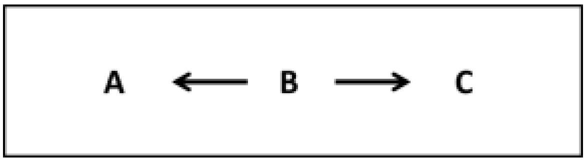
5. Clock transport

It is well known, that according to Special Relativity during an exchange of signals between two observers only a mutual consideration of the time needed in both directions is possible. Nevertheless, in the past effort was made to measure the one-way speed of light inside a system in motion. One of these attempts to perform a separate measurement was the examination of the effect that occurs, when clocks are moved at slow speed inside a moving observer. In this case a system in motion is defined, where two clocks after an Einstein Synchronization are lined up and one is following the other. To execute the experiment the clocks are moved in this system in a way, that after the end of the trial they have changed their positions. When the experiment is carried out at low speed the synchronization should maintain its original values and after a further synchronization process a difference should appear.

Since some time it is clear, however, that the effects measured by both clocks is changing exactly corresponding to their position inside the system and therefore leading to a null result (see i.e. [19,40]). This important verification and the necessary calculations are presented here, first simply by means of an example and afterwards in a general way. Further in this chapter the well-known twin paradox will be discussed, and it will appear as a special case of the clock transport.

5.1 Clock transport in direction of motion

To define an appropriate experimental set-up it is assumed, that in a laboratory 3 observers A, B and C are lined up equidistant.



First the case is considered that the observers are at rest. To start the experiment observer B is sending out synchronized clocks with the same speed to A and C. After the arriving of the clocks at A and C it is found that these - depending on the speed they were moved - are running slow compared to the clocks at rest because of time dilatation. Further A and C after exchanging of experiment data conclude that the moved clocks arrived at the same time at their positions.

It is now considered that the laboratory is accelerated and afterwards moving with a constant speed. The existing clocks shall then be synchronized. If an effect that could be measured inside the system would occur, it must be possible to find it out in one (or both) of the ways presented in the following:

1. Observers A and C find differences in the arriving time of the clocks sent out by observer B in comparison to the results of the experiments in a system at rest.
2. The moved clocks show differences when they arrive at A and C compared to the situation of a system at rest.

It shall be presented in the following, that inside a system at rest compared to a system in motion the same results will be achieved. This simplified statement can be extended to the proposition that it is valid also for any arbitrary inertial system, which means it is a system not accelerated and without rotation. The statement is therefore valid universally.

5.1.1 Qualitative Considerations

Fig. 5.1 shows the situation, that in a laboratory at rest (left) and in motion (right) at the time zero a light signal is emitted from position B in direction to the back end (A) and the front end (C). These signals are reaching A and C at the positions c_1 and a_1 as shown in the diagram. In this presentation further the situation with moving clocks starting from point B is added.

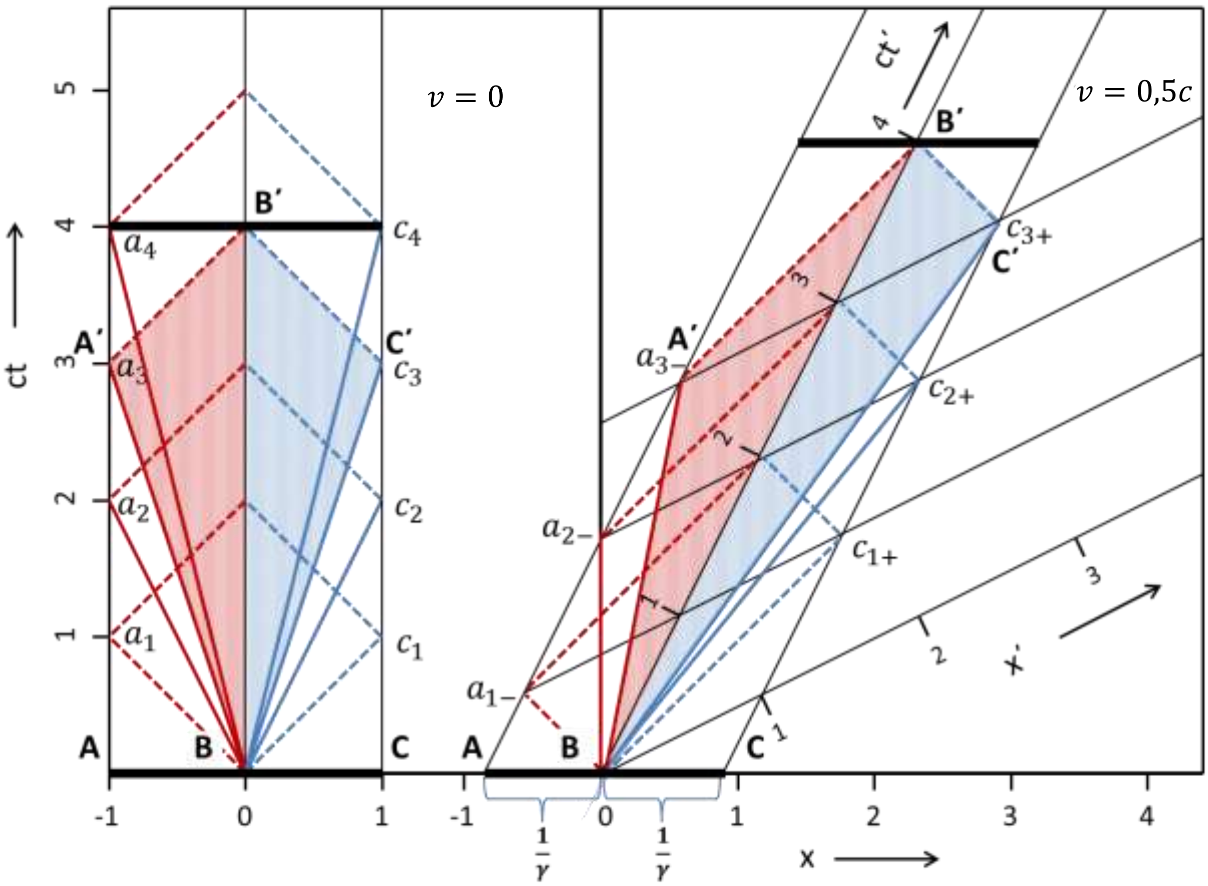


Fig. 5.1: Space-time-diagram for clock transport
Dotted lines: Signal exchange

First the laboratory at rest shall be looked at (left-hand side of the diagram). When the clocks are starting at time zero with a velocity of $1/2c$ they are reaching the positions c_2 as well as a_2 after 2 time-units, when the speed is $1/4c$ then 4 time-units are necessary for the positions c_4 and a_4 etc. All possible times for receiving the signals can be realized depending on the velocities of the moved clocks.

When a moving system is considered, however, for an observer at rest some differences in the situation would occur (right-hand side of the diagram), i.e. differences in the times to reach c_n and a_n, further the distance 1 is changing to $1/\gamma$ etc. These changes are described by the Lorentz-Transformation.

In the following the situation for an observer in motion shall be discussed. This is presented in Fig. 5.1 by means of marked zones (blue: in moving direction, red: opposite direction). The following relation applies for the system at rest

$$v = \left(0{,}5 \pm \frac{1}{3}\right) \cdot c \tag{5.01}$$

and for the observers in motion

$$v_{c3+} = \frac{0{,}5 + 0{,}\overline{3}}{1 + 0{,}5 \cdot 0{,}\overline{3}} c = 0{,}714c \tag{5.02}$$

$$v_{a3-} = \frac{0{,}5 - 0{,}\overline{3}}{1 - 0{,}5 \cdot 0{,}\overline{3}} c = 0{,}2c \tag{5.03}$$

To simplify the calculations the following definitions shall be introduced: The values for time, space and speed of light c are scaled to 1, the results of the velocities are therefore defined as fractions of c.

The arrival time and the factor γ is

$$\begin{aligned} t_{c3+} &= 4{,}041 \qquad & \gamma_{c3+} &= 1{,}429 \\ t_{a3-} &= 2{,}887 \qquad & \gamma_{a3-} &= 1{,}021 \end{aligned} \tag{5.04}$$

The passed (subjective) time for the observers is

$$\mathit{system\ in\ motion}: \quad \frac{t_{c3+}}{\gamma_{c3+}} = \frac{t_{a3-}}{\gamma_{a3-}} = 2{,}828 \tag{5.05}$$

This result is consistent with the values of the system at rest, because

$$t_{c3} = t_{a3} = 3 \qquad \gamma_{c3} = \gamma_{a3} = 1{,}061 \tag{5.06}$$

is valid and so the same result is obtained.

$$\mathit{system\ at\ rest}: \quad \frac{t_{c3}}{\gamma_{c3}} = \frac{t_{a3}}{\gamma_{a3}} = 2{,}828 \tag{5.07}$$

The presented deductions show that the subjectively measured time period for the transition to A and C of the moved observers is identical. Further the presentation makes clear, that the time measured for the arrival of the simultaneously moved clocks by the observers

A and C in their synchronized system is also the same. This makes it impossible inside a uniformly proceeding system, which is moving without acceleration or rotation, to take measurements with clocks or any other devices and find conclusions out of the received results about the velocity compared to other systems or to find deviations in the synchronization.

5.1.2 General derivation

The presented issue will now be verified in a general form. First it is necessary to define the following parameters:

System at rest	System in motion	
-	v_0	Velocity of the system in motion
Δv	v_+, v_-	Travelling speed of the moved observers
-	$\Delta t_A, \Delta t_A$	Synchronization difference to system at rest
t_0	t_+, t_-	Arrival time of moved observers
t'_0	t'_+, t'_-	Subjective travelling time of moved observers
γ_Δ	γ_+, γ_-	Lorentz-factor of moved observers

These parameters are presented in a modified Minkowski-diagram (see Fig. 5.2). The experimental set-up is the following:

From position B in the middle of a laboratory at rest, signals are sent to the positions at both ends A and C and arrive here at the time t' (left side of the diagram, positions marked with A′ and C′). At the same time 2 synchronized clocks start moving from the position B with an arbitrary velocity Δv which is the same for both. They arrive at their positions at time t'' (marked with A″ and C″); directly afterwards signals are sent back to position B. In the right part of the diagram the situation is presented for an observer in motion. The differences in moving direction and opposite to it are in conformance with the Lorentz equations.

In the following it is demonstrated that the observers taking part in this experiment are not able to detect differences in the measurements of the elapsing time. In detail these are the considerations:

1. The observers in motion cannot decide on basis of their measurements whether the system is moving or not.
2. The observers at rest find during their measurements - independent of the velocity of the moving system - the same time periods for the arriving of the moving observers.

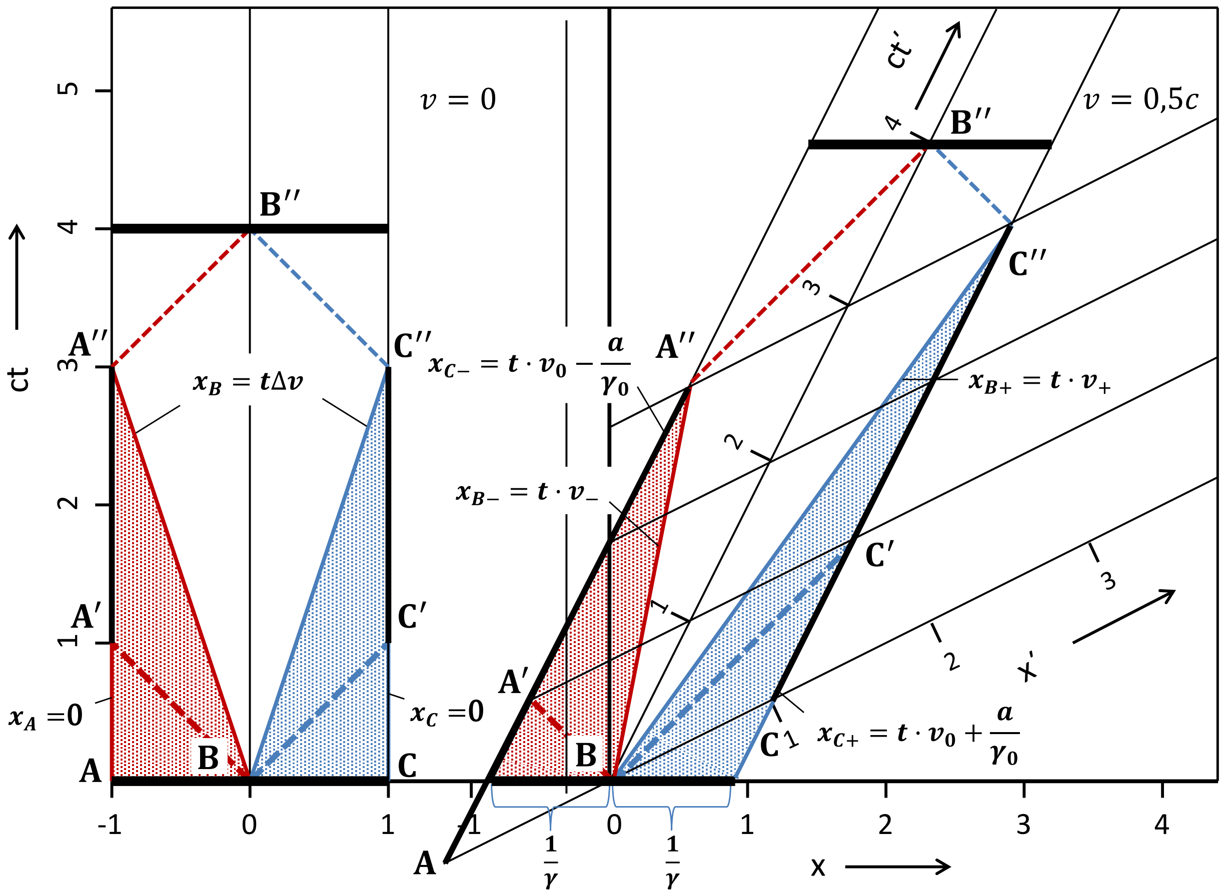


Fig. 5.2: Space-time-diagram for clock transport with defined parameters
Dotted lines: Signal exchange

The different issues are now dealt with separately.

5.1.3 Identical time schedules for the arriving of moved observers

The following issues shall be reviewed:

a) The synchronization differences in a moving system Δt_A and Δt_C for the observers A and C relating to B

b) The time periods t_- and t_+ the observers in motion need to reach the positions A and C

c) The difference between both values. When the result (multiplied by γ_0) is corresponding to the values of the system at rest, then the measuring results are not distinguishable from each other.

a) Synchronization differences

To determine the synchronization differences, it is first necessary to identify the travelling time a light signal needs starting from B to the positions A´ resp. C´. This is

$$\Delta t_{B\to A'} = \frac{a}{c\gamma_0(1+\frac{v_0}{c})} \tag{5.08}$$

$$\Delta t_{B\to C'} = \frac{a}{c\gamma_0(1-\frac{v_0}{c})} \tag{5.09}$$

The value which is necessary to reach the starting point is subtracted

$$\Delta t_{A'\to A} = \Delta t_{C'\to C} = \frac{a}{c}\gamma_0 \tag{5.10}$$

Thus, the synchronization leads to values

$$\Delta t_A = \frac{a}{c\gamma_0(1+\frac{v_0}{c})} - \frac{a}{c}\gamma_0 = -\frac{\gamma_0 a v}{c^2} \tag{5.11}$$

and

$$\Delta t_C = \frac{a}{c\gamma_0(1-\frac{v_0}{c})} - \frac{a}{c}\gamma_0 = \frac{\gamma_0 a v}{c^2} \tag{5.12}$$

b) Time for observers in motion

The time a moved observer needs to reach the positions A′ resp. C′ in a system at rest is

$$t_0 = \frac{a}{\Delta v} \tag{5.13}$$

To determine this in a system in motion the values of x_{B+} and x_{C+} (with $t \to t_+$) resp. x_{B-} and x_{C-} (with $t \to t_-$) are set equal and this results in (see Fig. 5.2)

$$t_+ = \frac{a}{\gamma_0(v_+ - v_0)} \tag{5.14}$$

$$t_- = \frac{a}{\gamma_0(v_0 - v_-)} \tag{5.15}$$

c) Consideration of differences

In the following the differences between Δt_A and t_- resp. Δt_C and t_+ are considered. In a system at rest this is

$$\Delta t_{A\to A''} = \Delta t_{C\to C''} = \frac{a}{\Delta v} \tag{5.16}$$

In a system in motion this changes to

$$\Delta t_{A\to A''} = \Delta t_{C\to C''} = \gamma_0 \frac{a}{\Delta v} \tag{5.17}$$

If

$$t_- = \Delta t_A + \Delta t_{A\to A''} \tag{5.18}$$

with

$$\frac{a}{\gamma_0(v_0 - v_-)} = \frac{a}{c\gamma_0(1+\frac{v_0}{c})} - \frac{a}{c}\gamma_0 + \gamma_0\frac{a}{\Delta v} \tag{5.19}$$

$$t_+ = \Delta t_C + \Delta t_{C\to C''} \tag{5.20}$$

$$\frac{a}{\gamma_0(v_+ - v_0)} = \frac{a}{c\gamma_0(1-\frac{v_0}{c})} - \frac{a}{c}\gamma_0 + \gamma_0\frac{a}{\Delta v} \tag{5.21}$$

is valid, no differences can be detected inside a system.

To simplify the calculation the equations shall be multiplied with c/a and the values of the velocities are replaced by their quotient to the speed of light c

$$v'_+ = \frac{v_+}{c} \quad v'_- = \frac{v_-}{c} \quad v'_0 = \frac{v_0}{c} \quad \Delta v' = \frac{\Delta v}{c} \tag{5.22}$$

Eq. 5.19 is developing to

$$\frac{1}{\gamma_0(v'_0 - v'_-)} = \frac{1}{\gamma_0(1 + v'_0)} - \gamma_0 + \frac{\gamma_0}{\Delta v'} \tag{5.23}$$

and Eq. 5.21 changes to

$$\frac{1}{\gamma_0(v'_+ - v'_0)} = \frac{1}{\gamma_0(1 - v'_0)} - \gamma_0 + \frac{\gamma_0}{\Delta v'} \tag{5.24}$$

Inserting the values

$${\gamma_0}^2 = \frac{1}{1 - {v'_0}^2} \tag{5.25}$$

then after simple transformation of Eq. 5.23

$$(1 + v'_-)(1 - v'_0) = -v'_0 + \frac{v'_0}{\Delta v'} + v'_- - \frac{v'_-}{\Delta v'} \tag{5.26}$$

can be derived. Further

$$v'_- = \frac{v'_0 - \Delta v'}{1 - v'_0 \cdot \Delta v'} \tag{5.27}$$

and from Eq. 5.24

$$(1 - v'_+)(1 + v'_0) = -v'_+ + \frac{v'_+}{\Delta v'} + v'_0 - \frac{v'_0}{\Delta v'} \tag{5.28}$$

$$v'_+ = \frac{v'_0 + \Delta v'}{1 + v'_0 \cdot \Delta v'} \tag{5.29}$$

is valid. These results correspond exactly to the definitions of v'_-. and v'_+. It is thus shown that inside a system the observers A and C are not able to find differences in the arriving time of a moved observer. The subjective time periods are completely independent whether the system is moving or not.

5.1.4 Identical time periods at arrival for moving observers

The time period a moving observer needs to reach the positions A or C in a system at rest is

$$t_0 = \frac{a}{\Delta v} \tag{5.30}$$

and in the moving system

$$t_+ = \frac{a}{\gamma_0(v_+ - v_0)} \tag{5.31}$$

$$t_- = \frac{a}{\gamma_0(v_0 - v_-)} \tag{5.32}$$

The time subjectively measured by the moving observer is here

$$t'_0 = \frac{a}{\gamma_\Delta \Delta v} \tag{5.33}$$

$$t'_+ = \frac{a}{\gamma_+ \gamma_0 (v_+ - v_0)} \tag{5.34}$$

$$t'_- = \frac{a}{\gamma_- \gamma_0 (v_0 - v_-)} \tag{5.35}$$

If the subjectively measured time is identical then the relation applies

$$t'_0 = t'_+ = t'_- \tag{5.36}$$

First this is discussed for the case $t'_0 = t'_+$. Thus

$$\frac{a}{\gamma_+ \gamma_0 (v_+ - v_0)} = \frac{a}{\gamma_\Delta \Delta v} \tag{5.37}$$

must be valid. This leads to

$$\frac{\gamma_\Delta}{\gamma_+ \gamma_0} = \frac{(v_+ - v_0)}{\Delta v} \tag{5.38}$$

To simplify the calculation again the values of the velocities are replaced by their quotient to the speed of light c

$$v'_+ = \frac{v_+}{c} \quad v'_- = \frac{v_-}{c} \quad v'_0 = \frac{v_0}{c} \quad \Delta v' = \frac{\Delta v}{c} \tag{5.39}$$

When in equation (5.38) the values of γ are inserted, then

$$\frac{(1 - {v'_+}^2)(1 - {v'_0}^2)}{1 - \Delta v'^2} = \frac{(v'_+ - v'_0)^2}{\Delta v'^2} \tag{5.40}$$

and

$$(1 - v'_+ v'_0)^2 \Delta v'^2 = (v'_+ - v'_0)^2 \tag{5.41}$$

When v_+ is replaced by

$$v'_+ = \frac{v'_0 + \Delta v'}{1 + v'_0 \cdot \Delta v'} \tag{5.42}$$

then

$$\left(1 - \frac{v'_0 + \Delta v'}{1 + v'_0 \cdot \Delta v'} v'_0\right)^2 \Delta v'^2 = \left(\frac{v'_0 + \Delta v'}{1 + v'_0 \cdot \Delta v'} - v'_0\right)^2 \tag{5.43}$$

If this equation is expanded completely, then 20 terms will occur which will add up to zero. The same procedure can be applied to $t'_0 = t'_-$. With

$$\frac{\gamma_\Delta}{\gamma_- \gamma_0} = \frac{(v'_0 - v'_-)}{\Delta v'} \tag{5.44}$$

and

$$v_- = \frac{v'_0 - \Delta v'}{1 - v'_0 \cdot \Delta v'} \tag{5.45}$$

the same result will be realized. Thus, it is shown that the subjective measurements of the moving observers do not differ from the results achieved at rest.

It is now generally verified that inside a moving system no possibility exists to find deviations caused by "slow clock transport" when using synchronized clocks in comparison to a reference system at rest.

5.2 Twin paradox

One of the best-known examples connected with the theory of Special Relativity is the twin paradox. This issue covers a long history in literature (see i.e. a comprehensive summary in [41]). In general, a pair of twins is looked at, where one is at rest (remaining at earth) while the other is leaving with a fast spaceship and comes back later. This twin will be aged less compared to the one who remained on earth. The paradox occurs because according to Special Relativity both twins should be considered as equal and therefore the travelling twin after his return should find the remaining twin also in a condition aged less.

The solution to overcome the contradictions is possible because the twin in the spaceship is changing the inertial system during his trip.

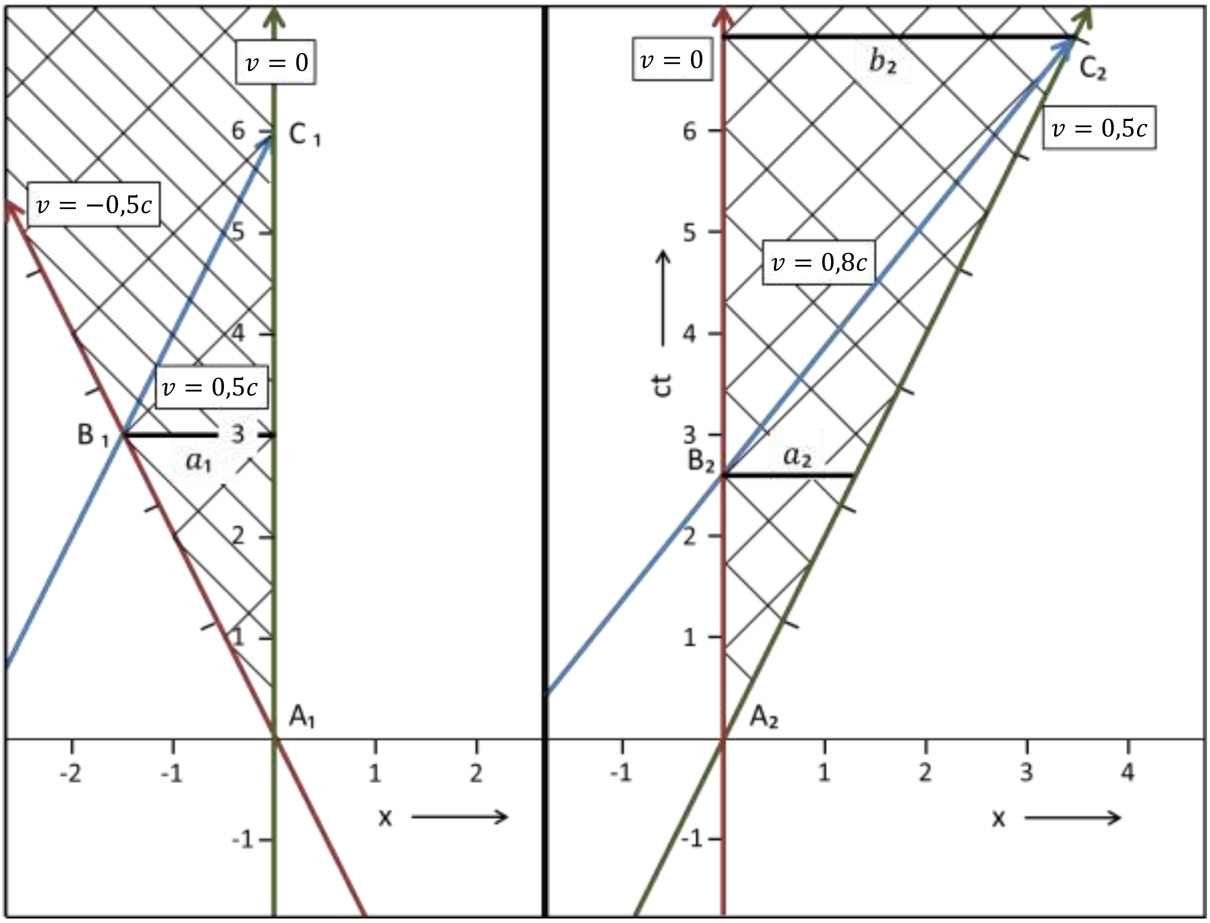


Fig. 5.3: Presentation of the twin paradox
Left: Observer A at rest, B in motion
Right : Observer A in motion, B at rest (at the beginning)

In Fig. 5.3 this case is presented on the left side of the diagram. On the right side the situation is presented, that the observers change their perspective and the one who was first considered as at rest is moving and vice versa. To avoid influences during changing of the direction, the experimental set-up is modified in a way that 3 observers take part (in Fig. 5.3 marked with the colors green, red, and blue) and each of the observers is in possession of a precise clock [41]. At the positions A_1 and B_1 resp. A_2 and B_2 the clocks are synchronized and at the end of the trial the results are evaluated. In this presentation the problem finally has the same status as the issue of a slow clock transport.

If the situations are comparable, then the subjective measuring results must be the same for all observers taking part in the trial. This shall be demonstrated in the following. The important issues are the total travelling time from the start to the end of the journey, and the subjective time periods for the moving observers, which must be identical from the start to the returning point and from that to the end. The total time for the observer at rest is defined as t_0 as shown in the left part of the diagram. The other parameters are presented in the following table.

System at rest	System in motion	
t_T	t'_T	Total time from start (A) to the end of journey (C)
t_1	t'_1	Time for the first part of the journey (A→ B)
t_2	t'_2	Time for the second part of the journey (B→ C)
-	v'_1	Velocity for $A_1 \rightarrow B_1$, $B_1 \rightarrow C_1$, $A_2 \rightarrow C_2$
-	v'_2	Velocity for $B_2 \rightarrow C_2$
-	γ_1	Lorentz factor for v'_1
-	γ_2	Lorentz factor for v'_2

Remark:

The velocities are always taken as ratio to the speed of light, i.e.

$$v'_1 = \frac{v_1}{c} \qquad v'_2 = \frac{v_2}{c} \tag{5.50}$$

<u>a) Total time</u>

<u>Left</u>: The total time t_T is defined as

$$t_T = t_0 \tag{5.51}$$

and for t'_T is valid

$$t'_T = t_1 + t_2 = \frac{t_0}{\gamma_1} \tag{5.52}$$

where in this case because of symmetry reasons applies

$$t'_1 = t'_2 = \frac{t'_T}{2} = \frac{t_0}{2\gamma_1} \tag{5.53}$$

Right: Because the subjective time periods t_1 shall be the same in both cases it must be valid

$$t_1 = \frac{t_0}{2\gamma_1} \tag{5.54}$$

The time t_2 can be derived using relations concerning b_2 (see Fig. 5.3, right), because for v'_1 and v'_2 applies

$$v'_1(t_1 + t_2) = v'_2 t_2 \tag{5.55}$$

$$t_2 = \frac{v'_1 t_1}{v'_2 - v'_1} \tag{5.56}$$

Further for v'_2 because of the same velocities during the round trip for the relativistic addition of velocities according to Eq. (4.21) applies

$$v'_2 = \frac{2v'_1}{1 + {v'_1}^2} \tag{5.57}$$

This leads to

$$t_T = t_1 + t_2 = \frac{t_0}{2\gamma_1} + \frac{v'_1 t_0}{2\gamma_1(v'_2 - v'_1)} = \frac{t_0}{2\gamma_1}\left(1 + \frac{v'_1}{v'_2 - v'_1}\right) \tag{5.58}$$

After insertion of Eq. (5.57) in Eq. (5.58) follows with

$$\gamma_1 = \sqrt{\frac{1}{1 - {v'_1}^2}} \tag{5.59}$$

$$t_T = \frac{t_0}{2}\gamma_1\left(1 - {v'_1}^2\right)\left(1 + \frac{v'_1}{\frac{2v'_1}{1 + {v'_1}^2} - v'_1}\right) \tag{5.60}$$

$$t_T = \frac{t_0}{2}\gamma_1\left(1 - {v'_1}^2\right)\left(1 + \frac{v'_1\left(1 + {v'_1}^2\right)}{v'_1 - {v'_1}^3}\right) \tag{5.61}$$

$$t_T = \gamma_1 t_0 \tag{5.62}$$

Because of

$$t'_T = \frac{t_T}{\gamma_1} \tag{5.63}$$

it applies

$$t'_T = t_0 \tag{5.64}$$

The measurements of subjective times are thus the same.

b) Single times

First it is necessary to calculate time t_2, which is subjectively elapsing for the observer in motion between B_2 and C_2.

According to Eq. (5.56) and (5.54) for the observer at rest applies

$$t_2 = \frac{v_1' t_0}{2\gamma_1 (v_2' - v_1')} \tag{5.65}$$

This leads to

$$t'_2 = \frac{v_1' t_0}{2\gamma_2 \gamma_1 (v_2' - v_1')} \tag{5.66}$$

When the subjective time periods for the left- and right-hand side of the diagram shall be the same then

$$\frac{t_0}{2\gamma_1} = \frac{v_1' t_0}{2\gamma_2 \gamma_1 (v_2' - v_1')} \tag{5.67}$$

This can be derived easily. First

$$\gamma_2 = \frac{v_1'}{v_2' - v_1'} \tag{5.68}$$

and using Eq. (5.57)

$$\gamma_2 = \frac{1 + {v_1'}^2}{1 - {v_1'}^2} \tag{5.69}$$

applies. Because of

$$\gamma_2 = \sqrt{\frac{1}{1 - {v_2'}^2}} \tag{5.70}$$

it applies

$$\frac{1 - {v_1'}^2}{1 + {v_1'}^2} = \sqrt{1 - \frac{4{v_1'}^2}{\left(1 + {v_1'}^2\right)^2}} \tag{5.71}$$

$$1 - {v_1'}^2 = \sqrt{\left(1 + {v_1'}^2\right)^2 - 4{v_1'}^2} \tag{5.72}$$

$$1 - {v_1'}^2 = \sqrt{1 - 2{v_1'}^2 + {v_1'}^4} \tag{5.73}$$

which is obviously the same. It is thus shown that the subjective measured times for the total distance and for the single parts of the trip are identical. The “paradox” is therefore not showing discrepancies.

5.3 Clock transport in arbitrary directions

When the clock transport in arbitrary spatial directions is considered the relation Eq. (4.20) must be used for relativistic addition of velocities.

$$v_T = \frac{\sqrt{v_1^2 + v_2^2 + 2v_1 v_2 cos\alpha - \left(\frac{v_1 v_2 sin\alpha}{c}\right)^2}}{1 + \frac{v_1 v_2 cos\alpha}{c^2}} \quad (4.20)$$

A simple example with $\alpha = 90°$ shows

$$v'_T = \sqrt{v_1'^2 + v_2'^2 - v_1'^2 v_2'^2} \quad (5.80)$$

This equation can be interpreted as a variant of the relation presented in Fig. 5.3 with the difference that all observers are moving with an additional speed of v_2. In this case the time dilatation during the trip from $A_1 \longrightarrow B_1$ is increasing in view of an observer at rest from γ_1 to $\gamma_1 \cdot \gamma_2$. This means that the following relation

$$\gamma_T = \gamma'_1 \gamma'_2 \quad (5.81)$$

must apply. This yield

$$\gamma_T = \frac{1}{\sqrt{1 - v_{ges}'^2}} \quad (5.82)$$

$$= \frac{1}{\sqrt{1 - (v_1'^2 + v_2'^2 - v_1'^2 v_2'^2)}} = \frac{1}{\sqrt{(1 - v_1'^2)(1 - v_2'^2)}} \quad (5.83)$$

which is obviously identical with Eq. (5.81). So, it is verified for this case also, that a linear combination of different motions will not lead to a possibility to measure differences of the elapsing time.

Summarizing the calculations, it was verified here, that no possibility exists to carry out measurements inside a system moving with constant speed and decide about its state of motion. All the discussed variants of the exchange of signals and the “slow clock transport” lead to a null result. Of course, this cannot be a surprise, because according to Special Relativity this is predicted.