10. Electromagnetism and Gravity

In the 19th century, electrical and magnetic effects were intensively researched. As already described in chapter 1.4, the invention of the first functional battery by Alessandro Volta made experimental investigations possible. The chapter also summarizes further details on the key developments and the many people involved.

The most important result is that all electromagnetic processes can be summarized in the representation of Maxwell's equations. These are listed in chapter 10.1, followed by a formal comparison with the conditions concerning gravity. To understand these relationships, a basic knowledge of vector calculus is required, the most important elements of which are summarized briefly in Appendix E.

10.1 Maxwell's equations

The system of Maxwell's equations consists of 4 laws. Their names and a brief explanation are given below. The formulaic representation and the basic meanings are summarized in Table 12.1. Table 12.2 shows the designation of the formula symbols and the associated dimensions.

1. *Gauss's law*
   In the physical fields of electrostatics and electrodynamics, Gauss's law describes the electrical flow through a closed surface. It is named after the mathematician Carl Friedrich Gauss, who developed the integral theorem named after him for a vector field.
2. *Gauss's law for magnetism*
   Analogous to the electric field, this describes the magnetic flux through a closed surface.
3. *Faraday's law*
   The law of induction, discovered by Michael Faraday, describes the structure of electric fields.
4. *Ampère's law with Maxwell's addition*
   Based on André-Marie Ampère's law, this describes the structure of a magnetic field.

For a better understanding of the relationships, the 4 Maxwell equations in Table 10.1 have been arranged in such a way that the sequence from the static electric field via the dynamic changes in electric and magnetic properties leads to the static magnetic field. The coupling of the electric and magnetic field constants represents the connection between the two parts. In the right half of the table, the respective meaning of the relationships has been added in short words.

(1) $\vec{\nabla} \cdot \vec{E} = \frac{\rho_{el}}{\varepsilon_0}$ $\vec{F} = \frac{Q_1\, Q_2}{4\pi\varepsilon_0 s^2} \cdot \vec{s}_0$ $\varepsilon_0 = 8{,}8542 \cdot 10^{-12} \frac{C^2}{Nm^2}$	**Electric Field** Source: Electric Charge Charges of the same type repel each other
(3) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	The change of a magnetic field $\vec{B}$ causes the build-up of an electric field $\vec{E}$ (in the form of a closed loop)
$\varepsilon_0 \mu_0 = \frac{1}{c^2}$	The field constants are coupled with the speed of light
(4) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}_{el} - \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$	The flow of an electric current $\vec{j}_{el}$ and the change of an electric field $\vec{E}$ cause the build-up of a magnetic field $\vec{B}$
(2) $\vec{\nabla} \cdot \vec{B} = 0$ $\vec{M} = \vec{m} \times \vec{B}$ $\mu_0 = 1{,}2566 \cdot 10^{-6} \frac{Ns^2}{C^2}$	**Magnetic Field** Free of sources (closed loop) Similar poles repel each other

Tab. 10.1: Maxwell's equations and their interpretation (definition of symbols in Table 10.3) The numbers of the laws precede the respective formula.

10.2 Comparison between electric field and gravity

Due to the formal similarity between the electric field and the gravitational field, it was assumed early on that Maxwell's equations should also apply here. Heaviside was the first to put forward this thesis in 1895. Today, there is a general consensus that this assumption is correct but only applies to the limit range of small masses and velocities [94]. For other conditions, especially when processes with large masses, such as black holes, are considered, other relationships apply and space curvature etc. must be taken into account, as is the case in the general theory of relativity, for example.

There is a formal difference between the representations of the electric field and gravity. Because the relationships for the electric field were derived for a homogeneous situation (as it is the case in a capacitor, for example, with Q as charge and s as plate distance), but gravitation for a spatial distribution (with m as mass, r as radius), the calculations for the forces differ:

Force in electric fields	Force in gravitational fields
$\vec{F} = \frac{Q_1\, Q_2}{4\pi\varepsilon_0 s^2} \cdot \vec{s}_0$ (10.01)	$\vec{F} = -G\, \frac{m_1\, m_2}{r^2} \cdot \vec{r}_0$ (10.02)

Since the following task is a comparison of the fields, it makes sense to standardize the representation and convert one of the quantities accordingly. If the gravitational constant is chosen for this, the result is

$$G' = \frac{1}{4\pi G}$$

If this modified formula is used, a comparison results in the form shown in Table 10.2. Further the Maxwell equations are presented here in a modified form, in which the magnetic field constant is not used and the coupling with the speed of light is considered instead (see Tab 10.1). In this way, it is not necessary to redefine corresponding quantities for the gravitational field. The resulting analogy to Maxwell's equations leads to the definition of a system of equations whose physical meaning is generally interpreted today as "Gravitoelectromagnetism (GEM)" [94].

Maxwell's equations for Electromagnetism	Maxwell's equations for Gravitoelectromagnetism
(1) $\vec{\nabla} \cdot \vec{E} = \frac{\rho_{el}}{\varepsilon_0}$	(1) $\vec{\nabla} \cdot \vec{E}_g = -\frac{\rho_g}{G'}$
(3) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	(3) $\vec{\nabla} \times \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$
(4) $\vec{\nabla} \times \vec{B} = \frac{\vec{J}_{el}}{\varepsilon_0 \cdot c^2} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$	(4) $\vec{\nabla} \times \vec{B}_g = -\frac{\vec{J}_g}{G' \cdot c^2} + \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t}$
(2) $\vec{\nabla} \cdot \vec{B} = 0$	(2) $\vec{\nabla} \cdot \vec{B}_g = 0$

Tab. 10.2: Application of Maxwell's equations to the gravitational field.
The numbers of the laws precede the respective formula.

It is assumed here that the speed of light c and the propagation speed of gravity are the same.

The equations shown correspond to each other with the difference that the prefixes for equations 1 and 4 are different. For equations 1, this firstly has the simple meaning that masses attract each other, while similar charges repel each other. With regard to equation 4, it follows in the same way that poles of the same direction in the GEM field do not repel each other as in electromagnetism but attract each other.

In this context, there is the interesting question of whether there is an equivalent for gravity for positive and negative charges. This could apply to the pair of matter/antimatter. Of great importance for theoretical considerations is whether matter and antimatter attract, repel or, as some theories predict, attract each other more weakly than pure matter. There was a first breakthrough in this regard in 2023, when investigations at CERN on antihydrogen atoms showed that they are attracted by the Earth's gravity [95]. This is one of the most interesting current experiments, the accuracy of which is to be further increased in order to clarify fundamental questions.

	Physical Variable	Dim.		Physical Variable	Dim.
$\vec{E}$	Electric field	$\frac{N}{C}$	$\vec{E}_g$	Gravitational field	$\frac{m}{s^2}$
$\vec{B}$	Magnetic flux	$\frac{Ns}{Cm}$	$\vec{B}_g$	Gravitomagnetic field	$\frac{1}{s}$
$\vec{M}$	Moment	Nm	$\vec{m}$	Magnetic dipole moment	$\frac{Cm^2}{s}$
$\vec{j}_{el}$	Electric current flow	$\frac{C}{m^2 s}$	$\vec{j}_g$	Mass flow	$\frac{kg}{m^2 s}$
ρ_{el}	Electrical charge density	$\frac{C}{m^2}$	ρ_g	Mass density	$\frac{kg}{m^2}$

Tab. 10.3: Definition of the used physical variables with dimensions.
For the definition and application of the Nabla operator $\vec{\nabla}$ see Appendix E

Despite the formal similarity between the variables shown in Table 10.2 and Table 10.3, there are substancial differences in their characteristics. This will first be considered for the electric field and gravitational field. If the difference in the respective attractive forces between a proton and an electron is calculated in a simple example, the formulae (10.01) and (10.02) can be used. The values for the specific quantities are listed in Table10.4. If the values are used, the result for this case is an extreme difference between the electric and gravitational forces of attraction, namely by a factor of $2{,}27 \cdot 10^{39}$!

Mass Proton	$m_P = 1{,}6726 \cdot 10^{-27}\text{kg}$	Mass Electron	$m_E = 9{,}1094 \cdot 10^{-31}\text{kg}$
Electr. Charge Proton	$\text{Q}_\text{P} = 1{,}6022 \cdot 10^{-27}\text{C}$	Electr. Charge Electron	$\text{Q}_E = -1{,}6022 \cdot 10^{-27}\text{C}$
Gravitational constant	$G = 6{,}6743 \cdot 10^{-11} \dfrac{\text{m}^3}{\text{kg s}^2}$	Electric constant	$\varepsilon_0 = 8{,}8542 \cdot 10^{-12} \dfrac{\text{C}^2}{\text{Nm}^2}$

Tab. 10.4: Values of the physical quantities used for the calculations.

Note on Table 10.4: The values for ε_0 are often given in the literature with the dimension As/Vm. This can be easily converted using the power P in watts [W] and results with the charge C (Coulomb) as As (Ampere-seconds)

$$\left[1\text{W} = 1\frac{\text{kg m}^2}{\text{s}^3} = 1\frac{\text{Nm}}{\text{s}} = 1\text{ VA}\right]$$

The difference between the electric field and the gravitational field lies not only in the magnitude of the attractive force, but above all in the fact that electric charges compensate each other in everyday life, i.e. every positive atomic nucleus is opposed by a negative electron. In addition, electric charges can be shielded. In the case of gravity, on the other hand, all masses add up and, according to current knowledge, the effective attractive forces cannot be influenced in any way.

Other important differences are that electric charges always occur as multiples of the elementary charges, whereas gravity has no known smallest indivisible unit. In addition, the kinetic energy of masses is dependent on the state of motion, whereas this does not apply to electric charges. Furthermore, permeability effects are unknown for gravity.

Despite the small effects, the gravitational balance developed by the Englishman H. Cavendish [1731-1810] made it possible to determine density differences in the earth and calculate the gravitational constant as early as 1798.

Direct experimental proof of the existence of gravitomagnetism in the form shown here has not yet been achieved on the Earth's surface due to the extremely small effects that occur. According to calculations by D. Giulini, a gyroscope set up at the North Pole would cause a precession at a speed of 0.6 milliarcseconds per day; given the current experimental conditions, this is still 1 to 2 orders of magnitude outside today's detection limits [96].

In cosmic dimensions, on the other hand, larger effects occur, whereby the shape of such a field can be determined by calculations. Fig. 10.1 shows the characteristics of a gravitomagnetic dipole field in a graphical representation, evaluated at points at an angular distance of 30°, which lie on a circle around the center [96]. In the center is the rotating star, whose angular momentum is symbolized by an upward-pointing arrow (vector). It generates the dipole field.

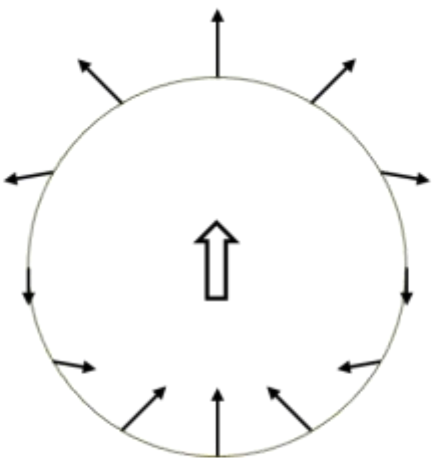

Fig. 10.1: Expression of a gravitomagnetic dipole field generated by a rotating star in the center [96].

Experimental proof was only possible with the launch of the Gravity Probe B into space in 2004, with which the interactions between 4 counter-rotating gyroscopes and the rotating Earth were investigated. After lengthy and complicated evaluations due to interference effects that occurred, results were published in 2011 that were obtained as part of investigations to verify the general theory of relativity [97]. These are the effects of spacetime curvature and the Lense-Thirring effect. For details, please refer to further literature [96, 97].

Finally, another interesting aspect should be considered. For years there have been investigations into the amplification of gravitomagnetic effects, similar to those observed when the permeability of magnetic fields is increased (e.g. by feeding an iron core into a magnetic coil). Such evidence would have enormous implications for the foundations of the theory of general relativity and is therefore subject to special observation. In one of the experiments, for example, a large quantity of rotating liquid helium was used in a superconducting Nb tube, and a gyroscope was placed in it. However, after initial positive results for increasing the gravitomagnetic effect, it became apparent that these could not be reproduced [98]. None of the experiments carried out so far have been successful and therefore no effects on the theory are recognizable.