7. Non-elastic processes

The situation concerning the elastic behavior during collisions was already discussed at length in chapter 6. The analysis of non-elastic processes is also of great importance for further considerations and shall now be examined in detail. At first the non-elastic collision will be scrutinized, where during the experimental situation two or more bodies are combined and an energy-absorption takes place. The reversing effect is observed during particle disintegration; in this case kinetic energy is set free because of conversion of mass into energy and carried away by the decay products. Non-elastic collision and particle disintegration can thus be interpreted as complementary processes.

7.1 Relativistic non-elastic collision

For the relativistic consideration of non-elastic collisions, the situation of observers with different velocities will be examined. For that purpose, a simple example shall be looked at and, after exact evaluation, the consequences derived will be discussed. The experimental conditions are as follows:

Two bodies are approaching each other and combine after axial contact, which means ideal plastic behavior is assumed. The collision shall be completely central and so no rotation will appear. In this case it is not necessary to use a vectorial calculation and the following calculation for the momentum can be used

$$p_3 = p_1 + p_2 = m_1\gamma_1 v_1 + m_2\gamma_2 v_2 = m_3\gamma_3 v_3 \tag{7.01}$$

where v_1 and v_2 are the velocities before and v_3 after the collision, the same definition is valid for the masses m_1, m_2 and m_3. If it is assumed that mass m_3 is at rest after the collision, then the values of p_1 and p_2 will neutralize each other because the conservation-principle of momentum must be respected. This means, that the absolute values of p_1 and p_2 are equal but the algebraic sign is different and so the total momentum after collision p_3 is zero.

However, the kinetic energy before and after the collision is not equal. This becomes clear when the equation of kinetic energy before the collision is considered (see also explanations in chapter 6.1)

$$E_{kin} = (\gamma_1 - 1)m_1 c^2 + (\gamma_2 - 1)m_2 c^2 \tag{7.02}$$

When again the situation is considered that mass m_3 is at rest after collision, then the kinetic energy is zero ether. Because kinetic energy is a scalar and not a vector like it is the case for momentum, it is compulsory that it must be transformed into another form. Otherwise, the conservation principle of energy would be violated. When it is assumed in this case that kinetic energy is transformed completely into mass the following equation is valid

$$\Delta m_3 = (\gamma_1 - 1)m_1 + (\gamma_2 - 1)m_2 \tag{7.03}$$

where Δm_3 is the increase of mass according to the transformation of kinetic energy.

To examine the situation an experiment with two different cases will be looked at, where in one instance an observer will be at rest and in another case moving. For simplification of the calculations, it is assumed that the masses of the bodies involved are equal, i.e. $m_1 = m_2 = m$. The cases will be marked with A and B; this identification will be continuously used for the relevant situations as index for the parameters depending on the velocities. It will be presumed in the first instance that the simple relation $m_3 = m_1 + m_2$ is valid. However, during the following considerations it will become clear that this assumption is leading to discrepancies, and it will be proven that Eq. (7.03) is valid in any case without restrictions.

A: Referring to an observer A at absolute rest the velocity is $v_{3A} = 0$

Because $m_1 = m_2$ was presumed this stand for the fact, that before the collision the two bodies are moving with equal speed but different directions, this means that beside $v_{3A} = 0$ also $v_{1A} = -v_{2A}$ is valid.

B: Referring to an observer B at absolute rest the velocity is $v_{1B} = 0$

All calculations refer to $v_{1B} = 0$.

The following relations apply:

	Observer A $v_{3A} = 0 \quad v_{1A} = -v_{2A}$	Observer B $v_{1B} = 0$
Momentum before collision	$p_{1A} = m\gamma_{1A}v_{1A}$ $p_{2A} = -m\gamma_{1A}v_{1A}$	$p_{1B} = 0$ $p_{2B} = m\gamma_{2B}v_{2B}$
Momentum after collision	$p_{3A} = 0$	$p_{3B} = 2m\gamma_{3B}v_{3B}$
Kinetic energy before collision	$\frac{E_{1A}}{c^2} = (\gamma_{1A} - 1)m$ $\frac{E_{2A}}{c^2} = (\gamma_{1A} - 1)m$	$\frac{E_{1B}}{c^2} = 0$ $\frac{E_{2B}}{c^2} = (\gamma_{2B} - 1)m$
Kinetic energy after collision	$\frac{E_{3A}}{c^2} = 0$	$\frac{E_{3B}}{c^2} = 2(\gamma_{3B} - 1)m$

In the presented table the results for momentum and kinetic energy are presented which apply for *identical experimental conditions* in view of the observers A and B. These will be discussed further in the next chapters using the relativistic addition of velocities for comparison.

7.1.1 Results based on relativistic addition of velocities

For observer A the simple case $v_{1\mathrm{A}} = -v_{2\mathrm{A}}$ is valid. The calculation of the velocity for observer B makes is necessary to use the relativistic addition of velocities, which was already described in chapter 4.1. Because of symmetry reasons the relation $v_{3\mathrm{B}} = v_{1\mathrm{A}}$ applies and this is leading to

$$v_{2\mathrm{B}} = \frac{2v_{1\mathrm{A}}}{1 + \left(\frac{v_{1\mathrm{A}}}{c}\right)^2} \tag{7.04}$$

Example:

Observer A	$v_{1\mathrm{A}} = 0{,}5c$	$v_{2\mathrm{A}} = -0{,}5c$	$v_{3\mathrm{A}} = 0$
Observer B	$v_{1\mathrm{B}} = 0$	$v_{2\mathrm{B}} = 0{,}8c$	$v_{3\mathrm{B}} = 0{,}5c$

7.1.2 Results based on relations for momentum

Observer A is considering the total value of the momentum before and after the collision as zero because of the relation $v_{1\mathrm{A}} = -v_{2\mathrm{A}}$ and thus

$$p_{3\mathrm{A}} = p_{1\mathrm{A}} + p_{2\mathrm{A}} = m\gamma_{1\mathrm{A}}v_{1\mathrm{A}} - m\gamma_{1\mathrm{A}}v_{1\mathrm{A}} = 0 \tag{7.05}$$

Observer B finds the following relations:

$$p_{1\mathrm{B}} = 0 \tag{7.06}$$

$$p_{2\mathrm{B}} = m\gamma_{2\mathrm{B}}v_{2\mathrm{B}} \tag{7.07}$$

$$p_{3\mathrm{B}} = 2m\gamma_{3\mathrm{B}}v_{3\mathrm{B}} \tag{7.08}$$

Because of the conservation principle of momentum, the values for p_{2B} and p_{3B} according to (7.01) must be equal, so

$$\gamma_{2\mathrm{B}}v_{2\mathrm{B}} = 2\gamma_{3\mathrm{B}}v_{3\mathrm{B}} \tag{7.09}$$

This equation allows the calculation of $v_{3\mathrm{B}}$ depending on $v_{2\mathrm{B}}$.

Because of the structure of the equation an analytical solution is not possible and so a numerical solution must be used. In annex D different approaches are presented; here the use of simple recursion, a procedure according to Newton and the bisection method were chosen to effectuate a solution. In all cases the results for $v_{3\mathrm{B}}$ were calculated using different values for $v_{2\mathrm{B}}$.

As expected, all iteration methods lead to the same values; the procedures using simple recursion and according to Newton share the advantage, that they converge very quickly for small values of v/c. However, as a drawback the convergence is reducing for increasing

v/c and the use is no longer possible when extremely high values are taken. Increasing to values higher than $v/c > 0.9c$ the bisection method is the only procedure which is still working.

Example:

Observer A	$v_{1A} = 0{,}5c$	$v_{2A} = -0{,}5c$	$v_{3A} = 0$
Observer B	$v_{1B} = 0$	$v_{2B} = 0{,}8c$	$v_{3B} = 0{,}5547c$

7.1.3 Results based on relations for energy

Observer A will consider the case that the kinetic energy of the colliding masses will be transformed completely into another form of energy (e.g. heat). This loss of energy has the value of

$$\frac{E_{1A} + E_{2A}}{c^2} = 2m(\gamma_{1A} - 1) \tag{7.10}$$

For observer B this is implicating that the difference between the kinetic energy before and after the collision is balanced and thus

$$2m(\gamma_{3B} - 1) = m(\gamma_{2B} - 1) - 2m(\gamma_{1A} - 1) \tag{7.11}$$

$$\gamma_{3B} = \frac{\gamma_{2B} - 2\gamma_{1A} + 3}{2} \tag{7.12}$$

This equation shows a simple analytical solution using

$$\frac{v_{3B}}{c} = \pm\sqrt{1 - \frac{1}{\gamma_{3B}^2}} = \pm\sqrt{1 - \frac{4}{(\gamma_{2B} - 2\gamma_{1A} + 3)^2}} \tag{7.13}$$

Example:

Observer A	$v_{1A} = 0{,}5c$	$v_{2A} = -0{,}5c$	$v_{3A} = 0$
Observer B	$v_{1B} = 0$	$v_{2B} = 0{,}8c$	$v_{3B} = 0{,}5293c$

(Negative results of the square root are not relevant because of plausibility reasons.)

7.1.4 Evaluation of the results

In Fig. 7.1 the deviations between the velocities according to the different calculations are presented.

Here the following definitions apply:

$$\delta = \frac{v_{3B} - v_{1A}}{v_{1A}} \tag{7.14}$$

where δ_p is the percental difference for the momentum (chapter 7.1.2) and δ_E for the energy (chapter. 7.1.3). It is clear at first sight that the height and also the position of the maxima are not sharing any similarities.

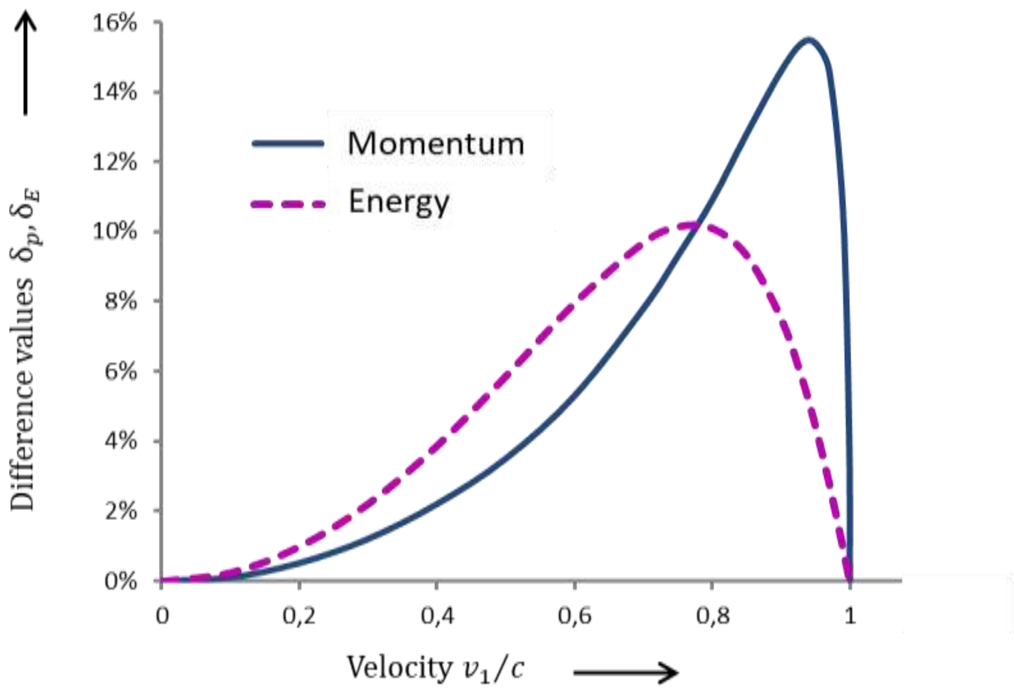

Fig. 7.1: Difference values δ_p and δ_E depending on v_1

It is obvious that for the non-elastic collision the consideration of the relations for relativistic addition of velocities and the conservation laws for momentum and energy using these calculations are leading to completely different results. This means that in these cases severe discrepancies would occur between the relativistic principles of identity and equivalence (for definition of the principles see chapter 1.6).

Up to now the velocity v_{3B} of the two combined masses was calculated based on the validity of the laws of momentum and energy without any further correction. To find a solution for the observed problems, in the following the attempt is made to examine the effect on momentum and energy which occurs, when the relativistic addition of velocities is supposed to be valid without further discussion. To realize this, the correction values C_p for the momentum and C_E for the energy are defined and used in the relevant relations.

a) Momentum

Equation Eq. (7.09) is modified to

$$C_p \cdot 2\gamma_{3\mathrm{B}} v_{3\mathrm{B}} = \gamma_{2\mathrm{B}} v_{2\mathrm{B}} \tag{7.15}$$

using the relation $v_{3\mathrm{B}} = v_{1\mathrm{A}}$ (see chapter 7.1.1)

$$C_p = \frac{\sqrt{1-\left(\frac{v_{1\mathrm{A}}}{c}\right)^2}}{2v_{1\mathrm{A}}} \frac{v_{2\mathrm{B}}}{\sqrt{1-\left(\frac{v_{2\mathrm{B}}}{c}\right)^2}} \tag{7.16}$$

Because of Eq. (7.04) is

$$C_p = \frac{\sqrt{1-\left(\frac{v_{1A}}{c}\right)^2}}{2v_{1A}} \frac{\frac{2v_{1A}}{1+\left(\frac{v_{1A}}{c}\right)^2}}{\sqrt{1-\left(\frac{\frac{2v_{1A}}{c}}{1+\left(\frac{v_{1A}}{c}\right)^2}\right)^2}}$$

$$= \sqrt{\frac{1-\left(\frac{v_{1A}}{c}\right)^2}{\left[1-\left(\frac{v_{1A}}{c}\right)^2\right]^2}} = \sqrt{\frac{1}{1-\left(\frac{v_{1A}}{c}\right)^2}} = \gamma_{1A} \tag{7.17}$$

This means that using unrestricted application of the relativistic addition of velocities the momentum is smaller by the factor γ_{1A} than required by the law of conservation of momentum.

b) Energy

Equation Eq. (7.11) is modified to

$$C_E \cdot\ 2(\gamma_{3B}-1) = (\gamma_{2B}-1) - 2(\gamma_{1A}-1) \tag{7.20}$$

With $v_{3B} = v_{1A}$ applies

$$C_E = \frac{(\gamma_{2B}-1)}{2(\gamma_{1A}-1)} - 1 \tag{7.21}$$

To develop a simple solution, first the term $\gamma_{2B}-1$ is considered. This can be transformed using Eq. (7.04) to

$$\gamma_{2B} - 1 = \frac{1}{\sqrt{1-\left(\frac{\frac{2v_{1A}}{c}}{1+\left(\frac{v_{1A}}{c}\right)^2}\right)^2}} - 1 \tag{7.22}$$

and

$$\gamma_{2B} - 1 = \pm \frac{1+\left(\frac{v_{1A}}{c}\right)^2}{1-\left(\frac{v_{1A}}{c}\right)^2} - 1 = 2(\gamma_{1A}^2 - 1) \tag{7.23}$$

For this calculation it was decided to take only positive values for the results of the square root, because negative values would lead to negative γ_{2B} and physical interpretation makes no sense in this case.

The result is inserted in Eq. (7.21)

$$C_E = \frac{2(\gamma_{1A}^2-1)}{2(\gamma_{1A}-1)} - 1 \tag{7.24}$$

$$C_E = \frac{(\gamma_{1A}+1)(\gamma_{1A}-1)}{(\gamma_{1A}-1)} - 1 = \gamma_{1A} \tag{7.25}$$

This calculation is leading to the same result as already obtained for the momentum.

7.1.5 Final approach for calculation

For final evaluation, the findings developed so far shall be summarized and reviewed first. When in case of nonelastic collision examinations concerning the conservation laws of momentum and energy with invariant mass (this means $m_3 = m_1 + m_2$; $\Delta m_3 = 0$) before and after collision are conducted, then it becomes clear that the gained results for the velocity v_3 are different to each other; further the calculated value using the equation of relativistic addition of velocities come to another different result. The values of v_3 for the combined body using conservation laws are both higher than the calculated result derived by relativistic addition.

This would mean that the concept of simple addition of mass before and after collision is no option because the basic principles concerning conservation of energy and momentum are violated. If the approach presented in Eq. (7.01) of complete conversion of kinetic energy into mass is used instead, then considering the special case $m_1 = m_2 = m$

$$\Delta m_3 = \frac{E_{1A} + E_{2A}}{c^2} = 2m(\gamma_{1A} - 1) \tag{7.30}$$

is valid for the generated mass Δm_3 by energy conversion (see also Eq. (7.04). For momentum, the relation Eq. (7.07) remains unchanged *before* collision

$$p_{2B} = m\gamma_{2B}v_{2B} \tag{7.07}$$

but Eq. (7.09) *after* collision is developing to

$$p_{3B} = 2m\gamma_{3B}v_{3B} \quad \Rightarrow \quad p_{3B} = m_3\gamma_{3B}v_{3B} \tag{7.31}$$

Because of $v_{1A} = v_{3B}$ derived from relativistic addition of velocities this leads to

$$p_{3B} = [2m(\gamma_{1A} - 1) + 2m]\gamma_{3B}v_{3B} = 2m\gamma_{3B}^2 v_{3B} \tag{7.32}$$

The consideration of complete transformation into mass can be looked at as reverse observation compared to the conditions during the disintegration of particles and may be designated as “negative mass defect”. This result is corresponding exactly to the value of the missing part of momentum and energy during collision and leads to the conclusion, that for relativistic considerations of the non-elastic collision always an increase of mass in the amount of the value presented by the transformation of kinetic energy must be presumed to prevent the occurrence of discrepancies.

This is comprehensible on an atomic scale, for macroscopic objects it is not conforming to the general understanding of processes, because e. g. during the generation of heat no transformation processes are observed. However, in this case because of the definition of heat – which means that a rising heat input is corresponding to increasing velocities of the apparent mass – the increase of energy can be interpreted as relativistic consideration of the oscillation-velocity of the participating atoms or molecules. When this issue is discussed in the literature, normally the transformation of kinetic energy into mass is placed first and then verified using the relevant equations, e.g. [47]. The approach presented here, however, provides clear evidence that the increase of mass caused by complete transformation of kinetic energy is required by the valid conservation laws.

7.2 Relativistic considerations of particle disintegration

As already mentioned before, the disintegration of particles can be interpreted as the reversion of the situation valid during non-elastic collision (see chapter 7.1). Because the mathematical correlations of both effects are exactly the same, it is not necessary to present the evaluations again. In this chapter the emphasis is laid on considerations of decay particles moving in different spatial directions and concerning the conditions, when the kinetic energy is not converted into mass as discussed before but is dissipated by electromagnetic radiation.

To avoid misinterpretations, it shall be generally defined that the dissipating particle is indicated with index 1, for the decay products the indices 3 and 4 (and increasing further if applicable) are used. An observer moving with a dissipating particle is additionally marked as f', for an observer at rest f is used (without marking).

7.2.1 Analysis of disintegration into 2 particles

For the investigation of the situation in arbitrary spatial directions it is necessary to use the analytical determination of aberration, which was already derived in chapter 2.3. The geometrical dependencies are presented in Fig. 7.2. The description is completely comparable and therefore the calculations will not be repeated. The only valid difference is concerning equation Eq. (2.43), where the relation between the velocity of the moving system and the speed of light is calculated. These must be replaced by the following relation

$$\text{Eq. (2.43)}: \quad \frac{b}{v} = \frac{d}{c} \qquad \Rightarrow \qquad \frac{b}{v_1} = \frac{d}{v_3} \tag{7.40}$$

where v_1 is the velocity of the moving system and v_3 is the speed of an arbitrary particle (the equations presented in the following can be derived in the same way for particle 4). It is necessary to calculate the velocity v_3 using Eq. (4.20) according to

$$v_3 = \frac{\sqrt{v_1^2 + v_3'^2 + 2v_1 v_3' cos\alpha_3' - \left(\frac{v_1 v_3' sin\alpha_3'}{c}\right)^2}}{1 + \frac{v_1 v_3' cos\alpha_3'}{c^2}} \tag{7.41}$$

where in this case v_3' is the velocity of the particle relative to the moving system and v_3 is the velocity in view of the observer at rest. The calculation leads to the following result [see also Eq. (2.48)]:

$$tan\alpha_3' = \pm \frac{sin\alpha_3}{\gamma\left(cos\alpha_3 - \frac{v_1}{v_3'}\right)} \tag{7.42}$$

Here α_3 is the angle, which an observer at rest will find between the motion of a particle relative to his system, while α_3' is the angle of the same particle in view of the moving observer**.** When the value of α_3' is given then the resulting value for α_3 can also easily be calculated. The only conversion necessary is the change of the algebraic sign (for details see chapter 2.3.4) and the result is

$$tan\alpha_3 = \pm \frac{sin\alpha_3'}{\gamma\left(cos\alpha_3' + \frac{v_1}{v_3'}\right)} \tag{7.43}$$

The validity of this relation can also easily be verified by numerical comparison. In table Tab. (7.1a) some examples for the calculation of the resulting angles for different velocities v_1 and v_3' are presented.

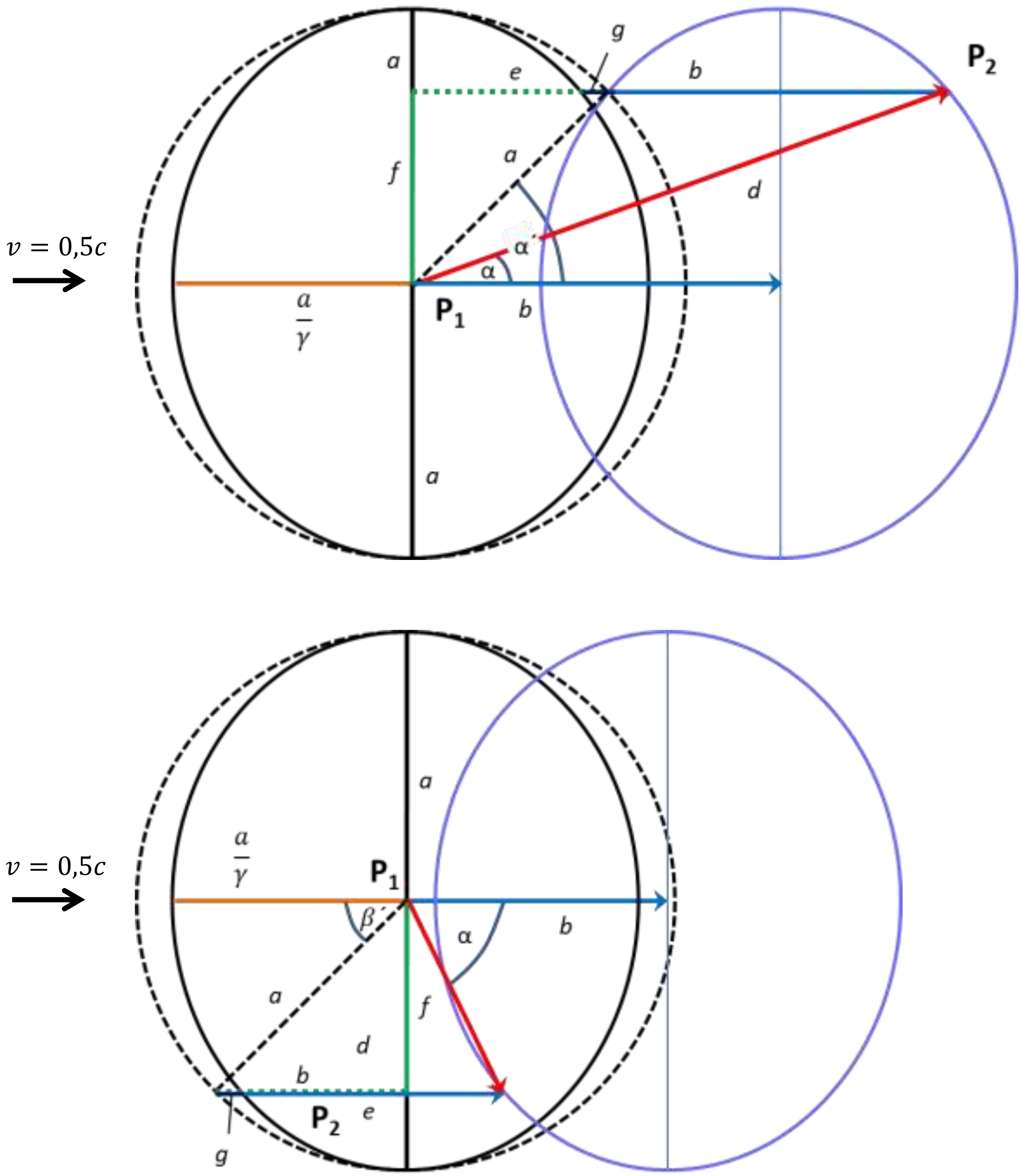

Fig. 7.2: Definition of parameters to determine the angle of an outgoing beam for a moving observer (examples for $v_1 = 0.5c$, $\alpha_3' = 45°$, $\alpha_4' = -135°$)
a) Signal emitted in moving direction, $\alpha_3 = 19{,}73°$
b) Signal emitted backwards, $\alpha_4 = -64.44°$

α'_3	v_3	α_3	$\tilde{p}_3$	$\tilde{p}_{3X}$	$\tilde{p}_{3Y}$	α_4	v_4	
0	0,57143	0	0,69631	0,69631	0	-180	0,42105	v_1=
45	0,55439	37,80	0,66612	0,52636	0,40825	-125,78	0,44953	0,1
90	0,50744	78,63	0,58890	0,11605	0,57735	-78,63	0,50744	v'_3=
135	0,44953	125,78	0,50324	-0,29425	0,40825	-37,80	0,55439	0,5
180	0,42105	180,00	0,46421	-0,46421	0	0	0,57143	

α'_3	$\tilde{p}_4$	$\tilde{p}_{4X}$	$\tilde{p}_{4Y}$	$\sum\tilde{p}_X$	$\sum\tilde{p}_Y$	$\tilde{E}_{kin,3}$	$\tilde{E}_{kin,4}$	$\sum\tilde{E}_{kin}$
0	0,46421	-0,46421	0	0,23210	0	0,218544	0,102492	0,321035
45	0,50324	-0,29425	-0,4085	0,23210	0	0,201548	0,119487	0,321035
90	0,58890	0,11605	-0,57735	0,23210	0	0,160518	0,160518	0,321035
135	0,66612	0,52636	-0,40825	0,23210	0	0,119487	0,201548	0,321035
180	0,69631	0,69631	0	0,23210	0	0,102492	0,218544	0,321035

α'_3	v_3	α_3	$\tilde{p}_3$	$\tilde{p}_{3X}$	$\tilde{p}_{3Y}$	α_4	v_4	
0	0,8	0	1,33333	1,33333	0,00000	-90	0,00000	v_1=
45	0,77059	19,73	1,20908	1,13807	0,40825	-64,44	0,41229	0,5
90	0,66144	40,89	0,88192	0,66667	0,57735	-40,89	0,66144	v'_3=
135	0,41229	64,44	0,45254	0,19526	0,40825	-19,73	0,77059	0,5
180	0	90	0	0	0	0	0,80000	

α'_3	$\tilde{p}_4$	$\tilde{p}_{4X}$	$\tilde{p}_{4Y}$	$\sum\tilde{p}_X$	$\sum\tilde{p}_Y$	$\tilde{E}_{kin,3}$	$\tilde{E}_{kin,4}$	$\sum\tilde{E}_{kin}$
0	0	0	0	1,33333	0	0,666667	0,000000	0,666667
45	0,45254	0,19526	-0,40825	1,33333	0	0,569036	0,097631	0,666667
90	0,88192	0,66667	-0,57735	1,33333	0	0,333333	0,333333	0,666667
135	1,20908	1,13807	-0,40825	1,33333	0	0,097631	0,569036	0,666667
180	1,33333	1,33333	0	1,33333	0	0,000000	0,666667	0,666667

α'_3	v_3	α_3	$\tilde{p}_3$	$\tilde{p}_{3X}$	$\tilde{p}_{3Y}$	α_4	v_4	
0	0,57143	0	0,69631	0,69631	0	0	0,42105	v_1=
45	0,55439	6,12	0,66612	0,66232	0,07107	-8,12	0,44953	0,5
90	0,50744	9,83	0,58890	0,58026	0,10050	-9,83	0,50744	v'_3=
135	0,44953	8,12	0,50324	0,49820	0,07107	-6,12	0,55439	0,1
180	0,42105	0	0,46421	0,46421	0	0	0,57143	

α'_3	$\tilde{p}_4$	$\tilde{p}_{4X}$	$\tilde{p}_{4Y}$	$\sum\tilde{p}_X$	$\sum\tilde{p}_Y$	$\tilde{E}_{kin,3}$	$\tilde{E}_{kin,4}$	$\sum\tilde{E}_{kin}$
0	0,46421	0,46421	0	1,16052	0	0,218544	0,102492	0,321035
45	0,50324	0,49820	-0,07107	1,16052	0	0,201548	0,119487	0,321035
90	0,58890	0,58026	-0,10050	1,16052	0	0,160518	0,160518	0,321035
135	0,66612	0,66232	-0,07107	1,16052	0	0,119487	0,201548	0,321035
180	0,69631	0,69631	0	1,16052	0	0,102492	0,218544	0,321035

Tab. 7.1a: Calculation for momentum and kinetic energy in a moving system.
Values marked grey: Approximation.
Values presented in frames: 180 °-angels.
Equations and dimensions: Tab. 7.1b and text.

$$v_3 = \frac{\sqrt{v_1^2 + v_3'^2 + 2v_1 v_3' cos\alpha_3' - \left(\frac{v_1 v_3' sin\alpha_3'}{c}\right)^2}}{1 + \frac{v_1 v_3' cos\alpha_3'}{c^2}} \cdot c$$			[-]
$$\alpha_3 = arctan\left[\frac{sin\alpha_3'}{\gamma\left(cos\alpha_3' + \frac{v_1}{v_3'}\right)}\right] \cdot \frac{180}{\pi}$$	[°]	$$\tilde{p}_3 = \frac{p_3}{mc} = \frac{v_3}{c}\gamma_3$$	
$$\tilde{p}_{3X} = \frac{p_{3X}}{mc} = \frac{v_3}{c}\gamma_3 cos\,(\alpha_3)$$	[-]	$$\tilde{p}_{3Y} = \frac{p_{3Y}}{mc} = \frac{v_3}{c}\gamma_3 sin\,(\alpha_3)$$	[-]
$$v_4 = \frac{\sqrt{v_1^2 + v_4'^2 + 2v_1 v_4' cos\alpha_4' - \left(\frac{v_1 v_4' sin\alpha_4'}{c}\right)^2}}{1 + \frac{v_1 v_4' cos\alpha_4'}{c^2}} \cdot c$$			[-]
$$\alpha_4 = arctan\left[\frac{sin\alpha_4'}{\gamma\left(cos\alpha_4' + \frac{v_1}{v_4'}\right)}\right] \cdot \frac{180}{\pi}$$	[°]	$$\tilde{p}_4 = \frac{p_4}{mc} = \frac{v_4}{c}\gamma_4$$	
$$\tilde{p}_{4X} = \frac{p_{4X}}{mc} = \frac{v_4}{c}\gamma_4 cos\,(\alpha_4)$$	[-]	$$\tilde{p}_{4Y} = \frac{p_{4Y}}{mc} = \frac{v_4}{c}\gamma_4 sin\,(\alpha_4)$$	[-]
$$\sum\tilde{p}_X = \tilde{p}_{3X} + \tilde{p}_{4X}$$	[-]	$$\sum\tilde{p}_Y = \tilde{p}_{3Y} + \tilde{p}_{4Y}$$	[-]
$$\tilde{E}_{kin,3} = \frac{E_{kin,3}}{mc^2} = \gamma_3 - 1$$	[-]	$$\tilde{E}_{kin,4} = \frac{E_{kin,4}}{mc^2} = \gamma_4 - 1$$	[-]
$$\sum\tilde{E}_{kin} = \tilde{E}_{kin,3} + \tilde{E}_{kin,4}$$	[-]		

Tab. 7.1b Equations and dimensions used in Tab. 7.1a

The equations used in table 7.1a and the connected dimensions are summarized in table 7.1b. To ensure a clear arrangement the values are presented in a normalized form as $\tilde{p}$ and $\tilde{E}$ with the dimension 1. This is also valid for the velocities; here the form v/c was chosen.

The values marked grey were calculated using an approximation process, because for $v_3' = v_1$ the developing equations contain a division by zero. The values of α_3 and α_4 > 90° were calculated using first standard calculations and then the results were reduced by 180°; this is marked in the table using a frame (for further details see also chapter 2.3).

For the calculations, the following preconditions apply:

It is presumed that a particle is disintegrated into 2 decay products of equal size, of which one is removing with an arbitrary angle α_3'. In this case the second "twin particle" will obey an angle of $\alpha_4' = \alpha_3' - 180°$ because of symmetry reasons. For these products, the angles α_3 and α_4 are calculated and the connected velocities v_3 and v_4 also. In a second step the values for momentum according to

$$p_3 = \gamma_1 m v_3 \quad bzw. \quad p_4 = \gamma_1 m v_4 \tag{7.44}$$

were determined. In a further step the fractions in moving direction (x) and perpendicular to it (y) according to

$$p_x = p \cdot cos(\alpha) \tag{7.45}$$

$$p_y = p \cdot sin(\alpha) \tag{7.46}$$

were calculated. When the angles α_3 and α_4 are added, the results in x-direction always show the same results, in y-direction they annihilate each other. Further the values for the kinetic energy were determined for particle 3 according to

$$E_{kin,3} = (\gamma_3 - 1)mc^2 \tag{7.47}$$

and for particle 4

$$E_{kin,4} = (\gamma_4 - 1)mc^2 \tag{7.48}$$

The summation of these values is producing the same result for all angles. It was possible to show with these calculations that for the disintegration into 2 decay particles the values for momentum and kinetic energy in all cases for an observer at rest and in a moving system are resulting in the same results and that it is not possible inside a system to decide whether this is moving or not.

7.2.2 Disintegration into 2 photons

It is well known from experimental results that a particle can disintegrate completely into photons without leaving matter. The π^0-pion for example is an extremely unstable particle with an average lifetime of approximately 10^{-18} s with the specific characteristic that it is disintegrating with almost 99% probability into 2 photons. When it is presumed that the disintegration is happening at a state of absolute rest the energy can be calculated using

$$E = m_0 c^2 = hf_3 + hf_4 \tag{7.50}$$

where h is Planck's quantum of action and f_3 as well as f_4 are the frequencies of the emitted photons. The momentum of one photon is

$$\vec{p} = h\frac{f}{c}\vec{e} \tag{7.51}$$

with $\vec{e}$ as unit vector in moving direction. Because of the conservation laws of energy and momentum the frequencies for both photons are the same and their moving directions are exactly opposite to each other. The momentum is zero before and after disintegration.

If an observer is monitoring a velocity v_1 before disintegration, then because of the relativistic mass increase the total energy of the particle is

$$E = \gamma_1 m_0 c^2 \tag{7.52}$$

After disintegration, the emitted photons must carry the total energy and the momentum of the particle. The total energy of the photons is

$$E = \gamma_1 h f_3 + \gamma_1 h f_4 \tag{7.53}$$

and the momentum of one photon

$$\vec{p} = \gamma_1 h \frac{f}{c} \vec{e} \tag{7.54}$$

When these relations are analyzed according to the ratio valid in moving direction, for an observer at rest the kinetic energy of the particle and the momentum has also to be carried away completely by the emitted photons. For the energy, the following relation applies

$$\gamma_1 m_0 c^2 = \gamma_1 h f_3 + \gamma_1 h f_4 \tag{7.55}$$

and for the momentum *in moving direction*

$$\gamma_1 m_0 v_1 = \gamma_1 h \frac{f_3}{c} - \gamma_1 h \frac{f_4}{c} \tag{7.56}$$

where f_3 is the emission in moving direction (positive) and f_4 opposite to it (negative). Using subtraction resp. addition of equations Eq. (7.55) and (7.56) then the values for the frequencies are

$$f_3 = \frac{m_0(c^2 + v_1 c)}{2h} \tag{7.57}$$

$$f_4 = \frac{m_0(c^2 - v_1 c)}{2h} \tag{7.58}$$

with

$$\frac{f_3}{f_4} = \frac{c + v_1}{c - v_1} \tag{7.59}$$

This relation is exactly corresponding to the macroscopic behavior of moving emitters which will be described in chapter 8.

For the derivation of the correlations in arbitrary spatial directions first the geometric dependencies for emitter and receiver must be examined. In Fig. 7.3 it is demonstrated, in which way observer A at the time A_1 and A_2 is sending specific signals. Depending on the distance to receiver B and on the velocity different angles in relation to the moving direction will appear. For simplification it will be assumed, that the receiver B, which is at rest, is far away and the time between 2 signals is comparatively short and thus for the angles the relation $\alpha_1 = \alpha_2 = \alpha$ can be presumed.

The time between the signals send by the moving emitter A is

$$\Delta t_A = \gamma \Delta t_0 \tag{7.60}$$

compared to the relations valid for an observer at rest. Beside the extension caused by time-dilatation, receiver B will also notice a geometric influence on time, because the emitter is either coming or going relative to his position between sending out the signals. In total this adds up to

$$\Delta t_B = \gamma \Delta t_0 \left(1 - \frac{v}{c} cos(\alpha)\right) \tag{7.61}$$

This is resulting for the frequency detected by receiver B

$$f_B = \frac{f_0}{\gamma \left(1 - \frac{v}{c} cos\ (\alpha)\right)} \tag{7.62}$$

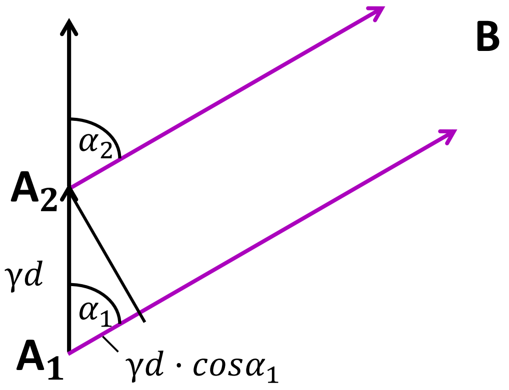

Fig. 7.3: Radiation geometry

To provide a final comparison between a particle at rest and moving, calculations for different angles for outgoing photons are made. In Tab. 7.2 different angles α_3' (in view of a moving observer) are defined; the corresponding angles of the “twin” photon are differing exactly by 180°, i.e. this means $\alpha_4' = \alpha_3' - 180°$. First the angles in view of the observer at rest are determined using the equations developed in chapter 2.3 and the value for α_3 is calculated. Further the corresponding frequencies are determined, in the next step the momentum in x- and y-direction is calculated (using cos resp. sin of the angle according to Eq. (7.45) and (7.46) presented in chapter 7.2.1). Finally, the total energy, which is released during disintegration of the particle, is calculated for any angle.

The starting value for f_0 was set to 1. To ensure a clear arrangement the values for momentum end energy are again presented in normalized form as $\tilde{p}$ and $\tilde{E}$; the dimension is in this case 1. Detailed definitions and the resulting dimensions are summarized in Tab. 7.2b.

The summation of the values for momentum in x-direction and total energy are always identical and correspond to the expected results; the values in y-direction add up to zero. Further it is easy to show, that the results found for angle $\alpha_3' = 0$ correspond exactly to equation (7.59), which was derived for the simple case for emission in moving direction and opposite. Thus it was possible to show, that also in this case no differences appear whether experiments are viewed by an observer at rest or referring to a moving system and so no violations of the principles of relativity occur.

α'_3	α_3	α_4	f_3	f_4	$\tilde{p}_{3X}$	$\tilde{p}_{4X}$	$\sum\tilde{p}_X$	$\tilde{p}_{3Y}$	$\tilde{p}_{4Y}$	$\sum\tilde{p}_Y$	$\tilde{E}$
0	0	-180	1,732	0,577	0,866	-0,289	0,577	0	0	0	2,309
15	8,69	-154,31	1,712	0,597	0,846	-0,269	0,577	0,129	-0,129	0	2,309
30	17,59	-130,21	1,655	0,655	0,789	-0,211	0,577	0,250	-0,250	0	2,309
45	26,90	-108,69	1,563	0,746	0,697	-0,120	0,577	0,354	-0,354	0	2,309
60	36,87	-90,00	1,443	0,866	0,577	0,000	0,577	0,433	-0,433	0	2,309
75	47,79	-73,92	1,304	1,005	0,438	0,139	0,577	0,483	-0,483	0	2,309
90	60,00	-60,00	1,155	1,155	0,289	0,289	0,577	1	-1	0	2,309
105	73,92	-47,79	1,005	1,304	0,139	0,438	0,577	0,483	-0,483	0	2,309
120	90	-36,87	0,866	1,443	0,000	0,577	0,577	0,433	-0,433	0	2,309
135	108,69	-26,90	0,746	1,563	-0,120	0,697	0,577	0,354	-0,354	0	2,309
150	130,21	-17,59	0,655	1,655	-0,211	0,789	0,577	0,25	-0,25	0	2,309
165	154,31	-8,69	0,597	1,712	-0,269	0,846	0,577	0,129	-0,129	0	2,309
180	180	0	0,577	1,732	-0,289	0,866	0,577	0	0	0	2,309

Tab 7.2a: Calculations of angles, momentum (moving direction: x, vertical: y), energy. Equations and dimensions see Tab. 7.2b

$\alpha_3 = 2 \cdot arctan\left[\left(\frac{c-v}{c+v}\right)^{1/2} tan\left(\frac{\alpha'_3}{2}\right)\right] \cdot \frac{180}{\pi}$ Eq. acc. Tab. 2.4, No. 4			[°]
$\alpha_4 = 2 \cdot arctan\left[\left(\frac{c-v}{c+v}\right)^{1/2} tan\left(\frac{\pi-\alpha'_3}{2}\right)\right] \cdot \frac{180}{\pi}$ Eq. acc. Tab. 2.4, No. 4			[°]
$f_3 = \frac{f_0}{\gamma\left(1-\frac{v}{c}\cos(\alpha_3)\right)}$	$[s^{-1}]$	$f_4 = \frac{f_0}{\gamma\left(1+\frac{v}{c}\cos(\alpha_4)\right)}$	$[s^{-1}]$
$\tilde{p}_{3X} = \frac{p_{3X}}{mc} = \frac{v}{c} f_3 \cos(\alpha_3)$	[-]	$\tilde{p}_{4X} = \frac{p_{4X}}{mc} = \frac{v}{c} f_4 \cos(\alpha_4)$	[-]
$\sum\tilde{p}_X = \tilde{p}_{3X} + \tilde{p}_{4X}$	[-]		
$\tilde{p}_{3Y} = \frac{p_{3Y}}{mc} = \frac{v}{c} f_3 \sin(\alpha_3)$	[-]	$\tilde{p}_{4Y} = \frac{p_{4Y}}{mc} = \frac{v}{c} f_4 \sin(\alpha_4)$	[-]
$\sum\tilde{p}_Y = \tilde{p}_{3Y} + \tilde{p}_{4Y}$	[-]	$\tilde{E} = \frac{E}{\gamma h} = f_3 + f_4$	[-]

Tab. 7.2b Equations and dimensions used for calculations in Tab. 7.2a