922464 | 4434160,57580526 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | $3,6266 \cdot 10^2$ | $2,0948 \cdot 10^1$ | | $6,2772 \cdot 10^6$ |
| $10^2$ | $3,0254 \cdot 10^1$ | $1,7713 \cdot 10^{-1}$ | $-8,0491 \cdot 10^{-16}$ | $5,3082 \cdot 10^4$ |
| $10^3$ | 3,5789 | $2,0650 \cdot 10^{-2}$ | $-1,0700 \cdot 10^{-14}$ | $6,1879 \cdot 10^3$ |
| $10^4$ | $3,6384 \cdot 10^{-1}$ | $2,1021 \cdot 10^{-3}$ | $-1,3300 \cdot 10^{-13}$ | $6,2990 \cdot 10^2$ |
| $10^5$ | $3,6443 \cdot 10^{-2}$ | $2,1059 \cdot 10^{-4}$ | $-1,4880 \cdot 10^{-12}$ | $6,3104 \cdot 10^1$ |
| $10^6$ | $3,6449 \cdot 10^{-3}$ | $2,1063 \cdot 10^{-5}$ | $1,1217 \cdot 10^{-11}$ | 6,3116 |
| $10^7$ | $3,6497 \cdot 10^{-4}$ | $2,1126 \cdot 10^{-6}$ | $-8,7121 \cdot 10^{-11}$ | $6,3313 \cdot 10^{-1}$ |
| $10^8$ | $3,6266 \cdot 10^{-5}$ | $2,0948 \cdot 10^{-7}$ | | $6,2772 \cdot 10^{-2}$ |
| $10^9$ | $3,6266 \cdot 10^{-6}$ | $2,0947 \cdot 10^{-8}$ | | $6,2772 \cdot 10^{-3}$ |
| $10^{10}$ | $3,6266 \cdot 10^{-7}$ | $2,0955 \cdot 10^{-9}$ | | $6,2772 \cdot 10^{-4}$ |
| $10^{11}$ | $3,6266 \cdot 10^{-8}$ | $2,0918 \cdot 10^{-10}$ | | $6,2772 \cdot 10^{-5}$ |
| $10^{12}$ | $3,6266 \cdot 10^{-9}$ | $2,1828 \cdot 10^{-11}$ | | $6,2771 \cdot 10^{-6}$ |
| $10^{13}$ | $3,6266 \cdot 10^{-10}$ | 0 | | $6,2771 \cdot 10^{-7}$ |
| $10^{14}$ | $3,6266 \cdot 10^{-11}$ | 0 | | $6,2399 \cdot 10^{-8}$ |
| $10^{15}$ | $3,6380 \cdot 10^{-12}$ | 0 | | 0 |
| $10^{16}$ | $3,6948 \cdot 10^{-13}$ | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 230,257805988392 | 10002,4827409090 | | 4434160,57039689 |
| $10^8$ | 230,257842254657 | 10002,4827411185 | | 4434160,63316912 |
| $10^9$ | 230,257845881283 | 10002,4827411395 | | 4434160,63944635 |
| $10^{10}$ | 230,257846243946 | 10002,4827411416 | | 4434160,64007407 |
| $10^{11}$ | 230,257846280212 | 10002,4827411418 | | 4434160,64013684 |
| $10^{12}$ | 230,257846283839 | 10002,4827411418 | | 4434160,64014312 |
| $10^{13}$ | 230,257846284201 | 10002,4827411418 | | 4434160,64014375 |
| $10^{14}$ | 230,257846284238 | 10002,4827411418 | | 4434160,64014381 |
| $10^{15}$ | 230,257846284241 | 10002,4827411418 | *b)* | 4434160,64014382 |
| $10^{16}$ | 230,257846284242 | 10002,4827411418 | | 4434160,64014382 |

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 196,011677951838 | 10002,2826191942 | 0,100000000000000 | 20684648,1809011 |
| $10^2$ | 226,263782574663 | 10002,4597525694 | 0,099999999999999 | 20737730,9547342 |
| $10^3$ | 229,842470286295 | 10002,4804028168 | 0,099999999999989 | 20743919,0319696 |
| $10^4$ | 230,206287313833 | 10002,4825049083 | 0,099999999999856 | 20744548,9428655 |
| $10^5$ | 230,242728572615 | 10002,4827154982 | 0,099999999998368 | 20744612,0480015 |
| $10^6$ | 230,246373417548 | 10002,4827365626 | 0,100000000009585 | 20744618,3601214 |
| $10^7$ | 230,246739825310 | 10002,4827386978 | 0,099999999922464 | 20744619,0002277 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | $3,6264 \cdot 10^2$ | $2,0949 \cdot 10^1$ | | $6,2775 \cdot 10^6$ |
| $10^2$ | $3,0252 \cdot 10^1$ | $1,7713 \cdot 10^{-1}$ | $-8,0491 \cdot 10^{-16}$ | $5,3083 \cdot 10^4$ |
| $10^3$ | 3,5787 | $2,0650 \cdot 10^{-2}$ | $-1,0700 \cdot 10^{-14}$ | $6,1881 \cdot 10^3$ |
| $10^4$ | $3,6382 \cdot 10^{-1}$ | $2,1021 \cdot 10^{-3}$ | $-1,3300 \cdot 10^{-13}$ | $6,2991 \cdot 10^2$ |
| $10^5$ | $3,6441 \cdot 10^{-2}$ | $2,1059 \cdot 10^{-4}$ | $-1,4880 \cdot 10^{-12}$ | $6,3105 \cdot 10^1$ |
| $10^6$ | $3,6448 \cdot 10^{-3}$ | $2,1064 \cdot 10^{-5}$ | $1,1217 \cdot 10^{-11}$ | 6,3121 |
| $10^7$ | $3,6641 \cdot 10^{-4}$ | $2,1352 \cdot 10^{-6}$ | $-8,7121 \cdot 10^{-11}$ | $6,4011 \cdot 10^{-1}$ |
| $10^8$ | $3,6265 \cdot 10^{-5}$ | $2,0949 \cdot 10^{-7}$ | | $6,2775 \cdot 10^{-2}$ |
| $10^9$ | $3,6265 \cdot 10^{-6}$ | $2,0949 \cdot 10^{-8}$ | | $6,2775 \cdot 10^{-3}$ |
| $10^{10}$ | $3,6265 \cdot 10^{-7}$ | $2,0955 \cdot 10^{-9}$ | | $6,2774 \cdot 10^{-4}$ |
| $10^{11}$ | $3,6265 \cdot 10^{-8}$ | $2,0918 \cdot 10^{-10}$ | | $6,2775 \cdot 10^{-5}$ |
| $10^{12}$ | $3,6265 \cdot 10^{-9}$ | $2,1828 \cdot 10^{-11}$ | | $6,2771 \cdot 10^{-6}$ |
| $10^{13}$ | $3,6263 \cdot 10^{-10}$ | 0 | | $6,2957 \cdot 10^{-7}$ |
| $10^{14}$ | $3,6266 \cdot 10^{-11}$ | 0 | | $6,3330 \cdot 10^{-8}$ |
| $10^{15}$ | $3,6380 \cdot 10^{-12}$ | 0 | | 0 |
| $10^{16}$ | $3,6948 \cdot 10^{-13}$ | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 230,246736063268 | 10002,4827386575 | | 20744618,9878669 |
| $10^8$ | 230,246772327840 | 10002,4827388670 | | 20744619,0506414 |
| $10^9$ | 230,246775954297 | 10002,4827388879 | | 20744619,0569189 |
| $10^{10}$ | 230,246776316943 | 10002,4827388900 | | 20744619,0575466 |
| $10^{11}$ | 230,246776353207 | 10002,4827388902 | | 20744619,0576094 |
| $10^{12}$ | 230,246776356834 | 10002,4827388902 | | 20744619,0576157 |
| $10^{13}$ | 230,246776357197 | 10002,4827388902 | | 20744619,0576163 |
| $10^{14}$ | 230,246776357233 | 10002,4827388902 | | 20744619,0576164 |
| $10^{15}$ | 230,246776357236 | 10002,4827388902 | *c)* | 20744619,0576164 |
| $10^{16}$ | 230,246776357237 | 10002,4827388902 | | 20744619,0576164 |

| N | $v_T$ | $t_T$ | $m_N$ | $x_N$ |
|---|---|---|---|---|
| 10 | 195,798241945411 | 10002,2826103399 | 0,100000000000000 | 100740248,476771 |
| $10^2$ | 226,016565669348 | 10002,4597418176 | 0,099999999999999 | 100793359,642360 |
| $10^3$ | 229,591238915968 | 10002,4803917961 | 0,099999999999989 | 100799551,023330 |
| $10^4$ | 229,954647601144 | 10002,4824938598 | 0,099999999999856 | 100800181,270485 |
| $10^5$ | 229,991048026237 | 10002,4827044481 | 0,099999999998368 | 100800244,409694 |
| $10^6$ | 229,994689292966 | 10002,4827255184 | 0,100000000009585 | 100800250,727051 |
| $10^7$ | 229,995062373247 | 10002,4827277682 | 0,099999999922464 | 100800251,401665 |
| | $\delta v_T$ | $\delta t_T$ | $\delta m_N$ | $\delta x_N$ |
| $\bar{x}$ | $3,6225 \cdot 10^2$ | $2,0950 \cdot 10^1$ | | $6,2812 \cdot 10^6$ |
| $10^2$ | $3,0218 \cdot 10^1$ | $1,7713 \cdot 10^{-1}$ | $-8,0491 \cdot 10^{-16}$ | $5,3111 \cdot 10^4$ |
| $10^3$ | 3,5747 | $2,0650 \cdot 10^{-2}$ | $-1,0700 \cdot 10^{-14}$ | $6,1914 \cdot 10^3$ |
| $10^4$ | $3,6341 \cdot 10^{-1}$ | $2,1021 \cdot 10^{-3}$ | $-1,3300 \cdot 10^{-13}$ | $6,3025 \cdot 10^2$ |
| $10^5$ | $3,6400 \cdot 10^{-2}$ | $2,1059 \cdot 10^{-4}$ | $-1,4880 \cdot 10^{-12}$ | $6,3139 \cdot 10^1$ |
| $10^6$ | $3,6413 \cdot 10^{-3}$ | $2,1070 \cdot 10^{-5}$ | $1,1217 \cdot 10^{-11}$ | 6,3174 |
| $10^7$ | $3,7308 \cdot 10^{-4}$ | $2,2498 \cdot 10^{-6}$ | $-8,7121 \cdot 10^{-11}$ | $6,7461 \cdot 10^{-1}$ |
| $10^8$ | $3,6225 \cdot 10^{-5}$ | $2,0950 \cdot 10^{-7}$ | | $6,2813 \cdot 10^{-2}$ |
| $10^9$ | $3,6225 \cdot 10^{-6}$ | $2,0949 \cdot 10^{-8}$ | | $6,2813 \cdot 10^{-3}$ |
| $10^{10}$ | $3,6225 \cdot 10^{-7}$ | $2,0955 \cdot 10^{-9}$ | | $6,2813 \cdot 10^{-4}$ |
| $10^{11}$ | $3,6225 \cdot 10^{-8}$ | $2,0918 \cdot 10^{-10}$ | | $6,2808 \cdot 10^{-5}$ |
| $10^{12}$ | $3,6225 \cdot 10^{-9}$ | $2,1828 \cdot 10^{-11}$ | | $6,2883 \cdot 10^{-6}$ |
| $10^{13}$ | $3,6226 \cdot 10^{-10}$ | 0 | | $6,2585 \cdot 10^{-7}$ |
| $10^{14}$ | $3,6238 \cdot 10^{-11}$ | 0 | | 0 |
| $10^{15}$ | $3,6096 \cdot 10^{-12}$ | 0 | | 0 |
| $10^{16}$ | $3,6948 \cdot 10^{-13}$ | 0 | | 0 |
| | $v_T$ | $t_T$ | | $x_N$ |
| $10^7$ | 229,994689292966 | 10002,4827276134 | | 100800251,355179 |
| $10^8$ | 229,994725518139 | 10002,4827278229 | | 100800251,417992 |
| $10^9$ | 229,994729140657 | 10002,4827278438 | | 100800251,424273 |
| $10^{10}$ | 229,994729502908 | 10002,4827278459 | | 100800251,424902 |
| $10^{11}$ | 229,994729539134 | 10002,4827278461 | | 100800251,424964 |
| $10^{12}$ | 229,994729542756 | 10002,4827278462 | | 100800251,424971 |
| $10^{13}$ | 229,994729543118 | 10002,4827278462 | | 100800251,424971 |
| $10^{14}$ | 229,994729543155 | 10002,4827278462 | | 100800251,424971 |
| $10^{15}$ | 229,994729543158 | 10002,4827278462 | *d)* | 100800251,424971 |
| $10^{16}$ | 229,994729543159 | 10002,4827278462 | | 100800251,424971 |

Tab. C.4: Calculation of relativistic rocket velocity according to program
Type: “A1”, $v_0' = -100\ \mathrm{km/s}$, $\Delta m_0 = 0.009\%$, $t_0 = 10000 s$
a) $v_0 = 0$, b) $v_0 = 369\ \mathrm{km/s}$, c) $v_0 = 2000\ \mathrm{km/s}$, d) $v_0 = 10000\ \mathrm{km/s}$

## C.4 Relativistic rocket equation according to J. Akeret

Since 1946 there is an analytical solution for the relativistic rocket equation by J. Akeret [90]. For this not only the momentum theorem and the relativistic velocity addition are necessary (as with the numerical derivation presented so far) but additionally the energy conservation theorem is used.

For the derivation of the equations, formula symbols are used which differ from the original text but are consistent with the representations used so far in this presentation. Functions related to the outflowing gas used for causing thrust are denoted by $f'$; relations referring to the moving rocket, on the other hand, are shown without this label. The actual mass of the rocket is $m$, and $dm'$ is the fraction of the propellant gas. This gives rise to the equations shown below.

a) The energy theorem provides:

$$d\left\{\frac{mc^2}{\sqrt{1-v^2/c^2}}\right\} = -\frac{dm' \cdot c^2}{\sqrt{1-v'^2/c^2}} \tag{C.21}$$

b) the relation for momentum:

$$d\left\{\frac{mv}{\sqrt{1-v^2/c^2}}\right\} = \frac{dm' \cdot v'}{\sqrt{1-v'^2/c^2}} \tag{C.22}$$

c) the relativistic addition theorem:

$$v' = \frac{v_0' - v}{1 - \frac{v \cdot v_0'}{c^2}} \tag{C.23}$$

where $v_0'$ has the meaning of the (constant) exit velocity of the gas relative to the rocket. The equations (C.21) and (C.22) can be further developed to

$$dm\frac{c^2}{\sqrt{1-v^2/c^2}} + mc^2 \cdot d\left\{\frac{1}{\sqrt{1-v^2/c^2}}\right\} = -dm'\frac{c^2}{\sqrt{1-v'^2/c^2}} \tag{C.24}$$

$$dm\frac{v}{\sqrt{1-v^2/c^2}} + m\frac{dv}{\sqrt{1-v^2/c^2}} + mv \cdot d\left\{\frac{1}{\sqrt{1-v^2/c^2}}\right\} = dm'\frac{v'}{\sqrt{1-v'^2/c^2}} \tag{C.25}$$

For the solution, the values of $v'$ and $dm'$ must be eliminated. To do this, first in equation (C.24) in the term on the right-hand side the value for $v'$ from equation (C.23) is inserted

$$\frac{c^2}{\sqrt{1-\frac{v'^2}{c^2}}} = \frac{c^2}{\sqrt{1-\frac{\left(\frac{v_0'-v}{1-v\cdot v_0'/c^2}\right)^2}{c^2}}}$$

$$= \frac{c^2 - v_0'v}{\sqrt{1-\frac{v^2}{c^2}-\frac{v_0'^2}{c^2}+\frac{v^2v_0'^2}{c^4}}} = \frac{c^2 - v_0'v}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{v_0'^2}{c^2}}} \tag{C.26}$$

In the same way follows

$$\frac{v'}{\sqrt{1-\frac{v'^2}{c^2}}} = \frac{v_0' - v}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{v_0'^2}{c^2}}} \tag{C.27}$$

Equations (C.26) and (C.27) are substituted into Eq. (C.24) and (C.25), respectively, and these are resolved to $dm'$ and equated. The result is:

$$m\left\{\frac{c^2 - vv_0'}{\sqrt{1-v^2/c^2}}\right\}dv + mv_0'(c^2 - v^2)\cdot d\left\{\frac{1}{\sqrt{1-v^2/c^2}}\right\} + dm\frac{v_0'(c^2-v^2)}{\sqrt{1-v^2/c^2}} = 0 \tag{C.28}$$

The two differentials with the dependence on $v$ must be unified and using the differential chain rule it follows

$$d\left\{\frac{1}{\sqrt{1-v^2/c^2}}\right\} = \frac{v}{c^2\left\{1-\frac{v^2}{c^2}\right\}^{3/2}}\,dv \tag{C.29}$$

After substituting in eq. (C.28) and separating the terms for mass and velocity, the final result is

$$\frac{dm}{m} = -\,\frac{dv}{v_0'(1 - v^2/c^2)} \tag{C.30}$$

The integration results in

$$ln(m) = -\,\frac{c}{2v_0'}\,ln\left\{\frac{c+v}{c-v}\right\} + C \tag{C.31}$$

With the initial value for mass $m_0$ and the final value $m$ the relativistic rocket equation according to J. Akeret arises

$$\frac{m}{m_0} = \left\{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right\}^{c/2v_0'} \tag{C.32}$$

or

$$\frac{v}{c} = \frac{1-\left(\frac{m}{m_0}\right)^{2v_0'/c}}{1+\left(\frac{m}{m_0}\right)^{2v_0'/c}} \tag{C.33}$$

In Section 6.4.2, calculations from this equation are contrasted with the classical rocket formula of K. E. Tsiolkovsky and the numerical relations derived in this annex.

# Annex D: Calculation of momentum for relativistic non-elastic collision

During ideal non-elastic, i.e. plastic collision 2 masses hit each other in central position and are moving forward as a combined body without rotation. An approximation procedure is developed to calculate the end-velocity of this body on basis of the principle of conservation of momentum, in a case where the validity of equation $m_3 = m_1 + m_2$ is postulated. This approach is relevant for theoretical analysis only because it can be shown, that in real cases an additional increase of mass $\Delta m_3$ because the conversion of potential energy into mass must be considered. For details it is referred to chapter 7.1.

In addition, the appearing simple equation makes it possible to perform a comparison between the approximation procedures recursion, Newton's calculus, and bijection. The latter proved to be superior to the others because it is the only calculation to cover all possible input values and is therefore used also in other calculations in Annex A – C.

Respecting the above-mentioned restrictions, for the relativistic momentum using relation $m_3 = m_1 + m_2$ referring to Eq. (7.01)

$$p_0 = m_1\gamma_1 v_1 + m_2\gamma_2 v_2 = (m_1 + m_2)\gamma_3 v_3 \qquad \text{(D. 01)}$$

applies, where $v_3$ can be calculated on basis of numerical approximation. In the following different procedures will be presented and the results are compared.

## D.1 Recursion procedure

The procedure with the smallest mathematical effort is the procedure using simple recursion. The equation for the development can be derived directly using Eq. (D.01) and shows the form

$$\frac{(v_3)_{k+1}}{c} = \frac{p_0}{c(m_1 + m_2)\gamma_{3k}} = \frac{p_0}{c(m_1 + m_2)}\sqrt{1 - \left(\frac{(v_3)_k}{c}\right)^2} \qquad \text{(D. 02)}$$

## D.2 Procedure according to Newton's calculus

Iteration according to Newton' calculus is generally using the sequence

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \qquad \text{(D. 03)}$$

When Eq. (D.01) is converted it applies first

$$\frac{m_1\gamma_1 v_1 + m_2\gamma_2 v_2}{m_1 + m_2} - \gamma_3 v_3 = 0 = f(v_3) \qquad \text{(D. 04)}$$

and then

$$f\left(\frac{v_3}{c}\right) = \frac{p_0}{c(m_1 + m_2)} - \frac{v_3}{c}\left(1 - \frac{{v_3}^2}{c^2}\right)^{-1/2} \tag{D.05}$$

Using

$$x = \frac{v_3}{c} \tag{D.06}$$

it yields

$$f(x) = \frac{p_0}{(m_1 + m_2)} - x(1 - x^2)^{-1/2} \tag{D.07}$$

and

$$f'(x) = -(1 - x^2)^{-3/2} \tag{D.08}$$

After inserting the result in Eq. (D.03) the iteration formula is finally

$$\frac{(v_3)_{k+1}}{c} = \frac{(v_3)_k}{c} + \frac{\frac{p_0}{(m_1 + m_2)} - (v_3)_k\left[1 - \left(\frac{(v_3)_k}{c}\right)^2\right]^{-1/2}}{c\left[1 - \left(\frac{(v_3)_k}{c}\right)^2\right]^{-3/2}} \tag{D.09}$$

## D.3 Bisection method

First the starting function is defined using Eq. (D.01)

$$f(v_3) = \gamma_3 v_3 = \frac{p_0}{(m_1 + m_2)} \tag{D.10}$$

where the value for $p_0$ is defined by the initial starting conditions. For the beginning of the calculation appropriate values for $(v_{3+})_0$ and $(v_{3-})_0$ are determined which are following the conditions

$$f(v_{3+})_0 > \frac{p_0}{(m_1 + m_2)} \tag{D.11}$$

and

$$f(v_{3-})_0 < \frac{p_0}{(m_1 + m_2)} \tag{D.12}$$

In the interval $[(v_{3-})_0; (v_{3+})_0]$ the function $f(v_3)$ must be continuous and differentiable, and further $f'(v_3) \neq 0$ is required, which means, that in the chosen interval no minima and maxima are allowed, because otherwise no exact solution exists. Then the mean value is formed

$$(v_3)_1 = \frac{(v_{3+})_0 + (v_{3-})_0}{2} \tag{D.13}$$

and $f(v_3)_1$ is calculated according to (D.10). The following equations apply:

$$f(v_3)_1 > \frac{p_0}{(m_1 + m_2)} :\Rightarrow (v_{3+})_1 = (v_3)_1 \; und \; (v_{3-})_1 = (v_{3-})_0 \tag{D.14}$$

$$f(v_3)_1 < \frac{p_0}{(m_1 + m_2)} :\Rightarrow (v_{3+})_1 = (v_{3+})_0 \; und \; (v_{3-})_1 = (v_3)_1 \tag{D.15}$$

The calculation is repeated with increasing index 1 to n until the required accuracy is achieved. Every step of the calculation is generating a bisection of the difference between $v_{3+}$ and $v_{3-}$. A standard calculation program (e.g. Microsoft Excel©) with the utilization of 15 digits is therefore requiring, because of the general estimation

$$2^{10} = 1024 \approx 10^3 \qquad (D.16)$$

following

$$10^{15} \approx 2^{50} \qquad (D.17)$$

the use of approximately 50 steps to reach maximum possible accuracy; in practice a utilization of 60 proved to be safe in any case. Because of the boundary condition $v_1 = 0$ the starting values can be determined easily and are $(v_{3-})_0 = 0$ resp. $(v_{3+})_0 = v_2$.

## D.4 Evaluation

In the following results for the discussed procedures are presented using different values for the velocities. According to the considerations in chapter 7.1 only cases will be viewed, where the masses are equal and one of the selected velocities (here $v_1$) is equal to zero. All iteration methods lead to the same values; the procedures using simple recursion and according to Newton share the advantage, that they converge very quickly for small values of $v/c$. However, as a drawback the convergence is reducing for increasing $v/c$ and starting with $p_0 = \gamma_2 v_2 \geq 2c$ (for $m_1 = m_2 = 1$) calculations are no longer possible. Bisection, however, shows a much better performance and is above approx. $v_2/c > 0{,}895$ the only remaining procedure which is still working.

In the following table examples for calculations with different conditions are presented. In all cases it is marked, from which iteration step on no differences between consecutive steps can be detected and so the procedure has reached its end (Status “x” in field “St”). If one of the procedures is not converging, then it is marked as “not ok” in the evaluation field. Further on the differences to the results of the relativistic addition of velocities $v_{3,Rel}$ are presented as percentage-value.

For the calculation, the following equations are used:

$$\frac{v_{3,Rel}}{c} = \frac{1 - \sqrt{1 - \left(\frac{v_2}{c}\right)^2}}{\frac{v_2}{c}} \qquad \gamma_{3,Rel} = \frac{1}{\sqrt{1 - \left(\frac{v_{3,Rel}}{c}\right)^2}} \qquad \frac{p_0}{c} = m_2 \gamma_2 \frac{v_2}{c}$$

Recursion:
$$\frac{(v_3)_{k+1}}{c} = \frac{p_0}{c(m_1 + m_2)} \sqrt{1 - \left(\frac{(v_3)_k}{c}\right)^2}$$

Newton

$$\frac{(v_3)_{k+1}}{c} = \frac{(v_3)_k}{c} + \left\{ \frac{p_0}{c(m_1 + m_2)} - \frac{(v_3)_k}{c} \left[1 - \left(\frac{(v_3)_k}{c}\right)^2\right]^{-1/2} \right\} \left[1 - \left(\frac{(v_3)_k}{c}\right)^2\right]^{3/2}$$

Bisection: $\frac{(v_3)_{k+1}}{c} = \frac{(v_{3-})_k + (v_{3+})_k}{2c}$

Condition $f(v_3)_{k+1} > \frac{p_0}{(m_1 + m_2)} :\Rightarrow (v_{3+})_{k+1} = (v_3)_{k+1}\ and\ (v_{3-})_{k+1} = (v_{3-})_k$

Condition $f(v_3)_{k+1} < \frac{p_0}{(m_1 + m_2)} :\Rightarrow (v_{3+})_{k+1} = (v_{3+})_k\ and\ (v_{3-})_{k+1} = (v_3)_{k+1}$

Appropriate starting values: For $\frac{(v_{3-})_0}{c} = -\frac{v_1}{c}$ and for $\frac{(v_{3+})_0}{c} = \frac{v_1}{c}$

Values in the fields for results (blue color): For Recursion, Newton and Bisection the last values of iteration.

$\frac{v_3}{v_{3,Rel}} - 1$ Comparison of results. Chosen was bisection ($v_3$) and relativistic addition of velocities ($v_{3,Rel}$)

For the presented calculations, the following values apply:

| Tab. D.1 | Tab D.2 | Tab D.3 |
|---|---|---|
| $m_1 = 1;\ m_2 = 1$<br>$v_1 = 0\ ;\ v_2 = 0{,}1c$ | $m_1 = 1;\ m_2 = 1$<br>$v_1 = 0\ ;\ v_2 = 0{,}8c$ | $m_1 = 1{:}\ m_2 = 1$<br>$v_1 = 0\ ;\ v_2 = 0{,}89c$ |

**Codes for calculation:**

| Coordinate | | Code |
|---|---|---|
| G1 | = | (1-SQRT(1-B2*B2))/B2 |
| G2 | = | 1/SQRT(1-G1*G1) |
| B3 | = | B2/SQRT(1-B2*B2) |
| B5 | = | IF(B6="ok";B70;"") |
| D5 | = | IF(D6="ok";D70;"") |
| F5 | = | IF(F6="ok";F70;"") |
| H5 | = | F5/G1-1 |
| B6 | = | IF(C70="";"not ok";"ok") |
| D6 | = | IF(E70="";"not ok";"ok") |
| F6 | = | IF(G70="";"not ok";"ok") |
| B8 | = | B70/D70-1 |
| D8 | = | D70/F70-1 |
| F8 | = | F70/B70-1 |
| G10 | = | B1 |
| H10 | = | B2 |
| B11 | = | B$3/(1+D$2)*SQRT(1-B10*B10) |
| C11 | = | IF(B11=B10);"x";"") |
| D11 | = | D10+(B$3/(1+D$2)-D10*(1-D10*D10)^-(1/2))*((1-D10*D10)^(3/2)) |
| E11 | = | IF(D11=D10);"x";"") |
| F11 | = | (G10+H10)/2 |
| G11 | = | IF(F11/SQRT(1-F11*F11)<B$3/(1+D$2);F11;G10) |
| H11 | = | IF(F11/SQRT(1-F11*F11)<B$3/(1+D$2);H10;F11) |
| I11 | = | IF(F11=F10;"x";"") |

The codes B11 to I11 to be copied as far as B70 to I70

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $v_1$/c = | 0 | | $m_2/m_1$= | | $v_{3,Rel}$/c = | 0,05012563 | | |
| 2 | $v_2$/c = | 0,1 | | 1 | | $\gamma_{3,Rel}$ = | 1,00125866 | | |
| 3 | $p_0$/c = | 0,1005037815 | | | | | | | |
| 4 | | Recursion | | Newton | | Bisection | | | |
| 5 | $v_3$/c = | 0,0501885613 | | 0,0501885613 | | 0,0501885613 | $v_3/v_{3,Rel}$ = | 0,1% | |
| 6 | | nicht ok | | ok | | ok | | | |
| 7 | | Recursion/Newton | | Newton/Bisection | | Bisection/Recursion | | | |
| 8 | | 0,0E+00 | | 7,1E-15 | | -7,2E-15 | | | |
| 9 | k | $v_3$/c | St | $v_3$/c | St | $(v_{3-}+v_{3+})/2c$ | $v_{3-}$/c | $v_{3+}$/c | St |
| 10 | 0 | 0 | | 0 | | | 0 | 0,1 | |
| 11 | 1 | 0,0502518908 | | 0,0502518908 | | 0,0500000000 | 0,0500000000 | 0,1000000000 | |
| 12 | 2 | 0,0501884013 | | 0,0501885616 | | 0,0750000000 | 0,0500000000 | 0,0750000000 | |
| 13 | 3 | 0,0501885617 | | 0,0501885613 | | 0,0625000000 | 0,0500000000 | 0,0625000000 | |
| 14 | 4 | 0,0501885613 | | 0,0501885613 | x | 0,0562500000 | 0,0500000000 | 0,0562500000 | |
| 15 | 5 | 0,0501885613 | | 0,0501885613 | x | 0,0531250000 | 0,0500000000 | 0,0531250000 | |
| 16 | 6 | 0,0501885613 | | 0,0501885613 | x | 0,0515625000 | 0,0500000000 | 0,0515625000 | |
| 17 | 7 | 0,0501885613 | x | 0,0501885613 | x | 0,0507812500 | 0,0500000000 | 0,0507812500 | |
| 18 | 8 | 0,0501885613 | x | 0,0501885613 | x | 0,0503906250 | 0,0500000000 | 0,0503906250 | |
| 19 | 9 | 0,0501885613 | x | 0,0501885613 | x | 0,0501953125 | 0,0500000000 | 0,0501953125 | |
| 20 | 10 | 0,0501885613 | x | 0,0501885613 | x | 0,0500976563 | 0,0500976563 | 0,0501953125 | |
| 21 | 11 | 0,0501885613 | x | 0,0501885613 | x | 0,0501464844 | 0,0501464844 | 0,0501953125 | |
| 22 | 12 | 0,0501885613 | x | 0,0501885613 | x | 0,0501708984 | 0,0501708984 | 0,0501953125 | |
| 23 | 13 | 0,0501885613 | x | 0,0501885613 | x | 0,0501831055 | 0,0501831055 | 0,0501953125 | |
| 24 | 14 | 0,0501885613 | x | 0,0501885613 | x | 0,0501892090 | 0,0501831055 | 0,0501892090 | |
| 25 | 15 | 0,0501885613 | x | 0,0501885613 | x | 0,0501861572 | 0,0501861572 | 0,0501892090 | |
| 26 | 16 | 0,0501885613 | x | 0,0501885613 | x | 0,0501876831 | 0,0501876831 | 0,0501892090 | |
| 27 | 17 | 0,0501885613 | x | 0,0501885613 | x | 0,0501884460 | 0,0501884460 | 0,0501892090 | |
| 28 | 18 | 0,0501885613 | x | 0,0501885613 | x | 0,0501888275 | 0,0501884460 | 0,0501888275 | |
| 29 | 19 | 0,0501885613 | x | 0,0501885613 | x | 0,0501886368 | 0,0501884460 | 0,0501886368 | |
| 30 | 20 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885414 | 0,0501885414 | 0,0501886368 | |
| 31 | 21 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885891 | 0,0501885414 | 0,0501885891 | |
| 32 | 22 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885653 | 0,0501885414 | 0,0501885653 | |
| 33 | 23 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885533 | 0,0501885533 | 0,0501885653 | |
| 34 | 24 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885593 | 0,0501885593 | 0,0501885653 | |
| 35 | 25 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885623 | 0,0501885593 | 0,0501885623 | |
| 36 | 26 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885608 | 0,0501885608 | 0,0501885623 | |
| 37 | 27 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885615 | 0,0501885608 | 0,0501885615 | |
| 38 | 28 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885612 | 0,0501885612 | 0,0501885615 | |
| 39 | 29 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885613 | 0,0501885612 | 0,0501885613 | |
| 40 | 30 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885612 | 0,0501885612 | 0,0501885613 | |
| 41 | 31 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885613 | 0,0501885613 | 0,0501885613 | |
| 42 | 32 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885613 | 0,0501885613 | 0,0501885613 | |
| 60 | 50 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885613 | 0,0501885613 | 0,0501885613 | |
| 61 | 51 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885613 | 0,0501885613 | 0,0501885613 | |
| 62 | 52 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885613 | 0,0501885613 | 0,0501885613 | x |
| 63 | 53 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885613 | 0,0501885613 | 0,0501885613 | x |
| 64 | 54 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885613 | 0,0501885613 | 0,0501885613 | x |
| 65 | 55 | 0,0501885613 | x | 0,0501885613 | x | 0,0501885613 | 0,0501885613 | 0,0501885613 | x |

Tab. D.1: Velocity $v_3$ after relativistic non-elastic collision, $v_1 = 0$ ; $v_2 = 0{,}1c$

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $v_1$/c = | 0 | | $m_2/m_1$= | | $v_{3,Rel}$/c = | 0,50000000 | | |
| 2 | $v_2$/c = | 0,8 | | 1 | | $\gamma_{3,Rel}$ = | 1,15470054 | | |
| 3 | $p_0$/c = | 1,3333333333 | | | | | | | |
| 4 | | Recursion | | Newton | | Bisection | | | |
| 5 | $v_3$/c = | 0,5547001962 | | 0,5547001962 | | 0,5547001962 | $v_3/v_{3,Rel}$ = | 10,9% | |
| 6 | | ok | | ok | | ok | | | |
| 7 | | Recursion/Newton | | Newton/Bisection | | Bisection/Recursion | | | |
| 8 | | 0,0E+00 | | 4,7E-15 | | -4,6E-15 | | | |
| 9 | k | $v_3$/c | St | $v_3$/c | St | $(v_{3-}+v_{3+})/2c$ | $v_{3-}$/c | $v_{3+}$/c | St |
| 10 | 0 | 0 | | 0 | | | 0 | 0,8 | |
| 11 | 1 | 0,6666666667 | | 0,6666666667 | | 0,4000000000 | 0,4000000000 | 0,8000000000 | |
| 12 | 2 | 0,4969039950 | | 0,5723540713 | | 0,6000000000 | 0,4000000000 | 0,6000000000 | |
| 13 | 3 | 0,5785370130 | | 0,5550845393 | | 0,5000000000 | 0,5000000000 | 0,6000000000 | |
| 14 | 4 | 0,5437707542 | | 0,5547003739 | | 0,5500000000 | 0,5500000000 | 0,6000000000 | |
| 15 | 5 | 0,5594891983 | | 0,5547001962 | | 0,5750000000 | 0,5500000000 | 0,5750000000 | |
| 16 | 6 | 0,5525584281 | | 0,5547001962 | | 0,5625000000 | 0,5500000000 | 0,5625000000 | |
| 17 | 7 | 0,5556494433 | | 0,5547001962 | x | 0,5562500000 | 0,5500000000 | 0,5562500000 | |
| 18 | 8 | 0,5542777868 | | 0,5547001962 | x | 0,5531250000 | 0,5531250000 | 0,5562500000 | |
| 19 | 9 | 0,5548878305 | | 0,5547001962 | x | 0,5546875000 | 0,5546875000 | 0,5562500000 | |
| 20 | 10 | 0,5546167828 | | 0,5547001962 | x | 0,5554687500 | 0,5546875000 | 0,5554687500 | |
| 21 | 11 | 0,5547372648 | | 0,5547001962 | x | 0,5550781250 | 0,5546875000 | 0,5550781250 | |
| 22 | 12 | 0,5546837205 | | 0,5547001962 | x | 0,5548828125 | 0,5546875000 | 0,5548828125 | |
| 23 | 13 | 0,5547075186 | | 0,5547001962 | x | 0,5547851563 | 0,5546875000 | 0,5547851563 | |
| 24 | 14 | 0,5546969418 | | 0,5547001962 | x | 0,5547363281 | 0,5546875000 | 0,5547363281 | |
| 25 | 15 | 0,5547016426 | | 0,5547001962 | x | 0,5547119141 | 0,5546875000 | 0,5547119141 | |
| 26 | 16 | 0,5546995534 | | 0,5547001962 | x | 0,5546997070 | 0,5546997070 | 0,5547119141 | |
| 27 | 17 | 0,5547004819 | | 0,5547001962 | x | 0,5547058105 | 0,5546997070 | 0,5547058105 | |
| 28 | 18 | 0,5547000692 | | 0,5547001962 | x | 0,5547027588 | 0,5546997070 | 0,5547027588 | |
| 29 | 19 | 0,5547002527 | | 0,5547001962 | x | 0,5547012329 | 0,5546997070 | 0,5547012329 | |
| 30 | 20 | 0,5547001711 | | 0,5547001962 | x | 0,5547004700 | 0,5546997070 | 0,5547004700 | |
| 31 | 21 | 0,5547002074 | | 0,5547001962 | x | 0,5547000885 | 0,5547000885 | 0,5547004700 | |
| 32 | 22 | 0,5547001913 | | 0,5547001962 | x | 0,5547002792 | 0,5547000885 | 0,5547002792 | |
| 33 | 23 | 0,5547001984 | | 0,5547001962 | x | 0,5547001839 | 0,5547001839 | 0,5547002792 | |
| 34 | 24 | 0,5547001952 | | 0,5547001962 | x | 0,5547002316 | 0,5547001839 | 0,5547002316 | |
| 35 | 25 | 0,5547001967 | | 0,5547001962 | x | 0,5547002077 | 0,5547001839 | 0,5547002077 | |
| 36 | 26 | 0,5547001960 | | 0,5547001962 | x | 0,5547001958 | 0,5547001958 | 0,5547002077 | |
| 37 | 27 | 0,5547001963 | | 0,5547001962 | x | 0,5547002017 | 0,5547001958 | 0,5547002017 | |
| 38 | 28 | 0,5547001962 | | 0,5547001962 | x | 0,5547001988 | 0,5547001958 | 0,5547001988 | |
| 39 | 29 | 0,5547001962 | | 0,5547001962 | x | 0,5547001973 | 0,5547001958 | 0,5547001973 | |
| 40 | 30 | 0,5547001962 | | 0,5547001962 | x | 0,5547001965 | 0,5547001958 | 0,5547001965 | |
| 41 | 31 | 0,5547001962 | | 0,5547001962 | x | 0,5547001962 | 0,5547001962 | 0,5547001965 | |
| 42 | 32 | 0,5547001962 | | 0,5547001962 | x | 0,5547001963 | 0,5547001962 | 0,5547001963 | |
| 60 | 50 | 0,5547001962 | x | 0,5547001962 | x | 0,5547001962 | 0,5547001962 | 0,5547001962 | |
| 61 | 51 | 0,5547001962 | x | 0,5547001962 | x | 0,5547001962 | 0,5547001962 | 0,5547001962 | x |
| 62 | 52 | 0,5547001962 | x | 0,5547001962 | x | 0,5547001962 | 0,5547001962 | 0,5547001962 | x |
| 63 | 53 | 0,5547001962 | x | 0,5547001962 | x | 0,5547001962 | 0,5547001962 | 0,5547001962 | |
| 64 | 54 | 0,5547001962 | x | 0,5547001962 | x | 0,5547001962 | 0,5547001962 | 0,5547001962 | x |
| 65 | 55 | 0,5547001962 | x | 0,5547001962 | x | 0,5547001962 | 0,5547001962 | 0,5547001962 | x |

Tab. D.2: Velocity $v_3$ after relativistic non-elastic collision, $v_1 = 0\,;\ v_2 = 0{,}8c$

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $v_1/c$ = | 0 | | $m_2/m_1$= | | $v_{3,Rel}/c$ = | 0,61128031 | | |
| 2 | $v_2/c$ = | 0,89 | | 1 | | $\gamma_{3,Rel}$ = | 1,26356090 | | |
| 3 | $p_0/c$ = | 1,9519233617 | | | | | | | |
| 4 | | Recursion | | Newton | | Bisection | | | |
| 5 | $v_3/c$ = | | | 0,6984528781 | | 0,6984528781 | $v_3/v_{3,Rel}$ = | 14,3% | |
| 6 | | not ok | | ok | | ok | | | |
| 7 | | Recursion/Newton | | Newton/Bisection | | Bisection/Recursion | | | |
| 8 | | 3,4E-02 | | 0,0E+00 | | -3,3E-02 | | | |
| 9 | k | $v_3/c$ | St | $v_3/c$ | St | $(v_{3-}+v_{3+})/2c$ | $v_{3-}/c$ | $v_{3+}/c$ | St |
| 10 | 0 | 0 | | 0 | | | 0 | 0,89 | |
| 11 | 1 | 0,9759616809 | | 0,9759616809 | | 0,4450000000 | 0,4450000000 | 0,8900000000 | |
| 12 | 2 | 0,2127032246 | | 0,9397078220 | | 0,6675000000 | 0,6675000000 | 0,8900000000 | |
| 13 | 3 | 0,9536286032 | | 0,8688424449 | | 0,7787500000 | 0,6675000000 | 0,7787500000 | |
| 14 | 4 | 0,2937506647 | | 0,7743135001 | | 0,7231250000 | 0,6675000000 | 0,7231250000 | |
| 15 | 5 | 0,9329042795 | | 0,7115556340 | | 0,6953125000 | 0,6953125000 | 0,7231250000 | |
| 16 | 6 | 0,3514676442 | | 0,6988106129 | | 0,7092187500 | 0,6953125000 | 0,7092187500 | |
| 17 | 7 | 0,9136953543 | | 0,6984531401 | | 0,7022656250 | 0,6953125000 | 0,7022656250 | |
| 18 | 8 | 0,3966306344 | | 0,6984528781 | | 0,6987890625 | 0,6953125000 | 0,6987890625 | |
| 19 | 9 | 0,8959116343 | | 0,6984528781 | | 0,6970507813 | 0,6970507813 | 0,6987890625 | |
| 20 | 10 | 0,4335537100 | | 0,6984528781 | x | 0,6979199219 | 0,6979199219 | 0,6987890625 | |
| 21 | 11 | 0,8794661312 | | 0,6984528781 | x | 0,6983544922 | 0,6983544922 | 0,6987890625 | |
| 22 | 12 | 0,4645201595 | | 0,6984528781 | x | 0,6985717773 | 0,6983544922 | 0,6985717773 | |
| 23 | 13 | 0,8642751101 | | 0,6984528781 | x | 0,6984631348 | 0,6983544922 | 0,6984631348 | |
| 24 | 14 | 0,4909276759 | | 0,6984528781 | x | 0,6984088135 | 0,6984088135 | 0,6984631348 | |
| 25 | 15 | 0,8502581396 | | 0,6984528781 | x | 0,6984359741 | 0,6984359741 | 0,6984631348 | |
| 26 | 16 | 0,5137129813 | | 0,6984528781 | x | 0,6984495544 | 0,6984495544 | 0,6984631348 | |
| 27 | 17 | 0,8373381377 | | 0,6984528781 | x | 0,6984563446 | 0,6984495544 | 0,6984563446 | |
| 28 | 18 | 0,5335439275 | | 0,6984528781 | x | 0,6984529495 | 0,6984495544 | 0,6984529495 | |
| 29 | 19 | 0,8254414098 | | 0,6984528781 | x | 0,6984512520 | 0,6984512520 | 0,6984529495 | |
| 30 | 20 | 0,5509184645 | | 0,6984528781 | x | 0,6984521008 | 0,6984521008 | 0,6984529495 | |
| 31 | 21 | 0,8144976752 | | 0,6984528781 | x | 0,6984525251 | 0,6984525251 | 0,6984529495 | |
| 32 | 22 | 0,5662205832 | | 0,6984528781 | x | 0,6984527373 | 0,6984527373 | 0,6984529495 | |
| 33 | 23 | 0,8044400793 | | 0,6984528781 | x | 0,6984528434 | 0,6984528434 | 0,6984529495 | |
| 34 | 24 | 0,5797542286 | | 0,6984528781 | x | 0,6984528965 | 0,6984528434 | 0,6984528965 | |
| 35 | 25 | 0,7952051896 | | 0,6984528781 | x | 0,6984528700 | 0,6984528700 | 0,6984528965 | |
| 36 | 26 | 0,5917650167 | | 0,6984528781 | x | 0,6984528832 | 0,6984528700 | 0,6984528832 | |
| 37 | 27 | 0,7867329748 | | 0,6984528781 | x | 0,6984528766 | 0,6984528766 | 0,6984528832 | |
| 38 | 28 | 0,6024547712 | | 0,6984528781 | x | 0,6984528799 | 0,6984528766 | 0,6984528799 | |
| 39 | 29 | 0,7789667662 | | 0,6984528781 | x | 0,6984528782 | 0,6984528766 | 0,6984528782 | |
| 40 | 30 | 0,6119916160 | | 0,6984528781 | x | 0,6984528774 | 0,6984528774 | 0,6984528782 | |
| 41 | 31 | 0,7718532028 | | 0,6984528781 | x | 0,6984528778 | 0,6984528778 | 0,6984528782 | |
| 42 | 32 | 0,6205171991 | | 0,6984528781 | x | 0,6984528780 | 0,6984528780 | 0,6984528782 | |
| 60 | 50 | 0,6671025921 | | 0,6984528781 | x | 0,6984528781 | 0,6984528781 | 0,6984528781 | |
| 61 | 51 | 0,7270581323 | | 0,6984528781 | x | 0,6984528781 | 0,6984528781 | 0,6984528781 | x |
| 62 | 52 | 0,6700717735 | | 0,6984528781 | x | 0,6984528781 | 0,6984528781 | 0,6984528781 | x |
| 63 | 53 | 0,7244527587 | | 0,6984528781 | x | 0,6984528781 | 0,6984528781 | 0,6984528781 | x |
| 64 | 54 | 0,6727542511 | | 0,6984528781 | x | 0,6984528781 | 0,6984528781 | 0,6984528781 | x |
| 65 | 55 | 0,7220808780 | | 0,6984528781 | x | 0,6984528781 | 0,6984528781 | 0,6984528781 | x |

Tab. D.3: Velocity $v_3$ after relativistic non-elastic collision, $v_1 = 0$ ; $v_2 = 0{,}89c$

# Annex E: Brief introduction to vector calculus

To understand the representation of Maxwell's equations in Chapter 10, a basic knowledge of vector calculus is required. The necessary relationships and basic elements for understanding field relationships are summarized here in brief. Only the absolutely necessary relationships are shown, and the following restrictions apply:

1. The representations apply to 3 dimensions; these are sufficient for the relationships in fields.
2. Only Cartesian (rectangular) coordinate systems are considered (e.g. no spherical or cylindrical coordinates).

First, the basic properties of vectors are presented and then the differential functions required to understand Maxwell's equations are explained.

## E.1 Scalar und Vector

In a coordinate system, physical quantities can be assigned to each point as a scalar or vector. Vectors are direction-dependent, scalars are not. Examples of scalar quantities are temperature, energy, and pressure. For directional quantities such as forces or fields, on the other hand, vectors are used which, in addition to the location in the coordinate system, also contain values for the magnitude and direction. For the representation of a vector $\vec{a}$ in Cartesian coordinates the following form is used:

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \qquad \text{(E.01)}$$

The amount of $\vec{a}$, for example for the magnitude of a force, is determined by

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad \text{(E.02)}$$

If the direction and magnitude of two vectors are the same, they are identical, but can be located at different points in the coordinate system.

## E.2 Vector addition

For the addition of two vectors $\vec{a}$ and $\vec{b}$ the rule applies:

$$\vec{a} + \vec{b} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} + \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} a_x + b_x \\ a_y + b_y \\ a_z + b_z \end{pmatrix} \tag{E.03}$$

This addition can also be performed graphically. For this purpose, a representation with arrows is used. The position in the diagram is the direction, the length of the arrow indicates the magnitude.

For the addition, the arrows $\vec{a}$ and $\vec{b}$ are joined together; the resulting line between the start and end points is the result of the addition in terms of magnitude and direction.

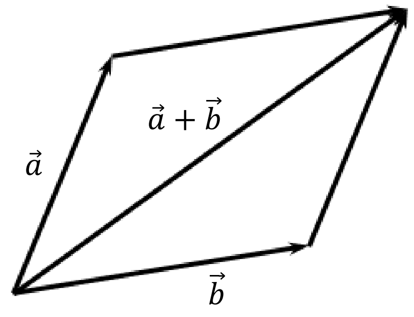



Fig.. E.1: Graphical vector addition

## E.3 Scalar product

The scalar product (or inner product) of two vectors is so called because the result of the multiplication is a scalar. This is in Cartesian coordinates

$$\vec{a} \cdot \vec{b} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \cdot \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = a_x b_x + a_y b_y + a_z b_z \tag{E.04}$$

or

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi \tag{E.05}$$

with $\varphi$ as the angle between $\vec{a}$ and $\vec{b}$. This operation is often used in physics when energy is to be calculated and the angle between the force and the direction of movement does not match. Force and direction are vectors, the resulting work is a scalar quantity. The meaning becomes clear when a mass in the Earth's gravitational field and an attacking force is considered. If the mass is moved upwards by the force ($\varphi = 0; \cos\varphi = 1$), energy is needed and the potential energy increases; if the force acts at $\varphi = 90°$, the mass remains at the same height and the energy does not change.

## E.4 Cross product

The cross product (also known as the vector product or outer product) of the vectors $\vec{a}$ and $\vec{b}$ in three-dimensional space is a certain vector that is perpendicular to the plane spanned by them. The length is equal to the area of the parallelogram, i.e.

$$\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot |\sin\varphi| \tag{E.06}$$

In the three-dimensional Cartesian coordinate system, the cross product is calculated as follows

$$\vec{a} \times \vec{b} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \times \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix} \tag{E.07}$$

Examples of the application of the cross product are the Lorentz force or the torque. For example, the following relationship applies to the magnetic part of the Lorentz force

$$\vec{F}_B = q\vec{v} \times \vec{B} \tag{E.08}$$

with $q$ as the charge and $\vec{v}$ as its velocity and $\vec{B}$ as the magnetic field. The orientation of the resulting Lorentz force is perpendicular to both the velocity and the magnetic field (3-finger rule).

## E.5 Fields and Nabla operator

In physics, a field is defined as the spatial distribution of a physical quantity. In the simplest case, there is a scalar field, as is possible for temperature distributions or potentials. If a physical vector is dependent on the position of the location, it is referred to as a vector field. It can be visualized by field lines, whereby the tangent to the field line indicates the direction of the vector. The magnitude of the vector is represented by the density of the field lines. Electric and magnetic fields are examples of this. These fields are characterized by the fact that temporal changes in particular play a role, which must be represented by differentiation. The use of the Nabla operator is helpful here.

The Nabla operator $\vec{\nabla}$ is a vectorial differential operator. This means that it can be written in vector form and, when applied to a function, performs a differential operation that represents a 3-dimensional derivative. With its help, the quantities gradient, divergence, and rotation, which are still to be described, can be easily represented. It is defined for the 3-dimensional Cartesian coordinates $x$, $y$, $z$ as

$$\vec{\nabla} = \begin{pmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} \end{pmatrix} \tag{E.09}$$

## E.6 Gradient

A field based on a scalar function $f$ assigns an exact value to each point in the definition space. Examples of scalar fields in three-dimensional space are the distribution of temperatures, density, or potentials. Applying the Nabla operator to $f$ results in a vector field called the gradient (grad). The gradient points in the direction of the strongest ascent at each point in space and its magnitude indicates the increase in this direction. The representation is as follows:

$$\text{grad}\, f = \vec{\nabla} \cdot f \quad = \begin{pmatrix} \frac{\partial f_x}{\partial x} \\ \frac{\partial f_y}{\partial y} \\ \frac{\partial f_z}{\partial z} \end{pmatrix} \tag{E.10}$$

If the scalar field is a potential, the negative gradient of the field indicates the associated force field. This is clear in the case of the gravitational field: Two of the coordinates are equal to zero and a body falls in the direction in which the change in its potential reaches the maximum.

## E.7 Divergence

When applying the Nabla operator to a vector field $f$, the scalar product $\vec{\nabla} \cdot f$ results in a scalar field that indicates whether field lines appear or disappear at each point in space. Thus, at the location of a positive charge, the divergence of the electric field is greater than zero, as field lines arise at this point. Points with positive divergence are called sources, points with negative divergence are called sinks. The calculation results in

$$\text{div}\, \vec{f} = \vec{\nabla} \cdot \vec{f} = \; = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} \tag{E.11}$$

## E.8 Rotation

If we form $\vec{\nabla} \times f$, we obtain a vector function called rotation (rot), which characterizes the closed loop of the vector field $f$. If we consider, for example, the magnetic field of a current-carrying wire, the field lines run in a circle around this wire and are closed. The calculation is carried out as follows:

$$\text{rot}\, \vec{f} = \vec{\nabla} \times \vec{f} = \begin{pmatrix} \frac{\partial}{\partial y} f_z - \frac{\partial}{\partial z} f_y \\ \frac{\partial}{\partial z} f_x - \frac{\partial}{\partial x} f_z \\ \frac{\partial}{\partial x} f_y - \frac{\partial}{\partial y} f_x \end{pmatrix} \tag{E.12}$$

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