Annex B: Exchange of signals during and after acceleration

In this annex it is shown that the reception of signals from an accelerated system by an observer at rest at the beginning of the acceleration phase and by an observer in uniform motion leads to the same results. The analytic relations valid here were already derived in chapter 6.4.1 in the equations (6.60) to (6.80). However, there is also a numerical method for the solution of this problem, which will be presented in the following. There are advantages and disadvantages between the analytical and the numerical method, which become visible in a comparison, also with comparable results of the numerical method from Annex C.

B.1 Numerical solution

The following general correlation between velocity and acceleration within the moving system S apply

$$\Delta v = a(v) \cdot \Delta t(v) = a_S \cdot \Delta t_S \qquad \text{(B.01)}$$

The values of a_S and Δt_S are constant by definition. A numerical solution requires the multiple calculation of different steps; for this, first the relativistic velocity addition is used, then the determination of the increase of time and distance follows for each case.

1st step:

$$v_1 = \frac{v_0 + \Delta v}{1 + \frac{v_0 \Delta v}{c^2}} = \frac{v_0 + a_S \Delta t_S}{1 + \frac{v_0 a_S \Delta t_S}{c^2}} \qquad \text{(B.02)}$$

2nd step:

$$\Delta t_1 = \Delta t_S \cdot \frac{\gamma(v_1) + \gamma(v_0)}{2} \qquad \text{(B.03)}$$

3rd step:

$$\Delta x_1 = \Delta t_1 v_0 + \frac{v_1 + v_0}{2} \Delta t_1 \qquad \text{(B.04)}$$

It should be noted that the functions for $\gamma(v)$ and $v(\Delta v)$ are not linear and thus the formation of a mean value is only an approximation, and the error must be compensated by choosing suitably small intervals for Δt_S. These steps are now to be repeated N times and the single results added. In general, it applies

$$t_N = \Delta t_S \sum_{K=1}^{N} \frac{\gamma(v_K) + \gamma(v_{K-1})}{2} \qquad \text{(B.05)}$$

$$x_N = \sum_{K=1}^{N} \Delta t_K \frac{v_K + v_{K-1}}{2} \tag{B.06}$$

with

$$v_{K+1} = \frac{v_K + \Delta v}{1 + \frac{v_K \Delta v}{c^2}} - v_0 = \frac{v_K + a_S \Delta t_S}{1 + \frac{v_K a_S \Delta t_S}{c^2}} - v_0 \tag{B.07}$$

At any arbitrary time t_K a signal from the accelerated system S shall be transmitted back to observer A. In case of $v_0 \neq 0$ observer A is moving during signal propagation in view of B either in direction to S or in the opposite way. Because a_S and v_0 can be both positive and/or negative, for the calculation different regulations are necessary (see also the comprehensive presentations in chapter 2.1). If first the situation is discussed that a_S and v_0 are both positive, then observer B will find the situation according to type "b" referring to Fig. 2.2 as

$$\Delta t = \Delta t_S \left(1 + \frac{v_0}{c}\right) \tag{B.08}$$

When a_S and v_0 show in different directions, however, the algebraic sign is changing in equation Eq. (B.08) according to situation of type "d" from Fig. 2.2.

In summary, the following combinations arise for the time between two pulses $t_{K,R}$ perceived by observer B due to the increasing distance, into which any positive or negative values for the velocity v_0 can be inserted:

$a_S > 0$:
$$t_{K,R} = \frac{x_K - v_0 t_K}{c\left(1 + \frac{v_0}{c}\right)} \tag{B.09}$$

$a_S < 0$:
$$t_{K,R} = \frac{|(x_K - v_0 t_K)|}{c\left(1 - \frac{v_0}{c}\right)} \tag{B.10}$$

For $v_0 = 0$ both equations for any value of a_S simplify to

$$t_{K,R} = \frac{|x_K|}{c} \tag{B.11}$$

The total time from the start of the acceleration to the transmission and subsequent reception of the signal is then in all cases

$$t_{K,T} = t_K + t_{K,R} \tag{B.12}$$

Further, the signals received by observer A must be adjusted in view of B according to equation

$$t_{K,T}(v_0) = \frac{t_K + t_{K,R}}{\gamma(v_0)} \tag{B.13}$$

to cover the effect, that for A in view of B the time is running slower by the factor $\gamma(v_0)$ according to the Lorentz-equations.

With the relations presented here it is possible to determine the values for the reception times of observers moving relative to each other. For this purpose, first the time intervals are calculated, with which the accelerated system S transmits the signals. While these are subjectively Δt_S within the system S, from a non-accelerated observer the values for the time interval can be determined using the equations presented before. The calculation scheme can also be used to define the distance of S when transmitting the signals. Thus, the total times for the arrival of the signals can be determined for any arbitrarily moving observer.

Fig. B.1 shows the program flow chart for the numerical calculation of v_N, t_N, t_T and x_N according to the equations mentioned (the values for t_K and x_K are calculated throughout; since only the last results are considered, these correspond to t_N and x_N). In addition, the acceleration a_N is determined for an observer moving relative to the system S; the value deviates from the acceleration a_S, which can be measured subjectively in S. As already shown in chapter 6.4.1, the subjectively adjusted acceleration in S and the acceleration measured by an external observer moving relative to it with the velocity v must differ by the factor $\gamma^3(v)$. Therefore, to verify this theoretically expected effect, the value $\gamma^3 a_N$ was also calculated from the data. The results show a very good agreement between a_S and $\gamma^3 a_N$.

The used VBA program (Visual Basic) code is shown in Fig. B.1. In Tab. B.1 the formula symbols taken for the calculation program are assigned to those used in the text. The program was designed in such a way that the initial velocity v_0, as well as the subjectively valid acceleration a_S and total duration of the experiment t_S are to be specified. In addition, the number of intended iteration steps N can be freely selected, which provides an important influencing variable. With the VBA program, values up to $\mathrm{N} = 10^7$ were investigated. These calculations only make sense with such programs, since with a conventional spreadsheet each iteration step requires separate program fields, and this would lead to enormous file sizes.

Tab. B.2 shows in the parts a) to c) the results from calculations with the boundary conditions $a_S = 10\ \mathrm{m/s^2}$ and $t_S = 1000\mathrm{s}$. Values of $v_0 = 0$, $v_0 = 369\ \mathrm{km/s}$ and $v_0 = 0.5c$ were chosen as initial velocities. For all results, δ-values were calculated according to the scheme

$$\delta v_T = \frac{v_T(\mathrm{K})}{v_T(\mathrm{K}-1)} - 1 \tag{B.14}$$

and compared, where K in this case represents a potency of 10 according to the specifications in the table.

The calculations performed show that within a range of about 10^2 to 10^4 the differences between the results reach a minimum. This suggests that these zones have the largest confidence range. This is primarily dependent on the chosen calculation system; Microsoft Excel© was used as the method here, which has an accuracy of 15 digits. If computer systems with higher accuracy would be used, other results are to be expected. However, the overall quality of the calculations can only be verified in the comparison between the analytical and numerical methods, which will be carried out subsequently.

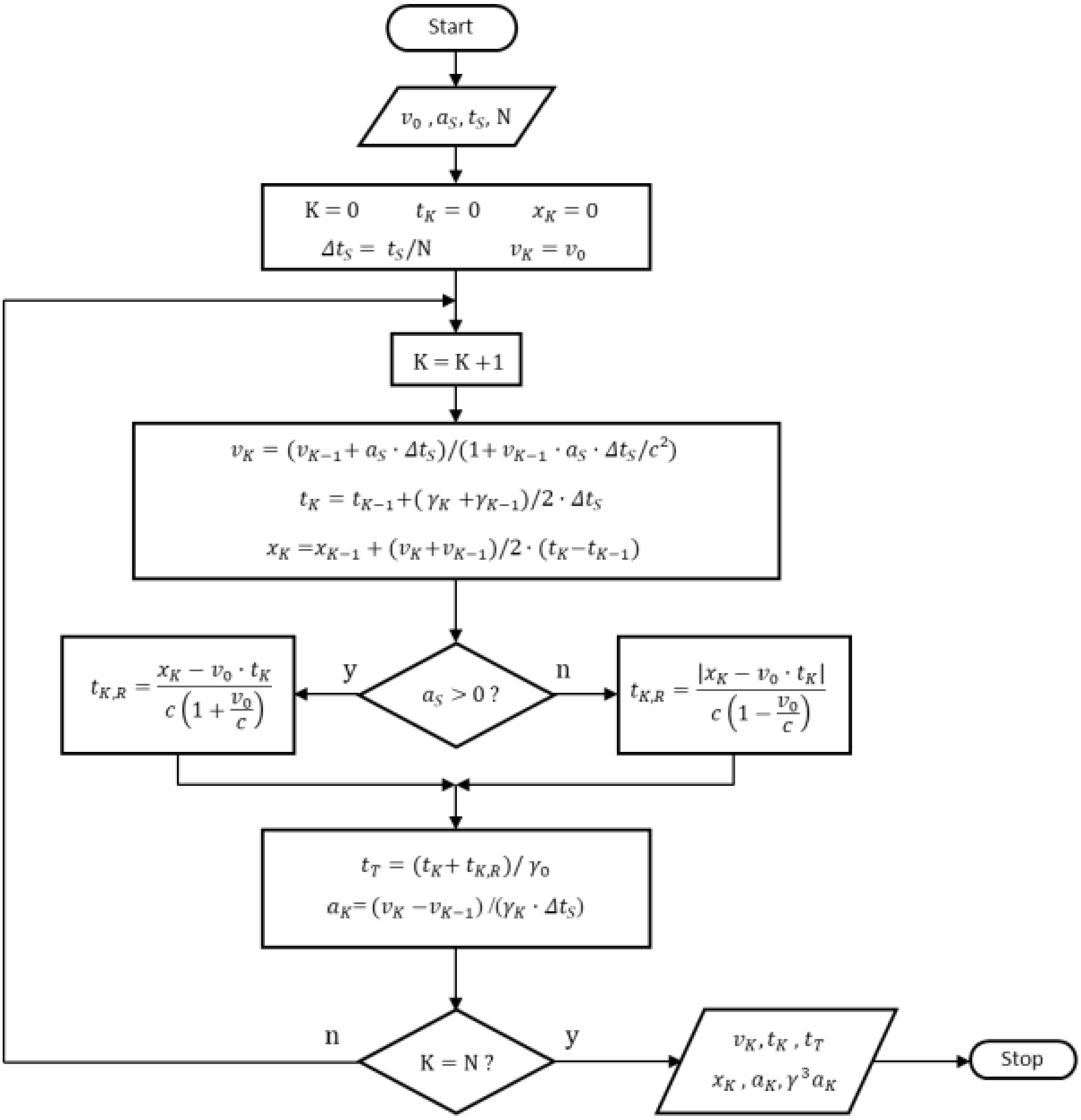

Fig. B.1: Flowchart of the calculation process

Symbol	VBA-Code	Symbol	VBA-Code	Symbol	VBA-Code
v_0	v0	a_S	a0	t_S	tS
Δt_S	dtS	t_K	tK	t_{K-1}	tKm1
x_K	xK	v_K	vK	v_{K-1}	vKm1
γ_K	GaK	γ_{K-1}	GaKm1	$t_{K,R}$	tKR
t_T	tT	a_K	aK	$\gamma^3 a_K$	aKGa3

Tab. B.1: Formula symbols and referring VBA-Codes

```
Sub B()
Dim c, v0, a0, aK, tS, dtS, tK, tKm1, xK, vK, vKm1, GaK, GaKm1 As Double
Dim aKGa3, tKR, tT, vT, K, N As Double
'Input
    v0 = 299792.458 / 2 'Initial velocity in km/s
    a0 = 10             'Acceleration in m/s²
    N = 1000            'Number of iteration steps
    tS = 1000           'Time for S between transmission of signals in s
'Start Calculation
    c = 299792.458       'Speed of light in km/s
    a0 = a0 / 1000       'Acceleration in km/s²
    dtS = tS / N
    tK = 0
    xK = 0
    vK = v0
    For K = 1 To N
        vKm1 = vK
        tKm1 = tK
        GaKm1 = 1 / (1 - (vKm1 / c) ^ 2) ^ 0.5
        vK = (vK + a0 * dtS) / (1 + vK * a0 * dtS / c ^ 2)
        GaK = 1 / (1 - (vK / c) ^ 2) ^ 0.5
        tK = tK + (GaKm1 + GaK) / 2 * dtS
        xK = xK + (vK + vKm1) / 2 * (tK - tKm1)
            If a0 > 0 Then
            tKR = (xK - tK * v0) / c / (1 + v0 / c)
            Else
            tKR = Abs((xK - tK * v0) / c / (1 - v0 / c))
            End If
        tT = (tK + tKR) * (1 - (v0 / c) ^ 2) ^ 0.5
    aK = (vK - vKm1) / (GaK * dtS) * 1000
    aKGa3 = aK * GaK ^ 3
    vT = vK - v0
    Next K
'Results in view of an observer moving with v0 at beginning of trial
    Debug.Print "vT", "vK", "tN", "xN", "aN", "aNGa3"
    Debug.Print vT, vK, tT, xK, aK, aKGa3
End Sub
```

Fig. B.2: VBA Program-Code for the calculation process presented in Fig. B1

Basically, it can be stated that all δ-values are very low at $v_0 = 0$ and then increase slightly at higher numbers. In particular, the values for t_T, which would be well suited for experimental verification, hardly differ between the individual values of v_0 within a range with constant acceleration a_S. Also, between the different acceleration values the differences are so small that a systematic influence cannot be assumed, but the effects are due to influences of the numerical calculation.

The deviations between the results for the selected iteration steps between 1 and 10^7 show that there are no systematic deviations. In the range of 10^3 the results show a high stability and the smallest differences; therefore, they are particularly suitable for comparative considerations.

The additional value of $v_0 = 369$ km/s was chosen because it corresponds to the velocity of the sun with respect to the cosmic background radiation and therefore, if an effect would show up in the calculations, it could be an appropriate basis for further considerations (see also chapter 1.7).

It is to be noted, however, that in none of these evaluations a noticeable difference becomes recognizable and thus the subjectively determined observations between differently moving observers agree. This is also true for the high velocity of $v_0 = 0{,}5c$.

In addition, it should be mentioned that the values of t_N, x_N etc. used here were named in this way exclusively because of the numerical calculation method and correspond to the analytically determined data for t_A and x_A, respectively. Accordingly, these values also refer to the measurement results of the observer A moving with the same speed as S at the beginning of an experiment.

K	N	v_T	t_N	t_T	x_N	$\gamma^3 a_N$
1	1	10,0000000000000	1000,00000027816	1000,01667848293	5000,00000139081	10,0000000111265
2	10	9,99999999632825	1000,00000018637	1000,01667839113	5000,00000047288	10,0000000011126
3	10^2	9,99999999629152	1000,00000018545	1000,01667839021	5000,00000046369	10,0000000001113
4	10^3	9,99999999629152	1000,00000018545	1000,01667839020	5000,00000046369	10,0000000000098
5	10^4	9,99999999629107	1000,00000018544	1000,01667839020	5000,00000046378	10,0000000000010
6	10^5	9,99999999628186	1000,00000018545	1000,01667839021	5000,00000046244	9,99999999991214
7	10^6	9,99999999612586	1000,00000018732	1000,01667839208	5000,00000044603	10,0000000000898
8	10^7	9,99999999923743	1000,00000024389	1000,01667844866	5000,00000204189	9,99999998587893
		δv_T	δt_N	δt_T	δx_N	$\delta \gamma^3 a_N$
1/2		$3{,}67 \cdot 10^{-10}$	$9{,}18 \cdot 10^{-11}$	$9{,}18 \cdot 10^{-11}$	$1{,}84 \cdot 10^{-10}$	$1{,}00 \cdot 10^{-9}$
2/3		$3{,}67 \cdot 10^{-12}$	$9{,}20 \cdot 10^{-13}$	$9{,}20 \cdot 10^{-13}$	$1{,}84 \cdot 10^{-12}$	$1{,}00 \cdot 10^{-10}$
3/4		0	0	$9{,}99 \cdot 10^{-15}$	0	$1{,}01 \cdot 10^{-11}$
4/5		$4{,}49 \cdot 10^{-14}$	$9{,}99 \cdot 10^{-15}$	0	$-1{,}80 \cdot 10^{-14}$	$8{,}88 \cdot 10^{-13}$
5/6		$9{,}21 \cdot 10^{-13}$	$-9{,}99 \cdot 10^{-15}$	$-9{,}99 \cdot 10^{-15}$	$2{,}68 \cdot 10^{-13}$	$8{,}88 \cdot 10^{-12}$
6/7		$1{,}56 \cdot 10^{-11}$	$-1{,}87 \cdot 10^{-12}$	$-1{,}87 \cdot 10^{-12}$	$3{,}28 \cdot 10^{-12}$	$-1{,}78 \cdot 10^{-11}$
7/8		$-3{,}11 \cdot 10^{-10}$	$-5{,}66 \cdot 10^{-11}$	$-5{,}66 \cdot 10^{-11}$	$-3{,}19 \cdot 10^{-10}$	$1{,}42 \cdot 10^{-9}$

a) $v_0 = 0,\ a_S = 10\mathrm{m/s^2},\ t_S = 1000\mathrm{s}$

K	N	v_T	t_N	t_T	x_N	$\gamma^3 a_N$
1	1	378,999984439478	1000,00077830515	1000,01667848259	374000,283305861	10,0000004216944
2	10	378,999984435807	1000,00077821336	1000,01667839113	374000,283372686	10,0000000421697
3	10^2	378,999984435772	1000,00077821244	1000,01667839021	374000,283373356	10,0000000042204
4	10^3	378,999984435760	1000,00077821243	1000,01667839020	374000,283373357	10,0000000004517
5	10^4	378,999984435533	1000,00077821243	1000,01667839020	374000,283373242	9,99999999988323
6	10^5	378,999984433259	1000,00077821243	1000,01667839020	374000,283372125	10,0000000010201
7	10^6	378,999984411821	1000,00077821244	1000,01667839018	374000,283362434	9,99999998396706
8	10^7	378,999984156412	1000,00077821289	1000,01667839010	374000,283206699	10,0000000408106
		δv_T	δt_N	δt_T	δx_N	$\delta \gamma^3 a_N$
1/2		$9{,}69 \cdot 10^{-12}$	$9{,}18 \cdot 10^{-11}$	$9{,}15 \cdot 10^{-11}$	$-1{,}79 \cdot 10^{-10}$	$3{,}80 \cdot 10^{-8}$
2/3		$9{,}10 \cdot 10^{-14}$	$9{,}20 \cdot 10^{-13}$	$9{,}20 \cdot 10^{-13}$	$-1{,}79 \cdot 10^{-12}$	$3{,}79 \cdot 10^{-9}$
3/4		$3{,}09 \cdot 10^{-14}$	$9{,}99 \cdot 10^{-15}$	$9{,}99 \cdot 10^{-15}$	$-2{,}66 \cdot 10^{-15}$	$3{,}77 \cdot 10^{-10}$
4/5		$6{,}01 \cdot 10^{-13}$	0	0	$3{,}07 \cdot 10^{-13}$	$5{,}68 \cdot 10^{-11}$
5/6		$6{,}00 \cdot 10^{-12}$	0	0	$2{,}99 \cdot 10^{-12}$	$-1{,}14 \cdot 10^{-10}$
6/7		$5{,}66 \cdot 10^{-11}$	$-9{,}99 \cdot 10^{-15}$	$2{,}00 \cdot 10^{-14}$	$2{,}59 \cdot 10^{-11}$	$1{,}71 \cdot 10^{-9}$
7/8		$6{,}74 \cdot 10^{-10}$	$-4{,}50 \cdot 10^{-13}$	$7{,}99 \cdot 10^{-14}$	$4{,}16 \cdot 10^{-10}$	$-5{,}68 \cdot 10^{-9}$

b) $v_0 = 369\ \mathrm{km/s},\ a_S = 10\mathrm{m/s^2},\ t_S = 1000\mathrm{s}$

K	N	v_T	t_N	t_T	x_N	$\gamma^3 a_N$
1	1	149903,728874916	1154,71016786646	1000,01667841339	173091029,842051	10,0001667931727
2	10	149903,728874913	1154,71016776046	1000,01667839043	173091029,861909	10,0000166791061
3	10^2	149903,728874913	1154,71016775940	1000,01667839021	173091029,862108	10,0000016684019
4	10^3	149903,728874922	1154,71016775940	1000,01667839022	173091029,862117	10,0000002015203
5	10^4	149903,728874803	1154,71016775925	1000,01667838996	173091029,862026	9,99999992987018
6	10^5	149903,728875378	1154,71016775999	1000,01667839124	173091029,862469	10,0000022582798
7	10^6	149903,728863738	1154,71016774494	1000,01667836518	173091029,853446	9,99998285456124
8	10^7	149903,729001552	1154,71016792619	1000,01667867908	173091029,962107	10,0000216670928
		δv_T	δt_N	δt_T	δx_N	$\delta \gamma^3 a_N$
1/2		$1{,}84 \cdot 10^{-14}$	$9{,}18 \cdot 10^{-11}$	$2{,}30 \cdot 10^{-11}$	$-1{,}15 \cdot 10^{-10}$	$1{,}50 \cdot 10^{-5}$
2/3		0	$9{,}18 \cdot 10^{-13}$	$2{,}20 \cdot 10^{-13}$	$-1{,}15 \cdot 10^{-12}$	$1{,}50 \cdot 10^{-6}$
3/4		$-5{,}84 \cdot 10^{-14}$	0	$-9{,}99 \cdot 10^{-15}$	$-5{,}20 \cdot 10^{-14}$	$1{,}47 \cdot 10^{-7}$
4/5		$7{,}93 \cdot 10^{-13}$	$1{,}30 \cdot 10^{-13}$	$2{,}60 \cdot 10^{-13}$	$5{,}26 \cdot 10^{-13}$	$2{,}72 \cdot 10^{-8}$
5/6		$-3{,}84 \cdot 10^{-12}$	$-6{,}41 \cdot 10^{-13}$	$-1{,}28 \cdot 10^{-12}$	$-2{,}56 \cdot 10^{-12}$	$-2{,}33 \cdot 10^{-7}$
6/7		$7{,}77 \cdot 10^{-11}$	$1{,}30 \cdot 10^{-11}$	$2{,}61 \cdot 10^{-11}$	$5{,}21 \cdot 10^{-11}$	$1{,}94 \cdot 10^{-6}$
7/8		$-9{,}19 \cdot 10^{-10}$	$-1{,}57 \cdot 10^{-10}$	$-3{,}14 \cdot 10^{-10}$	$-6{,}28 \cdot 10^{-10}$	$-3{,}88 \cdot 10^{-6}$

c) $v_0 = 0.5c,\ a_S = 10\text{m/s}^2,\ t_S = 1000\text{s}$

Tab. B.2: Results for v_T, t_N, t_T, x_N and $\gamma^3 a_N$ acc. to calculations of Program B presented in Fig. B.2 as a function of the number of iteration steps N. Values for v_T in km/s, t_T in s, x_N in km and a_N in m/s^2.

B.3 Improved accuracy by using a Taylor expansion

If the analytical calculations shown are to be carried out for very small values for time or speed, larger differences result depending on the calculation accuracy. This concerns in particular equation (6.74) for the distance covered during an experiment

$$x_A = \frac{c^2}{a_S}\left\{\left(1-\frac{v_A^2}{c^2}\right)^{-1/2} - 1\right\} = \frac{c^2}{a_S}\,(\gamma - 1) \qquad (6.74)$$

For small values for v_A, the effect arises that the value for γ deviates only slightly from 1 and the final result becomes inaccurate because of the difference formation to 1. In the present case, the spreadsheet program Microsoft Excel© was used which provides an accuracy of 15 digits, and thus for values for v_A below about 400 km/s, deviations occur which can become very high for small values. In this case, instead of using Eq. (6.74), it is recommended to use a Taylor expansion for γ that contains "1" as the first value. This is:

$$\gamma = \left(1-\frac{v_A^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2}\frac{v_A^2}{c^2} + \frac{3}{8}\frac{v_A^4}{c^4} + \frac{15}{48}\frac{v_A^6}{c^6} + + \cdots \qquad (\text{B}.15)$$

1. 2. 3. 4. Taylor − elements

The following table B.3 shows the effect on the results for different test times t_S or velocities of v_A using different approaches.

t_S	v_A	t_A	$x_A(1)$	$x_A(2)$	$x_A(3)$
1	0,0100000000000000	1,0000000000000	0,00399127477173885	0,0050000000000000	0,0050000000000000
10	0,0999999999999963	10,0000000000002	0,498909346467356	0,500000000000004	0,500000000000004
100	0,999999999996291	100,000000000185	50,0006947029584	50,0000000000463	50,000000000463
1.000	9,99999999629117	1000,00000018544	5000,00161862472	5000,0000046360	5000,0000046360
10.000	99,9999962911666	10000,0001854417	500000,004207119	500000,004636038	500000,004636042
20.000	199,999970329337	20000,0014835334	2000000,07470196	2000000,07417642	2000000,07417667
40.000	399,999762634824	40000,0118682683	8000001,18686648	8000001,18681095	8000001,18682679
60.000	599,999198893243	60000,0400554100	18000006,0086520	18000006,0081306	18000006,0083111
80.000	799,998101082647	80000,0949461719	32000018,9903299	32000018,9882180	32000018,9892321
100.000	999,996291182986	100000,185441779	50000046,3602845	50000046,3565675	50000046,3604362
200.000	1999,97032986004	200001,483536709	200000741,768963	200000741,520219	200000741,767795
1.000.000	9996,29281639030	1000185,45199287	5000463621,38578	5000459757,50365	5000463617,62560

Tab. B.3: Values of v_A, t_A and x_A depending on t_S acc. to different procedures
$x_A(1)$: Eq. (6.74)
$x_A(2)$: Eq. (B.15) Taylor elements 1–3
$x_A(3)$: Eq. (B.15) Taylor elements 1–4
Optimal values for x_A marked in green. Results in km and s.

For t_S values up to 20,000s, the calculation according to $x_A(3)$ using the first 4 Taylor elements has the highest accuracy, up to 1,000s the solution with $x_A(2)$ is also sufficiently accurate. For values from approx. 40,000s, Eq. (6.74) is preferable (or further Taylor elements would have to be added).

B.4 Comparison of results of the different methods

Finally, the results calculated from the different methods will be compared. In addition to the numerical and analytical methods presented here, the numerically obtained results from Annex C based on the relativistic rocket equation have been added. While in the first two calculations a constant acceleration is made a prerequisite, the same situation arises in the relativistic rocket equation for the special case that the ejection of the propellant mass is kept constant in relation to the remaining mass of the rocket.

Tab. B.4 shows the values determined according to the different methods $v_T = v_N - v_0$, t_A, t_T and x_N for the initial velocities $v_0 = 0$ as well as 369 km/s and 0.5c. The values listed in A were calculated analytically using the equations Eq. (6.60) to (6.74). For the velocities $v_0 = 0$ and 369 km/s the Taylor expansion was used as described in Tab. B3, details are presented in the table. The values for B are the numerical results corresponding to Annex B, and C are from Annex C, type "B1". The comparison shows that the velocities v_T for A and B agree very well, but this deviates somewhat for variant C, especially for higher initial values. Moreover, for A, slightly higher values for x_N result in the range of small velocities. In general, however, it can be said that the agreement of the results is good despite the completely different approaches.

Furthermore, for a comprehensive overlook the values for $\gamma^3 a_N$ were added. It is shown in all cases that they correspond very exactly to the value of a_S subjectively valid for the accelerated observer.

a)	St.	v_A	t_A	t_T	x_A	a_S
A	(1)	0	0		0	10
A	(2)	9,99999999629117	1000,00000018544		5000,0000046361	10
A	Diff.	9,99999999629117	1000,00000018544	1000,01667839020	5000,0000046361	
	N	v_T	t_N	t_T	x_N	$\gamma^3 a_N$
B	10^2	9,99999999629152	1000,00000018545	1000,01667839021	5000,0000046369	10,0000000001113
B	10^3	9,99999999629152	1000,00000018545	1000,01667839020	5000,0000046369	10,0000000000098
B	10^4	9,99999999629107	1000,00000018544	1000,01667839020	5000,0000046378	10,0000000000010
B	10^5	9,99999999628186	1000,00000018545	1000,01667839021	5000,0000046244	9,99999999991214
	N	δv_T	δt_N	δt_T	δx_N	$\delta\gamma^3 a_N$
B	10^2	$-3{,}52 \cdot 10^{-14}$	$-8{,}22 \cdot 10^{-15}$	$-6{,}78 \cdot 10^{-15}$	$-1{,}69 \cdot 10^{-14}$	$-1{,}11 \cdot 10^{-11}$
B	10^3	$-3{,}52 \cdot 10^{-14}$	$-8{,}22 \cdot 10^{-15}$	0	$-1{,}69 \cdot 10^{-14}$	$-9{,}84 \cdot 10^{-13}$
B	10^4	$9{,}77 \cdot 10^{-15}$	0	0	$-3{,}50 \cdot 10^{-14}$	$-9{,}65 \cdot 10^{-14}$
B	10^5	$9{,}31 \cdot 10^{-13}$	$-8{,}22 \cdot 10^{-15}$	$-6{,}78 \cdot 10^{-15}$	$2{,}34 \cdot 10^{-13}$	$8{,}79 \cdot 10^{-12}$
	N	v_T	t_N	t_T	x_N	$\gamma^3 a_N$
C	10^2	9,99999999608546	1000,00000018545	1000,01667839021	5000,00000054144	9,99999999883114
C	10^3	9,99999999607074	1000,00000018544	1000,01667839020	5000,0000053831	9,99999999868366
C	10^4	9,99999999607296	1000,00000018544	1000,01667839020	5000,0000053928	9,99999999869584
C	10^5	9,99999999610921	1000,00000018545	1000,01667839021	5000,0000055217	9,99999999845206
	N	δv_T	δt_N	δt_T	δx_N	$\delta\gamma^3 a_N$
C	10^2	$2{,}06 \cdot 10^{-11}$	$-8{,}22 \cdot 10^{-15}$	$-6{,}78 \cdot 10^{-15}$	$3{,}24 \cdot 10^{-7}$	$1{,}17 \cdot 10^{-10}$
C	10^3	$2{,}20 \cdot 10^{-11}$	0	0	$3{,}24 \cdot 10^{-7}$	$1{,}32 \cdot 10^{-10}$
C	10^4	$2{,}19 \cdot 10^{-11}$	0	0	$3{,}24 \cdot 10^{-7}$	$1{,}30 \cdot 10^{-10}$
C	10^5	$1{,}82 \cdot 10^{-11}$	$-8{,}22 \cdot 10^{-15}$	$-6{,}78 \cdot 10^{-15}$	$3{,}24 \cdot 10^{-7}$	$1{,}55 \cdot 10^{-10}$

b)	St.	v_A	t_A	t_T	x_A	a_S
A	(1)	369	36900,0279516977		6808057,73563331	10
A	(2)	378,99998443578	37900,0287299112		7182058,01900707	10
A	Diff.	9,99998443578	1000,0007782135	1000,01667839124	374000,28337376	
	N	v_T	t_N	t_T	x_N	$\gamma^3 a_N$
B	10^2	9,99998443577	1000,0007782124	1000,01667839021	374000,28337336	10,0000000042204
B	10^3	9,99998443576	1000,0007782124	1000,01667839020	374000,28337336	10,0000000004517
B	10^4	9,99998443553	1000,0007782124	1000,01667839020	374000,28337324	9,99999999988323
B	10^5	9,99998443326	1000,0007782124	1000,01667839020	374000,28337213	10,0000000010201
	N	δv_T	δt_N	δt_T	δx_N	$\delta\gamma^3 a_N$
B	10^2	$2{,}09 \cdot 10^{-14}$	$1{,}04 \cdot 10^{-12}$	$1{,}03 \cdot 10^{-12}$	$2{,}87 \cdot 10^{-12}$	$-4{,}22 \cdot 10^{-10}$
B	10^3	$5{,}17 \cdot 10^{-14}$	$1{,}05 \cdot 10^{-12}$	$1{,}04 \cdot 10^{-12}$	$1{,}08 \cdot 10^{-12}$	$-4{,}52 \cdot 10^{-11}$
B	10^4	$6{,}52 \cdot 10^{-13}$	$1{,}05 \cdot 10^{-12}$	$1{,}04 \cdot 10^{-12}$	$1{,}08 \cdot 10^{-12}$	$1{,}17 \cdot 10^{-11}$
B	10^5	$6{,}65 \cdot 10^{-12}$	$1{,}05 \cdot 10^{-12}$	$1{,}04 \cdot 10^{-12}$	$1{,}38 \cdot 10^{-13}$	$-1{,}02 \cdot 10^{-10}$
	N	v_T	t_N	t_T	x_N	$\gamma^3 a_N$
C	10^2	9,99998435551	1000,0007782124	1000,01667839008	374000,28333340	9,99999992287606
C	10^3	9,99998435367	1000,0007782124	1000,01667839007	374000,28333240	9,99999991716842
C	10^4	9,99998435376	1000,0007782124	1000,01667839006	374000,28333254	9,99999991716842
C	10^5	9,99998435645	1000,0007782124	1000,01667839007	374000,28333389	9,99999992496930
	N	δv_T	δt_N	δt_T	δx_N	$\delta\gamma^3 a_N$
C	10^2	$2{,}12 \cdot 10^{-10}$	$1{,}04 \cdot 10^{-12}$	$1{,}17 \cdot 10^{-12}$	$-5{,}74 \cdot 10^{-10}$	$7{,}71 \cdot 10^{-9}$
C	10^3	$2{,}17 \cdot 10^{-10}$	$1{,}05 \cdot 10^{-12}$	$1{,}18 \cdot 10^{-12}$	$1{,}10 \cdot 10^{-10}$	$8{,}28 \cdot 10^{-9}$
C	10^4	$2{,}16 \cdot 10^{-10}$	$1{,}05 \cdot 10^{-12}$	$1{,}19 \cdot 10^{-12}$	$1{,}10 \cdot 10^{-10}$	$8{,}28 \cdot 10^{-9}$
C	10^5	$2{,}09 \cdot 10^{-10}$	$1{,}05 \cdot 10^{-12}$	$1{,}18 \cdot 10^{-12}$	$1{,}07 \cdot 10^{-10}$	$7{,}50 \cdot 10^{-9}$

c)	St.	v_A	t_A	t_T	x_A	a_S
A	(1)	149896,229000000	17308525,6327320		1390379100217,26	10
A	(2)	149903,728874913	17309680,3428997		1390552191247,12	10
A	Diff.	7,499874913	1154,7101678	1000,01667838490	173091029,86	
	N	v_T	t_N	t_T	x_N	$\gamma^3 a_N$
B	10^2	7,499874913	1154,7101678	1000,01667839021	173091029,86	10,0000016684019
B	10^3	7,499874922	1154,7101678	1000,01667839022	173091029,86	10,0000002015203
B	10^4	7,499874803	1154,7101678	1000,01667838996	173091029,86	9,99999992987018
B	10^5	7,499875378	1154,7101678	1000,01667839124	173091029,86	10,0000022582798
	N	δv_T	δt_N	δt_T	δx_N	$\delta \gamma^3 a_N$
B	10^2	$-4{,}22 \cdot 10^{-15}$	$-4{,}29 \cdot 10^{-12}$	$-5{,}31 \cdot 10^{-12}$	$-7{,}33 \cdot 10^{-12}$	$-1{,}67 \cdot 10^{-7}$
B	10^3	$-6{,}27 \cdot 10^{-14}$	$-4{,}29 \cdot 10^{-12}$	$-5{,}32 \cdot 10^{-12}$	$-7{,}38 \cdot 10^{-12}$	$-2{,}02 \cdot 10^{-8}$
B	10^4	$7{,}30 \cdot 10^{-13}$	$-4{,}16 \cdot 10^{-12}$	$-5{,}06 \cdot 10^{-12}$	$-6{,}85 \cdot 10^{-12}$	$7{,}01 \cdot 10^{-9}$
B	10^5	$-3{,}10 \cdot 10^{-12}$	$-4{,}81 \cdot 10^{-12}$	$-6{,}34 \cdot 10^{-12}$	$-9{,}41 \cdot 10^{-12}$	$-2{,}26 \cdot 10^{-7}$
	N	v_T	t_N	t_T	x_N	$\gamma^3 a_N$
C	10^2	7,499850523	1154,7101677	1000,01667833597	173091029,84	9,99996914760263
C	10^3	7,499849967	1154,7101677	1000,01667833473	173091029,84	9,99996690196617
C	10^4	7,499849989	1154,7101677	1000,01667833478	173091029,84	9,99996743452330
C	10^5	7,499850786	1154,7101677	1000,01667833657	173091029,84	9,99996654696513
	N	δv_T	δt_N	δt_T	δx_N	$\delta \gamma^3 a_N$
C	10^2	$3{,}25 \cdot 10^{-6}$	$2{,}28 \cdot 10^{-11}$	$4{,}89 \cdot 10^{-11}$	$1{,}01 \cdot 10^{-10}$	$3{,}09 \cdot 10^{-6}$
C	10^3	$3{,}33 \cdot 10^{-6}$	$2{,}35 \cdot 10^{-11}$	$5{,}02 \cdot 10^{-11}$	$1{,}04 \cdot 10^{-10}$	$3{,}31 \cdot 10^{-6}$
C	10^4	$3{,}32 \cdot 10^{-6}$	$2{,}34 \cdot 10^{-11}$	$5{,}01 \cdot 10^{-11}$	$1{,}04 \cdot 10^{-10}$	$3{,}26 \cdot 10^{-6}$
C	10^5	$3{,}22 \cdot 10^{-6}$	$2{,}25 \cdot 10^{-11}$	$4{,}83 \cdot 10^{-11}$	$9{,}99 \cdot 10^{-11}$	$3{,}35 \cdot 10^{-6}$

Tab. B.4: Calculated values for v_T, t_A, t_T, x_N and $\gamma^3 a_N$ using different procedures
A: Analytically acc. to calculation using Eq. (6.60) to (6.74)
B: Numerically acc. to VBA-Code from Fig. B.2
C: Numerically acc. to VBA-Code from Fig. C.2, Type “B1”
$a_S = 10\mathrm{m/s}^2$. $\Delta t_S = 1.000\mathrm{s}$. Results in km and s.
a) $v_0 = 0$, results for x_A calculated using Eq. $(\mathrm{B}.15)$, $x_A(3)$ and $x_A(2)$
b) $v_0 = 369\ \mathrm{km/s}$, results for x_A calculated using Eq. $(\mathrm{B}.15)$, $x_A(3)$
c) $v_0 = 0.5c$, results for x_A calculated using Eq. (6.74)