Annex A: Relativistic elastic collision

In this attachment the necessary calculations for the elastic collision are presented (see also chapter 6.3). For this purpose, the equations

$$p = m_1\gamma_1 v_1 + m_2\gamma_2 v_2 = m_1\gamma_3 v_3 + m_2\gamma_4 v_4 \tag{A.01}$$

$$\frac{E_0}{c^2} = (\gamma_1 - 1)m_1 + (\gamma_2 - 1)m_2 = (\gamma_3 - 1)m_1 + (\gamma_4 - 1)m_2 \tag{A.02}$$

are used. Eq. (A.02) is transformed to

$$\gamma_4 = \frac{\frac{E_0}{c^2} - (\gamma_3 - 1)m_1}{m_2} + 1 \tag{A.03}$$

with

$$v_4 = \pm c \cdot \sqrt{1 - \frac{1}{\gamma_4^2}} \tag{A.04}$$

Further Eq. (A.03) and Eq. (A.04) are inserted in Eq. (A.01)

$$f(v_3) = m_1\gamma_3 v_3 \pm c\left(\frac{E_0}{c^2} - (\gamma_3 - 1)m_1 + m_2\right)\left[1 - \left(\frac{\frac{E_0}{c^2} - (\gamma_3 - 1)m_1}{m_2} + 1\right)^{-2}\right]^{1/2} \tag{A.05}$$

This relation is depending solely on the defined values for v_1 and v_2. Using the principle of bisection, the values for v_3 and in a second step also v_4 can now be determined (for comparisons of different calculation methods see annex D). First the appropriate starting values $(v_{3+})_0$ and $(v_{3-})_0$ must be identified for which the following conditions apply:

$$f(v_{3+})_0 > p \tag{A.06}$$

$$f(v_{3-})_0 < p \tag{A.07}$$

In the interval $[(v_{3-})_0; (v_{3+})_0]$ the function $f(v_3)$ must be continuous and differentiable and further $f'(v_3) \neq 0$ is required. This means, that in the chosen interval minima and maxima are not allowed, because otherwise no exact solution exists. Now the mean value is determined using

$$(v_3)_1 = \frac{(v_{3+})_0 + (v_{3-})_0}{2} \tag{A.08}$$

and $f(v_3)_1$ is calculated according to Eq. (A.05). The following equations apply:

$$f(v_3)_1 > p \quad \Rightarrow \quad \begin{cases} (v_{3+})_1 = (v_3)_1 \\ (v_{3-})_1 = (v_{3-})_0 \end{cases} \tag{A.09}$$

$$f(v_3)_1 \leq p \quad \Rightarrow \quad \begin{cases} (v_{3+})_1 = (v_{3+})_0 \\ (v_{3-})_1 = (v_3)_1 \end{cases} \tag{A.10}$$

The calculation is repeated with increasing index 1 to K until the required accuracy is achieved. Caused by the appearance of the indication $\pm$ in the relations Eq. (A.04) and Eq. (A.05), which is caused by the determination of the square root, the calculation of v_4 provides 2 different results, which must be interpreted using plausibility considerations according to the applicable situation.

If a simple spreadsheet is used for the calculation (cf. Chap. A.2), the input parameters are limited due to the previously discussed boundary conditions. For the calculations, the starting conditions must be chosen so that the values for v_1 are positive in all cases. It is also assumed that, through appropriate index selection, the values of v_1 are always greater than v_2 and the values for the calculated momentum in Eq. (A.01) are $p > 0$. If the actual default values deviate from these prerequisites, adjustments are necessary whose definition is shown below.

A.1 Program flow of the calculation process

In the following it is described which process steps a program must execute in order to carry out the necessary calculations (cf. Fig. A.1). To ensure an unrestricted selection of the output parameters, their determination is first carried out via the subprogram "Parameter Input" and after completion of the calculations the reconversion is carried out by means of the subprogram "Parameter Output" (Fig. A.2).

The specification of the input-parameters is determined by the following criteria:

1. For the consideration of the velocities of objects with mass m_1 and m_2 the precondition $v_1 > v_2$ is necessary. The reason for this is, that the calculation starts with the determination of v_3 (of the object with mass m_1 after collision); values with $v_1 < v_2$ would represent a situation that object m_2 is moving faster than m_1 and this would mean that the incident could not take place.
2. Further the general conditions $v_1 > 0$ and $p > 0$ must apply. These preconditions are necessary to guarantee an undisturbed execution of the program because the presence of the square root in the formula would otherwise lead to interpretation problems. In the case discussed here, only positive values must be obeyed instead of plus and minus as possible results.

The definition of these preconditions for the execution are severe restrictions at first sight, but they are representing no limit for the calculations. This is the case because several possibilities exist to modify the starting conditions as

1. The algebraic sign for velocities v_1 and v_2 can be determined as desired, under the condition that they are changed simultaneously.
2. The index between $v_1; m_1$ and $v_2; m_2$ can be changed.

When an appropriate combination of these conditions is used, this will cover all possible situations. To show this, first the case $v_1 > 0$ shall be discussed. Instead of the theoretically possible $2^3 = 8$ combinations defined by the 3 starting conditions $v_2 > 0, v_1 > v_2$ and $p > 0$ only 4 alternatives are remaining. This can be explained by discussing the following situations:

- For case $v_2 > 0$ in combination with $v_1 > 0$ the resulting total momentum is always positive and so it is not necessary to consider it further. A negative momentum can only occur when the velocities show different algebraic signs (or are both negative).
- The discussed case $v_1 > 0$ in combination with $v_2 < 0$ is obviously always leading to the result $v_1 > v_2$.

These cases can be excluded from further considerations. The remaining variants can be summarized as follows:

Condition 1	Condition 2	Action	Code
$v_2 > 0$	$v_1 > v_2$	No action necessary	F1
$v_2 > 0$	$v_1 < v_2$	Change of index	F2
$v_2 < 0$	$p > 0$	No action necessary	F1
$v_2 < 0$	$p < 0$	Change of index and algebraic sign	F4

Tab. A.1: Input-parameter depending on starting conditions for $v_1 > 0$

For the situation $v_1 < 0$ the determination follows the same procedure with the only difference, that first a general change of the algebraic sign is necessary. It must be obeyed that in this case the algebraic sign of the momentum is changing also. Finally, the following cases apply:

Condition 1	Condition 2	Action	Code
$v_2 > 0$	$p > 0$	Change of index and algebraic sign	F4
$v_2 > 0$	$p < 0$	Change algebraic sign	F3
$v_2 < 0$	$v_1 < v_2$	Change of index	F2
$v_2 < 0$	$v_1 < v_2$	Change algebraic sign	F3

Tab. A.2: Input-parameter depending on starting conditions for $v_1 < 0$

The values obtained in this way shall be named V_1, V_2, M_1, M_2 and can be used for further calculations. Following this procedure all possible combinations of appearing masses and velocities can be addressed. After finishing the calculations, the results for V_3, V_4, M_1, M_2 must be converted into the needed values using a reverse scheme reapplying the code defined during the Input-process.

It shall be mentioned that according to the transformation described above the results for V_4 always show positive values and only after a transformation, which may be necessary according to the preconditions, shifting to a negative result is possible. This is important because the values are calculated according to Eq. (A.04) and, thus, concerning the square root with

$$V_4 = +\sqrt{1 - \frac{1}{\gamma_4^2}} \tag{A.11}$$

only the positive result must be used.

The values determined in this way allow the initial values for $(V_{3+})_0$ and $(V_{3-})_0$ required for the calculations to be established in a straightforward manner. It can be easily shown that for all cases the conditions $(V_{3+})_0 = V_1$ as well as $(V_{3-})_0 = -V_1$ fulfill the requirements and always lead to usable results.

For the further calculations here (as in the other cases) the method of bisection was chosen. For the definition of the parameter for the termination of the calculations the possibility is given here that the values of $(v_3)_{\mathrm{K}-1}$ and $(v_3)_{\mathrm{K}}$ or $(v_4)_{\mathrm{K}-1}$and $(v_4)_{\mathrm{K}}$ are compared with each other and with equality the calculation process is terminated. However, if one of these queries is chosen, the situation may arise that - if the values are close to zero - the other has not yet been calculated exactly. To avoid this problem the fact was used that from a number of approx. 60 iteration steps with the available accuracy of 15 digits the possible limit accuracy is reached (see discussion in appendix D). To avoid any problem a fixed number of 80 iteration steps to stop the process was defined.

All necessary process steps are represented with the help of program flow charts, namely in Fig. A.1 for the general flow and in Fig. A.2 for the described subprograms. Subsequently, a VBA program code created for the calculations (Fig. A.3) as well as the assignment of the formula characters used (Tab. A.3) is reproduced.

In the following, a simple spreadsheet calculation program is shown in chapter A.2, which can be used to perform the same calculations. However, the already mentioned boundary conditions $v_1 > v_2$, $v_1 > 0$ and $p > 0$ must be observed or, if necessary, manually adjusted.

As already mentioned in chapter 6.3, the results from VBA program and spreadsheet are not completely identical, although they follow exactly the same calculation scheme. While this does not matter for large values, deviations are noticeable for very small values of v_1. These are caused by rounding errors during the calculation, which have different effects on the different procedures. However, this does not affect the general statement that in elastic relativistic impact no effects can occur which allow measurements to identify a system at absolute rest.

If cases with very low velocities shall be investigated numerically in more detail, computer systems with higher accuracy must be used to get reliable results.

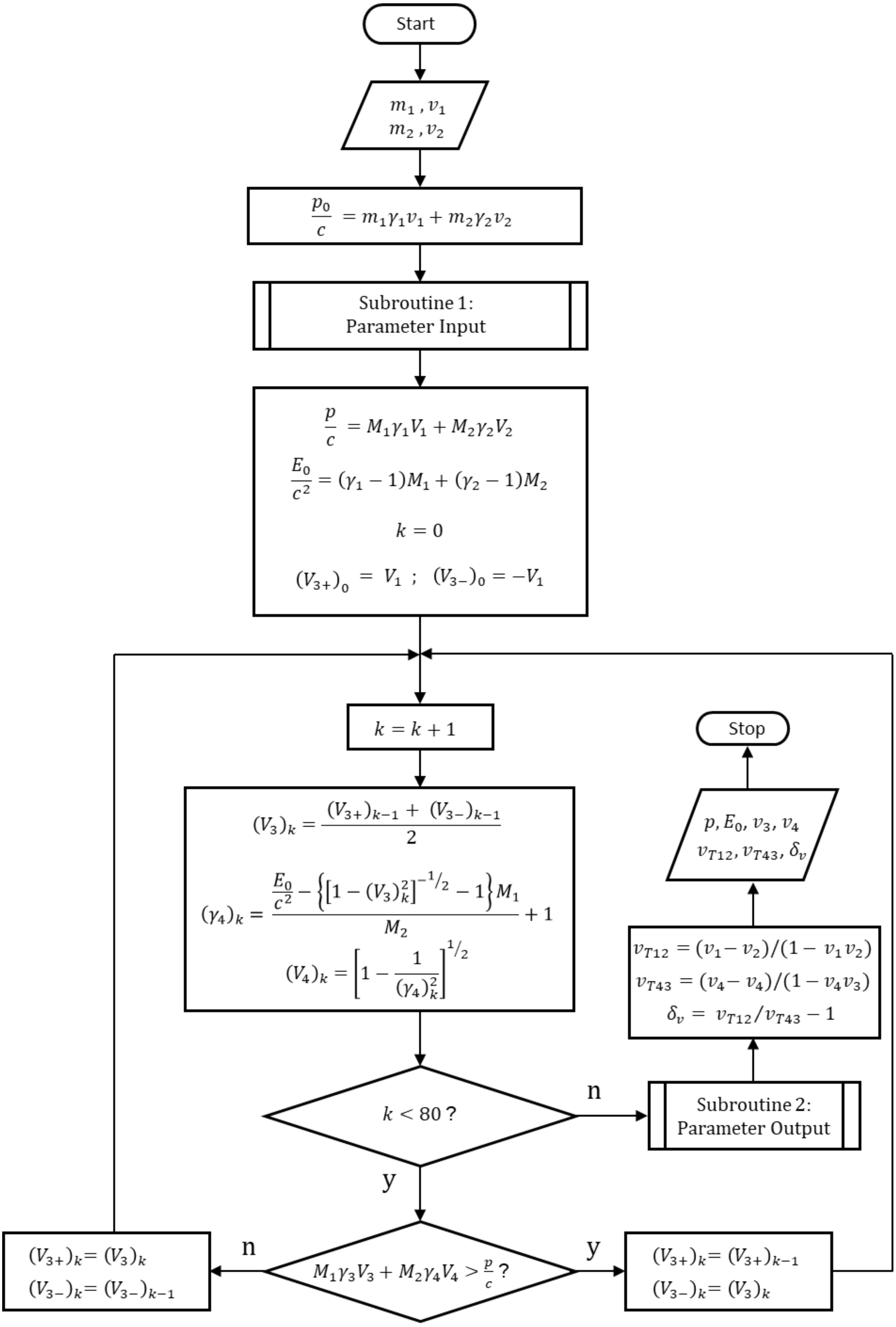

Fig. A.1: Flowchart of the calculation process

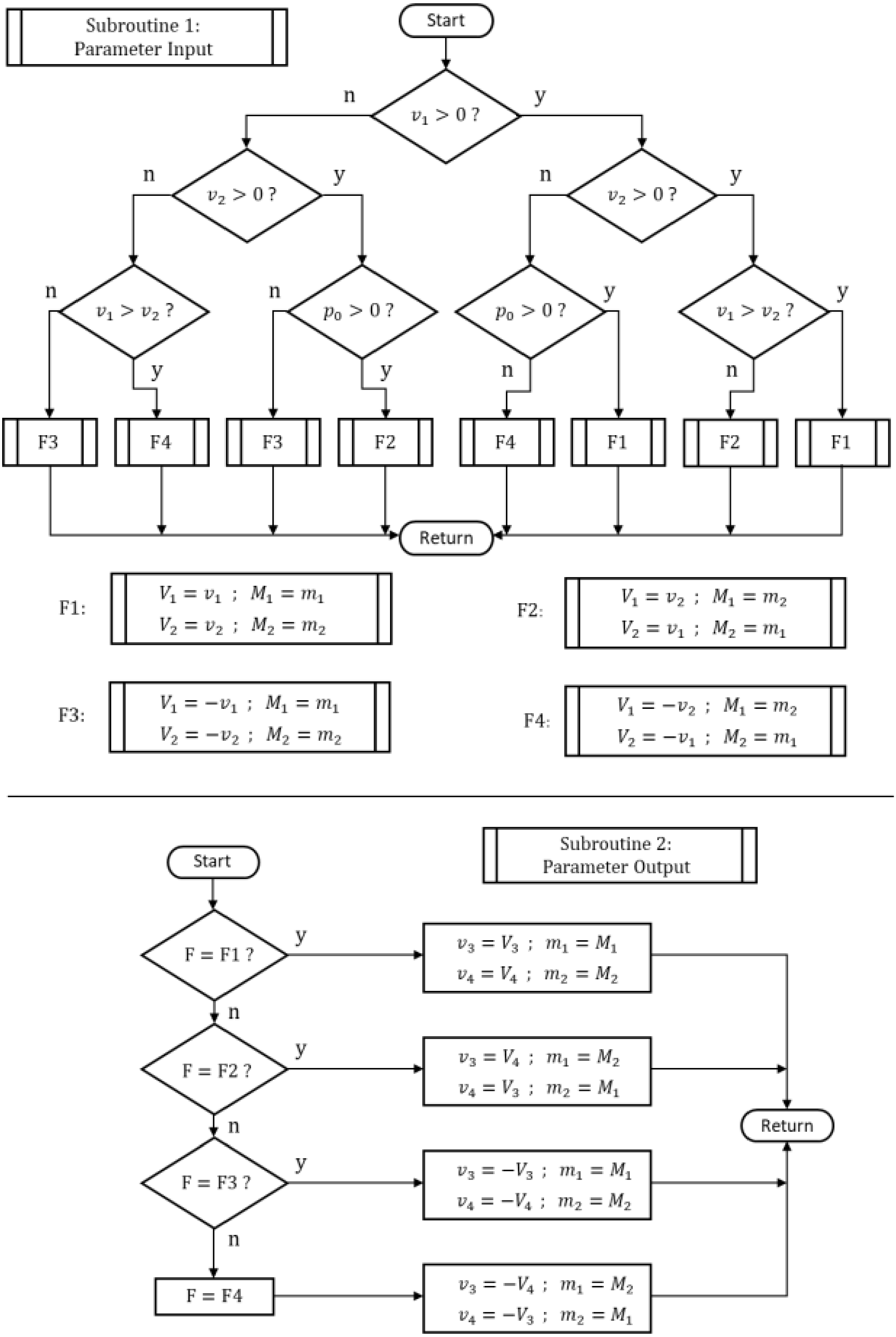

Fig. A.2: Subroutines for process in Fig. A.1

Symbol	VBA-Code	Symbol	VBA-Code	Symbol	VBA-Code
v_1	v1	v_2	v2	vc_1	vc1
vc_1	vc2	v_3	v3	vc_3	vc3
vc_{3-}	vc3m	vc_{3+}	vc3p	v_4	v4
vc_4	vc4	m_1	m1	mc_1	mc1
$v_T(v_1, v_2)$	vt12	$v_T(v_4, v_3)$	vt43	δ_v	Dv
m_2	m2	mc_2	mc2	p_0	p0
p	pc0	E_0	E0	γ_4	Ga4

Tab. A.3: Formula symbols and referring VBA-Codes

```
Sub A()
Dim v1, v2, vc1, vc2, v3, vc3, vc3m, vc3p, v4, vc4, vt12, vt43, Dv, m1,
mc1, m2, mc2, p0, pc0, E0, Ga4, Gav, K As Double
Dim F, F1, F2, F3, F4 As String
'Input
    v1 = 0.3
    v2 = -0.1
    m1 = 1
    m2 = 3
'Start calculation
    If v1 = v2 Then
        Debug.Print "Calculation not possible: v1 = v2"
        GoTo Out1:
        End If
    p0 = v1 * m1 / (1 - v1 ^ 2) ^ 0.5 + v2 * m2 / (1 - v2 ^ 2) ^ 0.5
'Subroutine 1
    If v1 > 0 Then
        GoTo P1:
    End If
    If v2 > 0 Then
        GoTo P2:
    End If
    If v1 > v2 Then
        F = "F4"
        Else
        F = "F3"
    End If
    GoTo Def1:
P2:
    If p0 > 0 Then
        F = "F2"
        Else
        F = "F3"
    End If
    GoTo Def1:
P1:
    If v2 > 0 Then
        GoTo P3:
```

```
    End If
    If p0 > 0 Then
        F = "F1"
        Else
        F = "F4"
    End If
    GoTo Def1:
P3:
    If v1 > v2 Then
        F = "F1"
        Else
        F = "F2"
    End If
    GoTo Def1:
Def1:
    If F = "F1" Then
        vc1 = v1
        vc2 = v2
        mc1 = m1
        mc2 = m2
    End If
    If F = "F2" Then
        vc1 = v2
        vc2 = v1
        mc1 = m2
        mc2 = m1
     End If
     If F = "F3" Then
        vc1 = -v1
        vc2 = -v2
        mc1 = m1
        mc2 = m2
    End If
    If F = "F4" Then
        vc1 = -v2
        vc2 = -v1
        mc1 = m2
        mc2 = m1
    End If
'End Subroutine 1
'Calculation
   pc0 = vc1 * mc1 / (1 - vc1 ^ 2) ^ 0.5 + vc2 * mc2 / (1 - vc2 ^ 2) ^ 0.5
   E0 = mc1 * ((1 - vc1 ^ 2) ^ -0.5 - 1) + mc2 * ((1 - vc2 ^ 2) ^ -0.5 - 1)
    vc3m = -vc1             'Values for start
    vc3p = vc1
    K = 0
Do
        K = K + 1
        vc3 = (vc3m + vc3p) / 2
        Ga4 = (E0 - ((1 - vc3 ^ 2) ^ -0.5 - 1) * mc1) / mc2 + 1
        vc4 = (1 - 1 / Ga4 ^ 2) ^ 0.5
        If (vc3 * mc1 / (1 - vc3 ^ 2) ^ 0.5 + vc4 * mc2 / (1 - vc4 ^ 2) ^
0.5) > pc0 Then
        vc3p = vc3
        Else
        vc3m = vc3
        End If
Loop Until K = 80
'Subroutine 2
    If F = "F1" Then
        v3 = vc3
        v4 = vc4
        m1 = mc1
```

```
        m2 = mc2
    End If
    If F = "F2" Then
        v3 = vc4
        v4 = vc3
        m1 = mc2
        m2 = mc1
     End If
     If F = "F3" Then
        v3 = -vc3
        v4 = -vc4
        m1 = mc1
        m2 = mc2
    End If
    If F = "F4" Then
        v3 = -vc4
        v4 = -vc3
        m1 = mc2
        m2 = mc1
    End If
'End Subroutine 2
    vt12 = (v1 - v2) / (1 - v1 * v2)
    vt43 = (v4 - v3) / (1 - v4 * v3)
    Dv = (vt12 / vt43) - 1
'Presentation of results: Calculated values in view of observer at rest
    Debug.Print "F =", F
    Debug.Print "v3 =", v3
    Debug.Print "v4 =", v4
    Debug.Print "vt12 =", vt12
    Debug.Print "vt43 =", vt43
    Debug.Print "Dv =", Dv
Out1:
End Sub
```

Fig. A.3: VBA Program-Code for the calculation process presented in Fig. A.1 and A.2

A.2 Spreadsheet calculation

The following equations are used for calculation:

$$p_0 = \frac{p}{c} = \frac{m_1\gamma_1 v_1 + m_2\gamma_2 v_2}{c} \qquad \frac{E_0}{c^2} = (\gamma_1 - 1)m_1 + (\gamma_2 - 1)m_2$$

$$\frac{(v_3)_k}{c} = \frac{(v_{3+})_{k-1} + (v_{3-})_{k-1}}{2 \cdot c} \qquad \gamma_4 = \frac{\frac{E_0}{c^2} - (\gamma_3 - 1)m_1}{m_2} + 1$$

$$\frac{v_4}{c} = \sqrt{1 - \frac{1}{\gamma_4^2}}$$

(Remark: Because of appropriate selection of basic conditions, only positive results of the square root must be considered.

Determination: $f(v_3)_1 > p : \Rightarrow \frac{(v_{3+})_k}{c} = \frac{(v_3)_k}{c}$ and $\frac{(v_{3-})_k}{c} = \frac{(v_{3-})_{k-1}}{c}$

Determination: $f(v_3)_1 < p: \Rightarrow \frac{(v_{3+})_k}{c} = \frac{(v_{3+})_{k-1}}{c}$ and $\frac{(v_{3-})_k}{c} = \frac{(v_3)_k}{c}$

Useful starting values: For $\frac{(v_{3-})_0}{c} = -\frac{v_1}{c}$ and for $\frac{(v_{3+})_0}{c} = \frac{v_1}{c}$

Values in the fields for results (blue color):

$$\frac{v_3}{c} = \frac{(v_3)_{k=80}}{c} \qquad \frac{v_4}{c} = \frac{(v_4)_{k=80}}{c}$$

$$\frac{v_T(\, v_1,\, v_2)}{c} = \frac{v_1 - v_2}{1 - \frac{v_1\, v_2}{c^2}} \qquad \frac{v_T(\, v_4,\, v_3)}{c} = \frac{v_4 - v_3}{1 - \frac{v_4\, v_3}{c^2}}$$

$$\delta_v = \frac{v_T(\, v_1,\, v_2)}{v_T(\, v_4,\, v_3)} - 1$$

As examples for $m_1 = 2;\ m_2 = 1$ the cases $v_1 = 0{,}5c$ and $v_2 = -0{,}5$c as well as $v_1 = 0{,}00001c$ and $v_2 = 0$ are shown.

Codes for calculation:

Coordinate		Code
B3	=	B1*D1*(1-B1^2)^-0,5+B2*D2*(1-B2^2)^-0,5
D3	=	D1*((1-B1^2)^-0,5-1)+D2*((1-B2^2)^-0,5-1)
B8	=	(E7+F7)/2
C8	=	(D$3-((1-B8^2)^-0,5-1)*D$1)/D$2+1
D8	=	(1-1/C8^2)^0,5
E8	=	IF((B8*D$1*(1-B8^2)^-0,5+D8*D$2*(1-D8^2)^-0,5)>B$3;E7;B8)
F8	=	IF((B8*D$1*(1-B8^2)^-0,5+D8*D$2*(1-D8^2)^-0,5)>B$3;B8;F7)
G9	=	IF(B9=B8;"x";"")
H9	=	IF(D9=D8;"x";"")
F1	=	B87
F2	=	D87
F3	=	(B1-B2)/(1-B1*B2)
F4	=	(F2-F1)/(1-F2*F1)
F5	=	F3/F4-1

Codes B8 to G8 to be copied as far as B87 to G87.

The status queries in columns G and H are used to determine whether the values for v_3 and v_4 still differ. For v_3 there are only slight deviations (Fig. A.4: step 51, Fig. A.5: step 52), v_4 shows strongly different behavior depending on the initial values; in these examples there are no further changes from step 49 (with interruptions), resp. already from step 19.

	A	B	C	D	E	F	G	H
1	v_1/c=	0,5	m_1=	2	v_3/c=	-0,209677419354839		
2	v_2/c=	-0,5	m_2=	1	v_4/c=	0,709302325581396		
3	p_0/c=	0,5773502692	E_0/c²=	0,4641016151	v_T/c (v_1, v_2)=	0,800000000000000		
4					v_T/c (v_4, v_3)=	0,800000000000000		
5					δ_V=	0,0E+00		
6	k	v_3/c	γ_4	v_4/c	v_{3-}/c	v_{3+}/c	St.	
7	0				-0,5	0,5	3	4
8	1	0,000000000000000	1,46410161513776	0,730406495763757	-0,500000000000000	0,000000000000000		
9	2	-0,250000000000000	1,39851049716047	0,699076919847366	-0,250000000000000	0,000000000000000		
10	3	-0,125000000000000	1,44829109242188	0,723362051517259	-0,250000000000000	-0,125000000000000		
11	4	-0,187500000000000	1,42799037361527	0,713863539464603	-0,250000000000000	-0,187500000000000		
12	5	-0,218750000000000	1,41446124643264	0,707230579852351	-0,218750000000000	-0,187500000000000		
13	6	-0,203125000000000	1,42151952770374	0,710722406262171	-0,218750000000000	-0,203125000000000		
14	7	-0,210937500000000	1,41806486850481	0,709022002845605	-0,210937500000000	-0,203125000000000		
15	8	-0,207031250000000	1,41981068269019	0,709883363160283	-0,210937500000000	-0,207031250000000		
16	9	-0,208984375000000	1,41894241344356	0,709455499378042	-0,210937500000000	-0,208984375000000		
17	10	-0,209960937500000	1,41850480254676	0,709239458600344	-0,209960937500000	-0,208984375000000		
18	11	-0,209472656250000	1,41872389812336	0,709347655434525	-0,209960937500000	-0,209472656250000		
19	12	-0,209716796875000	1,41861442290016	0,709293601181963	-0,209716796875000	-0,209472656250000		
20	13	-0,209594726562500	1,41866917864890	0,709320639342717	-0,209716796875000	-0,209594726562500		
21	14	-0,209655761718750	1,41864180530934	0,709307123021790	-0,209716796875000	-0,209655761718750		
22	15	-0,209686279296875	1,41862811523852	0,709300362791843	-0,209686279296875	-0,209655761718750		
23	16	-0,209671020507812	1,41863496055736	0,709303743079295	-0,209686279296875	-0,209671020507812		
24	17	-0,209678649902344	1,41863153796880	0,709302052978691	-0,209678649902344	-0,209671020507812		
25	18	-0,209674835205078	1,41863324928079	0,709302898039773	-0,209678649902344	-0,209674835205078		
26	19	-0,209676742553711	1,41863239362922	0,709302475511927	-0,209678649902344	-0,209676742553711		
27	20	-0,209677696228027	1,41863196580012	0,709302264245983	-0,209677696228027	-0,209676742553711		
47	40	-0,209677419355103	1,41863209000868	0,709302325581337	-0,209677419355103	-0,209677419354193		
48	41	-0,209677419354648	1,41863209000888	0,709302325581438	-0,209677419355103	-0,209677419354648		
49	42	-0,209677419354875	1,41863209000878	0,709302325581387	-0,209677419354875	-0,209677419354648		
50	43	-0,209677419354762	1,41863209000883	0,709302325581413	-0,209677419354875	-0,209677419354762		
51	44	-0,209677419354819	1,41863209000880	0,709302325581400	-0,209677419354875	-0,209677419354819		
52	45	-0,209677419354847	1,41863209000879	0,709302325581394	-0,209677419354847	-0,209677419354819		
53	46	-0,209677419354833	1,41863209000880	0,709302325581397	-0,209677419354847	-0,209677419354833		
54	47	-0,209677419354840	1,41863209000879	0,709302325581395	-0,209677419354840	-0,209677419354833		
55	48	-0,209677419354836	1,41863209000880	0,709302325581396	-0,209677419354840	-0,209677419354836		
56	49	-0,209677419354838	1,41863209000880	0,709302325581396	-0,209677419354840	-0,209677419354838		X
57	50	-0,209677419354839	1,41863209000880	0,709302325581396	-0,209677419354840	-0,209677419354839		X
58	51	-0,209677419354839	1,41863209000879	0,709302325581395	-0,209677419354839	-0,209677419354839	X	
59	52	-0,209677419354839	1,41863209000879	0,709302325581395	-0,209677419354839	-0,209677419354839	X	X
60	53	-0,209677419354839	1,41863209000879	0,709302325581395	-0,209677419354839	-0,209677419354839	X	X
61	54	-0,209677419354839	1,41863209000879	0,709302325581395	-0,209677419354839	-0,209677419354839	X	X
62	55	-0,209677419354839	1,41863209000880	0,709302325581396	-0,209677419354839	-0,209677419354839	X	

Fig. A.4: Results when using the spreadsheet calculation. $v_1 = 0{,}5c$, $v_2 = -0{,}5c$
Green fields: Input values. Steps between 20 and 40 hidden

	A	B	C	D	E	F	G	H
1	v_1/c=	0,00001	m_1=	2	v_3/c=	3,33333305742102E-06		
2	v_2/c=	0	m_2=	1	v_4/c=	1,33333338849358E-05		
3	p_0/c=	2,00000000E-05	E_0/c²=	1,00000008E-10	v_T/c (v_1, v_2)=	0,000010000000000		
4					v_T/c (v_4, v_3)=	0,000010000000828		
5					δ_V=	-8,3E-08		
6	k	v_3/c	γ_4	v_4/c	v_{3-}/c	v_{3+}/c	St.	
7	0				-0,5	0,5	3	4
8	1	0,00000000000000E+00	1,00000000010000	1,41421362087937E-05	0,00000000000000E+00	1,00000000000000E-05		
9	2	5,00000000000000E-06	1,00000000007500	1,22474492205951E-05	0,00000000000000E+00	5,00000000000000E-06		
10	3	2,50000000000000E-06	1,00000000009375	1,36930563961891E-05	2,50000000000000E-06	5,00000000000000E-06		
11	4	3,75000000000000E-06	1,00000000008594	1,31101090273823E-05	2,50000000000000E-06	3,75000000000000E-06		
12	5	3,12500000000000E-06	1,00000000009023	1,34338740912243E-05	3,12500000000000E-06	3,75000000000000E-06		
13	6	3,43750000000000E-06	1,00000000008818	1,32803375392606E-05	3,12500000000000E-06	3,43750000000000E-06		
14	7	3,28125000000000E-06	1,00000000008923	1,33591548737510E-05	3,28125000000000E-06	3,43750000000000E-06		
15	8	3,35937500000000E-06	1,00000000008871	1,33202712640041E-05	3,28125000000000E-06	3,35937500000000E-06		
16	9	3,32031250000000E-06	1,00000000008898	1,33398271082881E-05	3,32031250000000E-06	3,35937500000000E-06		
17	10	3,33984375000000E-06	1,00000000008885	1,33300694297534E-05	3,32031250000000E-06	3,33984375000000E-06		
18	11	3,33007812500000E-06	1,00000000008891	1,33349658128449E-05	3,33007812500000E-06	3,33984375000000E-06		
19	12	3,33496093750000E-06	1,00000000008888	1,33325345004281E-05	3,33007812500000E-06	3,33496093750000E-06		
20	13	3,33251953125000E-06	1,00000000008889	1,33337335592179E-05	3,33251953125000E-06	3,33496093750000E-06		
21	14	3,33374023437500E-06	1,00000000008889	1,33331340433020E-05	3,33251953125000E-06	3,33374023437500E-06		
22	15	3,33312988281250E-06	1,00000000008889	1,33334338046295E-05	3,33312988281250E-06	3,33374023437500E-06		
23	16	3,33343505859375E-06	1,00000000008889	1,33332672713907E-05	3,33312988281250E-06	3,33343505859375E-06		
24	17	3,33328247070313E-06	1,00000000008889	1,33333671915836E-05	3,33328247070313E-06	3,33343505859375E-06		
25	18	3,33335876464844E-06	1,00000000008889	1,33333338849358E-05	3,33328247070313E-06	3,33335876464844E-06		
26	19	3,33332061767578E-06	1,00000000008889	1,33333338849358E-05	3,33332061767578E-06	3,33335876464844E-06		X
27	20	3,33333969116211E-06	1,00000000008889	1,33333338849358E-05	3,33332061767578E-06	3,33333969116211E-06		X
47	40	3,33333305743509E-06	1,00000000008889	1,33333338849358E-05	3,33333305741690E-06	3,33333305743509E-06		X
48	41	3,33333305742599E-06	1,00000000008889	1,33333338849358E-05	3,33333305741690E-06	3,33333305742599E-06		X
49	42	3,33333305742144E-06	1,00000000008889	1,33333338849358E-05	3,33333305741690E-06	3,33333305742144E-06		X
50	43	3,33333305741917E-06	1,00000000008889	1,33333338849358E-05	3,33333305741917E-06	3,33333305742144E-06		X
51	44	3,33333305742031E-06	1,00000000008889	1,33333338849358E-05	3,33333305742031E-06	3,33333305742144E-06		X
52	45	3,33333305742087E-06	1,00000000008889	1,33333338849358E-05	3,33333305742087E-06	3,33333305742144E-06		X
53	46	3,33333305742116E-06	1,00000000008889	1,33333338849358E-05	3,33333305742087E-06	3,33333305742116E-06		X
54	47	3,33333305742102E-06	1,00000000008889	1,33333338849358E-05	3,33333305742087E-06	3,33333305742102E-06		X
55	48	3,33333305742095E-06	1,00000000008889	1,33333338849358E-05	3,33333305742095E-06	3,33333305742102E-06		X
56	49	3,33333305742098E-06	1,00000000008889	1,33333338849358E-05	3,33333305742098E-06	3,33333305742102E-06		X
57	50	3,33333305742100E-06	1,00000000008889	1,33333338849358E-05	3,33333305742100E-06	3,33333305742102E-06		X
58	51	3,33333305742101E-06	1,00000000008889	1,33333338849358E-05	3,33333305742101E-06	3,33333305742102E-06		X
59	52	3,33333305742101E-06	1,00000000008889	1,33333338849358E-05	3,33333305742101E-06	3,33333305742102E-06	X	X
60	53	3,33333305742101E-06	1,00000000008889	1,33333338849358E-05	3,33333305742101E-06	3,33333305742102E-06	X	X
61	54	3,33333305742102E-06	1,00000000008889	1,33333338849358E-05	3,33333305742102E-06	3,33333305742102E-06		X
62	55	3,33333305742102E-06	1,00000000008889	1,33333338849358E-05	3,33333305742102E-06	3,33333305742102E-06	X	X

Fig. A.5: Representation as in Fig. A.4. $v_1 = 0{,}00001c$, $v_2 = 0$
Values for v_4 already unchanged as of iteration step 19