12. Conclusions and proposals for modification

The Theory of Special Relativity postulated by A. Einstein in combination with the transformation equations derived by Larmor, Lorentz and Poincaré and further the relativistic increase of mass makes it possible to describe all conceivable relations between moving bodies in arbitrary inertial systems without contradictions. To prove this a wide selection of examples concerning this issue was already discussed in detail in the chapters presented before.

However, this concept is not sufficient to describe all observed cosmological cases. At the beginning of the second half of the 20th century it was found that a cosmic microwave background radiation exists, which is isotropic and constant in all directions. Therefore, based on the "Ether-theories" already developed at the end of the 19th century, new attempts were made to bring special relativity in accordance with a state of absolute rest. However, none of these theories were able to show results without severe discrepancies to experimental findings. The most important theories will be discussed briefly in the following. In addition, the Einstein synchronization already discussed in chapter 3.4. will be evaluated again.

Furthermore, it is proved that by using light pulses for a signal exchange between two observers moving arbitrarily to each other, additionally a superordinate system of absolute rest can be incorporated. With the use of the Lorentz transformation as only precondition this system can be integrated without contradiction. This is done first for the case that two observers are on a straight line in orientation to the system at rest, then for arbitrary constellations.

12.1 Alternative theories

In the following theories shall be presented, which are not in accordance with the calculus of the Lorentz-Transformation (LT). They were developed to avoid the principle of "relativity of simultaneously", which is integral part of LT. The main difference is the introduction of an absolute time which is concurrent valid in any arbitrary inertial system. Although all these theories in their initial form are not in compliance with experimental results, they are historically important and, because of the basic approach concerning violations of LT, are still basis for current research programs.

12.1.1 Simple addition of velocities

At the early beginning of discussion concerning speed of light and "ether-drift" it was generally assumed, that the velocity of an observer (together with the measuring device carried with him) and the speed of light must be simply added [12c]. Also, the theoretical approach connected with the Michelson-Morley-Experiment is based on this assumption, and for the calculation of light beams coming and going to mirrors the value was either higher or lower than the speed of light c.

Already in the year 1913, however, the examination of double star systems by W. de Sitter provided evidence, that the speed of light is independent of the speed of the object that is transmitting the signals [55]. It was now proven for the first time that this assumption is not in accordance with the facts.

12.1.2 Theory of „Neo-Lorentzianism“

Following a similar idea of H. Ives and developed further by J. S. Prokhovnik [74] it is assumed that in all parts of the universe a reference system S exists, which is at absolute rest. When a different inertial system is moving relative to it, the only related attribute valid for this system is, that space is contracting according to

$$x_A = \frac{x_S}{\gamma} \tag{12.01}$$

Consequently, for the coming and going of a light signal inside this system the following different velocities will appear

$$c_1 = c + u_A \tag{12.02}$$

$$c_2 = c - u_A \tag{12.03}$$

The characteristics of time can be calculated by the consideration of a closed loop for a signal

$$t_A = \frac{x_A}{c_1} + \frac{x_A}{c_2} = \frac{x_S(c - u_A + c + u_A)}{\gamma(c + u_A)(c - u_A)} = \frac{2x_S}{c}\gamma = \gamma t_S \tag{12.04}$$

This means that time dilatation is only a seemingly effect which is not real. Effects connected with this theory should be found easily using e.g. synchronization experiments and, because this is not the case, the theory must be rejected. However, the involved persons, mainly Herbert E. Ives (1882-1953), are still today of historical interest. He was all his life in strict opposition to Einstein and, apart from his different theoretical approach, tried hard to discredit him in any possible way. He denied his contribution to Special Relativity and even tried to show that the equation

$$E = mc^2 \tag{6.17}$$

was not originally developed by Einstein [75]. Nevertheless, he provided evidence with the Ives-Stilwell-experiment (co-working with G. R. Stilwell) that time-dilatation for moved bodies exists [17,18] and thus supported, surely without intention, the validity of the Lorentz-equations.

12.1.3 RMS-Test theory

The development of another alternative theory started with a proposal by H. Robertson [59] and was finalized by R. Mansouri and R. Sexl [24] and is today usually referred to as Robertson-Mansouri-Sexl- or RMS-Theory. In this case it is assumed, that a system of absolute rest (called “ether system”) exists. For the notation of this ether-system capital letters and for any arbitrary initial reference system small letters are used for calculation. The following general transformation equations are valid:

$$t = aT + \varepsilon x \tag{12.10}$$

$$x = b(X - vT) \tag{12.11}$$

where the factors a and b can be determined by measurements (e.g. Michelson-Morley- and Kennedy-Thorndike-experiments) and ε out of synchronization effects as

$$\frac{1}{a} = b = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \qquad \varepsilon = -v \tag{12.12}$$

Hence

$$t = \frac{T}{\gamma} - vx \tag{12.13}$$

$$x = \gamma(X - vT) \tag{12.14}$$

Equation (12.14) is obviously corresponding to the Lorentz-Transformation according to Eq. (1.08). Eq. (12.13) can be transformed to

$$t = \frac{T}{\gamma} - vx = \gamma T(1 - v^2) - vx = \gamma T - \gamma T v^2 - vx \tag{12.15}$$

If Eq. (12.11) is converted, then

$$T = \frac{X - \frac{x}{\gamma}}{v} \tag{12.16}$$

with

$$t = \gamma T - \gamma \frac{X - \frac{x}{\gamma}}{v} v^2 - vx = \gamma T - \gamma v X + vx - vx \tag{12.17}$$

and

$$t = \gamma(T - vX) \tag{12.18}$$

This means that the calculations follow exactly the Lorentz-Transformation. The RMS-Theory now predicts that during passing of a moving system a comparison of clocks inside both systems shows the result

$$\Delta t = -vx \tag{12.19}$$

Eq. (12.13) is transforming to

$$t = \frac{T}{\gamma} \tag{12.20}$$

A graphic presentation leads to the diagram shown in Fig. 12.1.

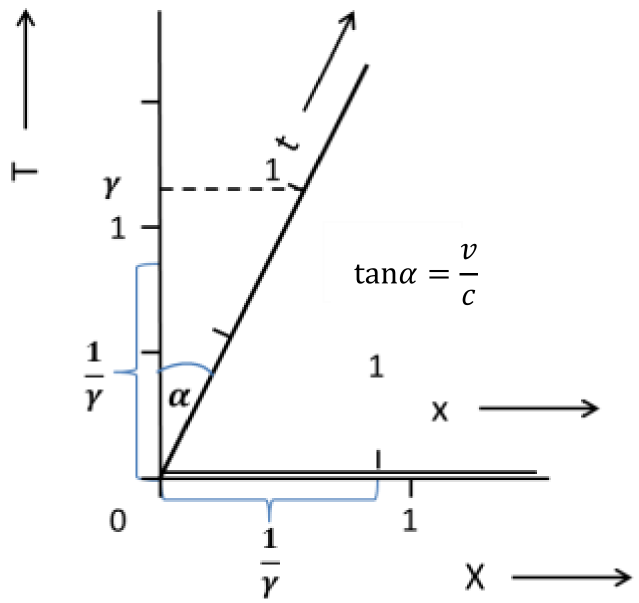

Fig. 12.1: Space-time diagram following Eq. (12.18) (according to [24])

It is obvious that in this case during a synchronization performed with light signals differences inside the moving system should occur; however, no experimental evidence could be provided up to now [54,75]. Although the theory shows severe shortcomings, it is further developed until today [54]. The reason is that new approaches using quantum gravitation resp. string theory are suggesting violations of the Lorentz-Transformation. In combination with the equation

$$y = d \cdot Y \qquad\qquad z = d \cdot Z \tag{12.21}$$

now effort is made to find small differences to the equations given by the Lorentz-Transformation

$$\frac{1}{a} = b = \left[1 - \left(\frac{v}{c}\right)^2\right]^{-\frac{1}{2}} \qquad\qquad d = 1 \tag{12.22}$$

The intention is that with increasing accuracy of experiments following the methods of Michelson-Morley, Kennedy-Thorndike, and Ives-Stilwell these differences will be detected and that it will be possible to integrate the results into a generally valid overall picture. Examples for new measurements with highest precision are given e.g. [76,77,78,79], however, up to now no violations of the Lorentz-Invariance could be detected.

12.1.4 Further alternatives

In the last years many alternative theories were developed, which are demanding variations of the Lorentz-Equations. These approaches are usually connected with a further development of the “Theory of General Relativity”, trying to find a general unifying theory

(GUT), which can possibly bridge the gap to quantum mechanics. These new theories are generally of high complexity, but despite of fierce struggle it was not possible to find a reasonable formalism during the last decades. Here the question is allowed, why such an effort is made and how this can be justified. To answer this, a remarkable statement shall be cited out of a publication by C. M. Will [64]. This is in principle dealing with the position of General Relativity, but because further developments of this theory are mainly connected with the search for violations of the Lorentz-Invariance, it is also valid for the relations discussed before:

"We find that general relativity has held up under extensive experimental scrutiny. The question then arises, why bother to continue to test it? One reason is that gravity is a fundamental interaction of nature, and as such requires the most solid empirical underpinning we can provide. Another is that all attempts to quantize gravity and to unify it with the other forces suggest that the standard general relativity of Einstein is not likely to be the last word. Furthermore, the predictions of general relativity are fixed; the theory contains no adjustable constants so nothing can be changed. Thus, every test of the theory is either a potentially deadly test or a possible probe for new physics. Although it is remarkable that this theory, born 80 years ago out of almost pure thought, has managed to survive every test, the possibility of finding a discrepancy will continue to drive experiments for years to come."

11.2 Interpretation of Einstein-synchronization

In chapter 3.4 the Einstein synchronization was already discussed shortly. Because of the paramount importance it shall be investigated again and a close look at this topic will be taken. In a first step the theoretically appearing synchronization differences for an observer at rest and in a moved system are established.

In the following space-time-diagram the synchronization differences ΔS and $\Delta S'$ experienced by an observer at rest A in view of a moved observer B are presented (Fig. 12.2). The diagram is standardized (which means a scaling of $\Delta t = \Delta x = 1$). In a diagram scaled this way, light pulses show a graphic orientation of 45° to t and x axis. The velocity used for B in this diagram is $v = x/t = 0{,}5c$.

The cases appear, that:

a) A is sending a signal which is reflected by B,

b) B is sending a signal which is reflected by A.

The equations necessary for the calculation of the synchronization differences are compiled in Tab. 11.1. For A the interpretation of diagram a) is simple and because of the appearing symmetry $\Delta t_0 = \Delta t_2 = \Delta t_1$ is valid.

The situation of part b) is different and the calculation more complex. Observer A is monitoring in his view, that the signal will be sent later from B, because time is running slower by factor γ, but that it is arriving earlier compared to the signal sent by him. The latter is caused by the effect, that B is increasing distance to A during the transmission of the signal (for exact definition and modes for calculation see chapter 2.).

The synchronization difference for A can be calculated as follows

$$\Delta S = \Delta t_S \frac{1 - \frac{1}{\gamma}}{1 - \frac{v}{c}} \qquad (12.30)$$

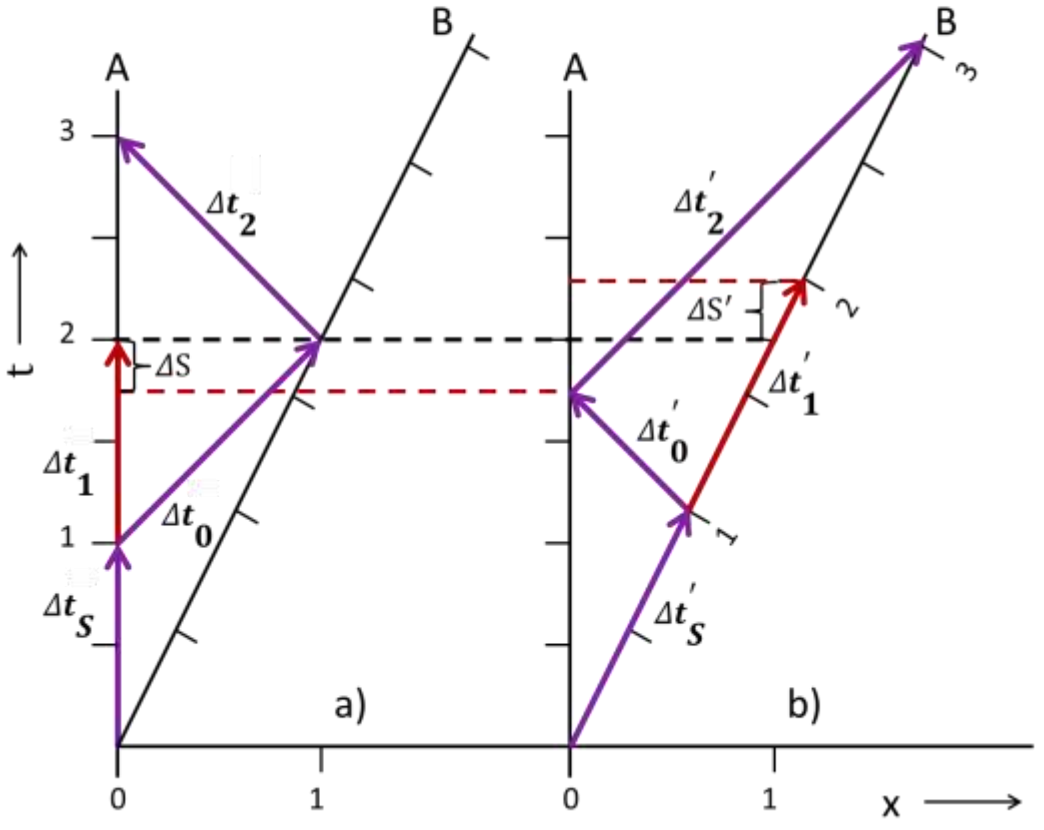

Fig. 12.2: Definition of synchronization differences ΔS resp. $\Delta S'$

Δt_S	$\Delta t'_S = \gamma \Delta t_S$
$\Delta t_0 = \Delta t_S \left[\frac{v}{c\left(1-\frac{v}{c}\right)}\right]$	$\Delta t'_0 = \Delta t_S \gamma \frac{v}{c}$
$\Delta t_2 = \Delta t_S \left[\frac{v}{c\left(1-\frac{v}{c}\right)}\right]$	$\Delta t'_2 = \Delta t_S \gamma \left[\frac{v\left(1+\frac{v}{c}\right)}{c\left(1-\frac{v}{c}\right)}\right]$
$\Delta S = \Delta t_S + \left[\frac{\Delta t_0 + \Delta t_2}{2}\right] - [\Delta t'_S + \Delta t'_0]$ $= \Delta t_S \frac{1-\frac{1}{\gamma}}{1-\frac{v}{c}}$	$\Delta S' = \Delta t'_S + \left[\frac{\Delta t'_0 + \Delta t'_2}{2}\right] - [\Delta t_S + \Delta t_0]$ $= \Delta t_S \frac{\gamma - 1}{1-\frac{v}{c}}$

Tab. 12.1: Equations for the calculation of ΔS resp. $\Delta S'$

In view of the moving observer B in b) a similar situation appears. In this case also the signal will arrive too early with

$$\Delta S' = \Delta t_S \frac{\gamma - 1}{1 - \frac{v}{c}} \tag{12.31}$$

Because time is running slower for the moving observer the subjective values for both are equal and it applies

$$\Delta S' = \gamma \Delta S \tag{12.32}$$

The Einstein-Synchronization now specifies the following:

At time t_S resp. t_S' a signal will be transmitted by observers A and B. When the signals are received by B resp. A, the clocks are considered as synchronized, when the following conditions apply:

$$t_1 = t_S + \frac{t_2 - t_0}{2} \tag{12.33}$$

and

$$t_1' = t_S' + \frac{t_2' - t_0'}{2} \tag{12.34}$$

For system a), the validity of the determination results directly from the representation in the diagram and there are no differences to the calculations carried out. For b), however, there are serious changes.

An essential statement is first that $\Delta t_1'$ is hereby uniquely determined and the division between the single times $\Delta t_0'$ and $\Delta t_2'$ does not play any role. Together with the statement that the speed of light is the same in all inertial systems, in this way the synchronization difference becomes a virtual quantity which cannot be determined from the moving system. Since this value would be measurable for a resting observer, however, at the transmission of impulses with superluminal velocities, there must be no information transmission faster than the light and also no system of absolute rest on the basis of these determinations. Herewith, a central statement of the special relativity theory is described.

So, it becomes clear that the Einstein synchronization is a definition and not covered by an observation.

The use of the Einstein synchronization has beside the possibility for the calculation of the Lorentz equations still another meaning. As already described in detail, from the point of view of an observer at rest it is not possible to describe the course of oscillation of an electromagnetic wave (e.g. light) without contradiction without using the principle of constant phase velocity in a moving system.

To avoid this, it is a simple means to use the definition of the Einstein synchronization in such a way that oscillation considerations are permitted in principle only within the respective inertial system. If one proceeds according to this principle, it follows that a state of absolute rest cannot be inserted; this leads to apparent contradictions, and then the principle, that a system of absolute rest can exist, must be rejected as erroneous. This will be an important consideration in a final study of the speed of light in chapter 13.1.

In the following, another important aspect on the subject of the speed of light will be dealt with. The statement: "The speed of light is the same in all inertial frames" must be considered and interpreted carefully.

However, if several test participants from different inertial systems moving against each other observe the *same* event, e.g. the signal exchange between different spatially separated points, different observations must occur. If the speed of light of the own system is taken as a basis for measurements and if the times and distances necessary for the signal exchange are determined for the way there and back, they come to different results. Path and time are *not* divided symmetrically. This effect is caused by the "relativity of simultaneity".

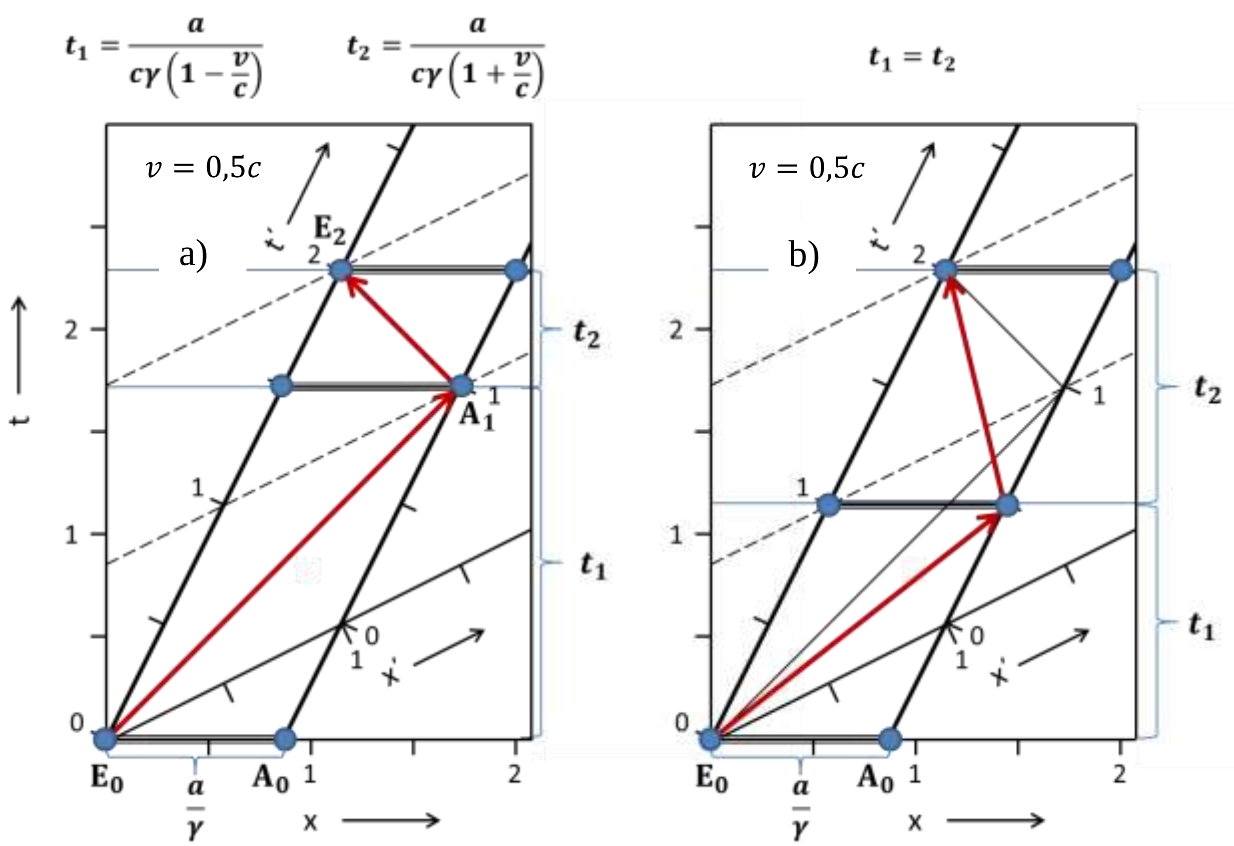

Fig. 12.3: Schematic presentation of a signal in a laboratory L between E and A from the point of view of an inertial system S moving relative to it ($v = 0{,}5c$).
a) Correct: $c =$ const. referred to S.
b) Not correct: $t_1 = t_2$ referred to S

To make this clear, the situation is shown in fig. 12.3. While the situation is always clear for an observer at rest (the outward and return paths are of equal length and the individual times are also equal), this does not apply to an observer from an inertial system S moving relative to the lab.

The determination of the Einstein synchronization, i.e. at the outward and return path for the signal exchange between two points (e.g. the ends of a laboratory A and E) time and path are in each case divided to the half, is valid only subjectively for the system L which is in rest to the laboratory. If from another inertial system S moving relative to it this determination would also apply and the times $t_1 = t_2$ would be equal, the situation would arise as shown in the right part of the diagram with signal velocities larger or smaller than c as well as measurable synchronization differences. Moreover, according to these considerations, a situation where the path is constant in both directions cannot even theoretically occur because the lab end moves away from the original point immediately after the signal is emitted and is at a different location on the return path. Instead, the situation as shown in the left partial picture applies. This means that the determination of a reference system can always only be subjective.

12.3 Integration of a system at absolute rest into the Lorentz-Equations

The approaches to combine a system of absolute rest with the Lorentz-equations presented in chapter 11.1 were obviously not successful. In the following it will be examined whether it is possible for two observers moving in arbitrary directions against each other to integrate an additional superior system which is at absolute rest. In this case the use of the Lorentz equations must lead to a consistent connection without discrepancies. First a simple comparison reveals the fact, that this must be possible because the discussed equations can be considered as a mathematical group. The implementation of the Lorentz equations in a system A→ B can therefore easily being carried out using A→ S→ B, where S could be a system with a basis at absolute rest.

Because of the importance of this proposition the validity of this correlation will be presented here in detail. To show this, the possible constellations between the observers will be treated separately in the following.

1. Observers A and B are moving on a straight line in relation to S

In the following the experimental relation shall be examined in an analytical way, where the system at rest S and an arbitrary reference system 1 with observer A, which is moving in relation to it with v_0 and the investigated system 2 with observer B (moving with v_1 compared to the reference system) are lined up and $v_0 < v_1$ applies. To simplify the calculation the values of the velocities shall be replaced by their quotient to the speed of light c.

The Lorentz equations between Reference System 1 and the investigated System 2 are given by

$$x_2 = \gamma_1(x_1 - v_1 t_1) \tag{12.40}$$

$$t_2 = \gamma_1(t_1 - v_1 x_1) \tag{12.41}$$

where x_1 and t_1 are coordinates of the Reference System 1 and x_2 and t_2 coordinates of the investigated System 2, which is traveling with speed v_1 compared to system 1. If a system which is at absolute rest is introduced, then system 1 will generally show a movement compared to this. In view of the system at rest the following relations apply

$$x_1 = \gamma_0(x_0 - v_0 t_0) \tag{12.42}$$

$$t_1 = \gamma_0(t_0 - v_0 x_0) \tag{12.43}$$

$$x_2 = \gamma_2(x_0 - v_2 t_0) \tag{12.44}$$

$$t_2 = \gamma_2(t_0 - v_2 x_0) \tag{12.45}$$

where v_0 is the speed between system at rest and Reference System 1, whereas v_2 represents the speed between the system at rest and system 2. Furthermore, the equation for relativistic addition of velocities applies

$$v_2 = \frac{v_0 + v_1}{1 + v_0 v_1} \tag{12.46}$$

Equations Eq. (12.42) and (12.43) are leading to the following relationship for the coordinates x_0 and t_0

$$x_0 = \gamma_0(x_1 + v_0 t_1) \tag{12.47}$$

$$t_0 = \gamma_0(t_1 + v_0 x_1) \tag{12.48}$$

In combination with (12.44) and (12.45) this yields

$$x_2 = \gamma_2\big(\gamma_0(x_1 + v_0 t_1) - v_2\gamma_0(t_1 + v_0 x_1)\big) \tag{12.49}$$

$$x_2 = \gamma_2\gamma_0\big((1 - v_0 v_2)x_1 - (v_2 - v_0)t_1\big) \tag{12.50}$$

$$t_2 = \gamma_2\big(\gamma_0(t_1 + v_0 x_1) - v_2\gamma_0(x_1 + v_0 t_1)\big) \tag{12.51}$$

$$t_2 = \gamma_2\gamma_0\big((1 - v_0 v_2)t_1 - (v_2 - v_0)x_1\big) \tag{12.52}$$

The equations (12.40) and (12.41) shall be identical with equations (12.50) resp. (12.51). To prove this a comparison of coefficients is carried out and the following equations apply

(12.40) ⇔ (12.50)	x_1:	$\gamma_1 = \gamma_2\gamma_0(1 - v_0 v_2)$	(12.53)
(12.40) ⇔ (12.50)	t_1:	$v_1\gamma_1 = \gamma_2\gamma_0(v_2 - v_0)$	(12.54)
(12.41) ⇔ (12.51)	t_1:	$\gamma_1 = \gamma_2\gamma_0(1 - v_0 v_2)$	(12.55)
(12.41) ⇔ (12.51)	x_1:	$v_1\gamma_1 = \gamma_2\gamma_0(v_2 - v_0)$	(12.56)

Obviously, the equations (12.53) and (12.55) as well as (12.54) and (12.56) are identical. Because of

$$v_1 = \frac{v_2 - v_0}{1 - v_0 v_2} \tag{12.57}$$

Eq. (12.54) can be replaced by Eq. (12.53) since

$$(v_2 - v_0)\gamma_1 = \gamma_2\gamma_0(v_2 - v_0)(1 - v_0 v_2) \tag{12.58}$$

It is now proved that all 4 equations are identical. To show the validity of the complete system it is necessary to validate only one of these equations.

If now both sides of the Eq. (12.54) are squared and the respective values for γ are inserted, it follows

$$\frac{v_1^2}{(1 - v_1^2)} = \frac{(v_2 - v_0\,)^2}{(1 - v_2^2)\cdot(1 - v_0^2)} \tag{12.59}$$

and

$$(1 - v_2 v_0)^2 v_1^2 = (v_2 - v_0)^2 \tag{12.60}$$

If for v_2 the equation (12.46) is used, then

$$\left(1 - \frac{v_0 + v_1}{1 + v_0 v_1} v_0\right)^2 v_1^2 = \left(\frac{v_0 + v_1}{1 + v_0 v_1} - v_0\right)^2 \tag{12.61}$$

If this equation is expanded completely, then 20 terms will occur which will add up to zero. It was thus shown for this case that the integration of a system at rest will not lead to any violations or to mathematical inconsistencies by using the Lorentz equations. Modified conditions taking $v_0 > v_1$ into consideration lead to the same result, because in any case only linear conditions are present which can be combined without restrictions.

When an arbitrary dependency between the combinations of velocities for the movement of observers in different directions is considered, however, the calculation will be more difficult. In this case the observers will not contact each other but approaching to a minimum before they increase the distance again. It was already shown in chapter 2.1.2, that for any observer in a system at rest (A) or in a moved system (B) there is no difference in their observation of the situation and that it is not possible for both of them to decide with measurements during a signal exchange, whether they are moving or at rest. If a system of absolute rest is integrated, which is different from zero to an observer A which was stipulated to be as at rest before, then the calculation will be more complex, but the situation can be simplified considerably if a suitable point of origin for the calculation is defined.

For simple calculation the fact is used that the direction vectors of both observers are passing along a straight line. If the vectors are moving along these lines the correlation between them are changing as a linear quantity, which means in a mathematical sense a constant is added which can be subtracted later after the calculation is finalized. Two different cases must be dealt with:

2. The straight lines of the direction vectors are intersecting

For this purpose, the fact is used that if a system at rest is assumed then no further requirements concerning the point of origin are necessary from which the examination would have to start. This means that out of the unlimited possibilities the point of origin can be defined in a way that A is distancing to it and is part of the directional vector; this line is defined as corresponding to the x-axis. Further the vectors of both observers are moved in such a way that they are intersecting. These are the conditions to determine the point of origin as zero-coordinates of x, y, t in view of the system at rest S, the values for the z axis are always zero due to the definition of the coordinates. In this case the correlations must follow the Lorentz equations.

For verification the following experiment shall be discussed: Starting from observer A observer B is departing with an arbitrary angle in relation to the x-axis. After a certain time Δt this observer is emitting a signal. The related coordinates will be determined by observer A and in the system at rest S. When these are identical after use of the Lorentz equations then the system at rest can be integrated without discrepancies.

The following calculations apply:

Observer A finds that the signals transmitted by observer B distancing with the velocity v_1 are arriving with the delay

$$t_1 = \gamma_1 \Delta t \tag{12.62}$$

The connected coordinates are

$$x_1 = v_1 t_1 cos\ \alpha' \tag{12.63}$$

$$y_1 = v_1 t_1 sin\ \alpha' \tag{12.64}$$

In view of system S the velocity of observer B is calculated according to Eq. (4.20), see also chapter 4.1:

$$v_2 = \frac{\sqrt{(v_0^2 + v_1^2 + 2v_0 v_1 cos\alpha') - (v_0 v_1 sin\alpha')^2}}{1 + v_0 v_1 cos\alpha'} \tag{12.65}$$

where in his view the velocity of A is equal to v_0. The angle α measured by S is following equation Eq. (7.43)

$$\alpha = arctan\left[\frac{sin\ \alpha'}{\gamma_0\left(cos\alpha' + \frac{v_0}{v_1}\right)}\right] \tag{12.66}$$

(For details see chapter 7.2). Analogous to the coordinates found before it is

$$t_2 = \gamma_2 \Delta t \tag{12.67}$$

$$x_2 = v_2 t_2 cos\alpha \tag{12.68}$$

$$y_2 = v_2 t_2 sin\alpha \tag{12.69}$$

Finally, the coordinates are calculated which can be found using the Lorentz equations and it applies

$$t_1' = \gamma_0 (t_2 - v_0 x_2) \tag{12.70}$$

$$x_1' = \gamma_0 (x_2 - v_0 t_2) \tag{12.71}$$

The following correlations must apply:

$$t_1' = t_1 \tag{12.72}$$

$$x_1' = x_1 \tag{12.73}$$

$$y_2 = y_1 \tag{12.74}$$

Eq. (12.74) shows that the values in y direction are the same in all systems, what is a direct requirement of the Lorentz transformation.

α'	t_1	x_1	y_1	v_2	α	t_2	x_2	y_2	t'_1	x'_1
0	1,154701	0,577350	0,000000	0,571429	0,00000	1,218544	0,696311	0,000000	1,154701	0,577350
15	1,154701	0,557678	0,149429	0,569508	12,45513	1,216566	0,676539	0,149429	1,154701	0,557678
30	1,154701	0,500000	0,288675	0,563786	25,01756	1,210770	0,618571	0,288675	1,154701	0,500000
45	1,154701	0,408248	0,408248	0,554386	37,79756	1,201548	0,526357	0,408248	1,154701	0,408248
60	1,154701	0,288675	0,500000	0,541551	50,91089	1,189531	0,406181	0,500000	1,154701	0,288675
75	1,154701	0,149429	0,557678	0,525691	64,48031	1,175536	0,266234	0,557678	1,154701	0,149429
90	1,154701	0,000000	0,577350	0,507445	78,63457	1,160518	0,116052	0,577350	1,154701	0,000000
105	1,154701	-0,149429	0,557678	0,487753	93,50218	1,145500	-0,034130	0,557678	1,154701	-0,149429
120	1,154701	-0,288675	0,500000	0,467905	109,19583	1,131505	-0,174078	0,500000	1,154701	-0,288675
135	1,154701	-0,408248	0,408248	0,449528	125,78294	1,119487	-0,294253	0,408248	1,154701	-0,408248
150	1,154701	-0,500000	0,288675	0,434472	143,24177	1,110266	-0,386467	0,288675	1,154701	-0,500000
165	1,154701	-0,557678	0,149429	0,424533	161,41618	1,104469	-0,444435	0,149429	1,154701	-0,557678
180	1,154701	-0,577350	0,000000	0,421053	0,00000	1,102492	-0,464207	0,000000	1,154701	-0,577350
0	1,154701	0,577350	0,000000	0,800000	0,00000	1,666667	1,333333	0,000000	1,154701	0,577350
15	1,154701	0,557678	0,149429	0,796896	6,50446	1,655309	1,310617	0,149429	1,154701	0,557678
30	1,154701	0,500000	0,288675	0,787340	13,06431	1,622008	1,244017	0,288675	1,154701	0,500000
45	1,154701	0,408248	0,408248	0,770588	19,73390	1,569036	1,138071	0,408248	1,154701	0,408248
60	1,154701	0,288675	0,500000	0,745356	26,56505	1,500000	1,000000	0,500000	1,154701	0,288675
75	1,154701	0,149429	0,557678	0,709783	33,60502	1,419606	0,839213	0,557678	1,154701	0,149429
90	1,154701	0,000000	0,577350	0,661438	40,89339	1,333333	0,666667	0,577350	1,154701	0,000000
105	1,154701	-0,149429	0,557678	0,597477	48,45800	1,247060	0,494121	0,557678	1,154701	-0,149429
120	1,154701	-0,288675	0,500000	0,515079	56,30993	1,166667	0,333333	0,500000	1,154701	-0,288675
135	1,154701	-0,408248	0,408248	0,412289	64,43855	1,097631	0,195262	0,408248	1,154701	-0,408248
150	1,154701	-0,500000	0,288675	0,289259	72,80788	1,044658	0,089316	0,288675	1,154701	-0,500000
165	1,154701	-0,557678	0,149429	0,149449	81,35612	1,011358	0,022716	0,149429	1,154701	-0,557678
180	1,154701	-0,577350	0,000000	0,000000	0,00000	1,00000	0,000000	0,000000	1,154701	-0,577350
0	1,005038	0,100504	0,000000	0,571429	0,00000	1,218544	0,696311	0,000000	1,005038	0,100504
15	1,005038	0,097079	0,026012	0,569508	2,15163	1,216566	0,692356	0,026012	1,005038	0,097079
30	1,005038	0,087039	0,050252	0,563786	4,22175	1,210770	0,680763	0,050252	1,005038	0,087039
45	1,005038	0,071067	0,071067	0,554386	6,12440	1,201548	0,662320	0,071067	1,005038	0,071067
60	1,005038	0,050252	0,087039	0,541551	7,76517	1,189531	0,638285	0,087039	1,005038	0,050252
75	1,005038	0,026012	0,097079	0,525691	9,03827	1,175536	0,610295	0,097079	1,005038	0,026012
90	1,005038	0,000000	0,100504	0,507445	9,82643	1,160518	0,580259	0,100504	1,005038	0,000000
105	1,005038	-0,026012	0,097079	0,487753	10,00607	1,145500	0,550222	0,097079	1,005038	-0,026012
120	1,005038	-0,050252	0,087039	0,467905	9,46232	1,131505	0,522233	0,087039	1,005038	-0,050252
135	1,005038	-0,071067	0,071067	0,449528	8,11836	1,119487	0,498198	0,071067	1,005038	-0,071067
150	1,005038	-0,087039	0,050252	0,434472	5,97964	1,110266	0,479755	0,050252	1,005038	-0,087039
165	1,005038	-0,097079	0,026012	0,424533	3,18024	1,104469	0,468161	0,026012	1,005038	-0,097079
180	1,005038	-0,100504	0,000000	0,421053	0,00000	1,102492	0,464207	0,000000	1,005038	-0,100504

Tab. 12.2: Comparison of calculations using Lorentz-Transformation. Values marked grey: Approximation (otherwise division by zero); Presentation in frames: 180 °+angle Equations for $t_1 \rightarrow$ Eq. (12.33) to $x'_1 \rightarrow$ Eq. (12.42) see text.

An analytical solution of these equations is complex, a direct numerical comparison not. In tab. 12.2 the results for the calculation of different angles between A and B and varying velocities are presented. No differences occur and Eq. (12.72), (12.73) and (12.74) are unrestrictedly valid.

3. The straight lines of the direction vectors are not intersecting

In the case where the direction vectors of observers A and B are not intersecting, this means in the terminology of analytical geometry, that the straight lines are "out of square". For the solution of this problem first the position must be determined where the distance between the straight lines for both observers reach a minimum. In this case, here (and only here) the angle of the connecting line is matching the value of 90° in relation to the straight lines for both observers.

This connecting line is now selected as basis for the z-axis, which played no role in the interpretation up to now. The intersection point with the x-axis is now defined as origin of the coordinate system and the direction of the y-axis is perpendicular to both. When observer B has reached the minimum distance to the center of origin with distance z_{min} on the z-axis then $x = y = 0$ applies. Now the fact is used that the values in z-direction do not change during Lorentz transformation and that therefore a projection by factor z_{min} is possible. The situation appearing now is identical to the case, where the direction vectors showed intersection. So, in a final statement it can be noticed, that it was possible to prove that a system of absolute rest can be integrated in any arbitrary inertial system without violation of the Lorentz equations or showing any other discrepancies.