Annex C: Relativistic rocket equation

For numerical calculation, the equations derived in chapter 6.4.2

$$p_K + p_K' = (m_{K-1} - \Delta m_{K-1}) v_K \gamma_k + \Delta m_{K-1} v_K' \gamma_K' = m_{K-1} v_{K-1} \gamma_{K-1} \tag{6.84}$$

and

$$v_K' = \frac{v_K + v_0'}{1 + \frac{v_K v_0'}{c^2}} \tag{6.85}$$

are used. For the determination of v_K, as already presented in other chapters, the method of bisection was chosen (see also the comparison of different numerical calculation methods in annex D). The basis is the momentum calculation of the total system, consisting of the momentum of the rocket p_K as well as that of the propulsion gas p_K' with mass Δm_{K-1} moving in the opposite direction, and the determination of the corresponding rocket velocity v_K. Due to the law of conservation of momentum, the total value must be constant before and after the velocity increase of the rocket including the consideration of mass ejection.

First, suitable starting values for $(v_+)_0$ and $(v_-)_0$ must be defined; it makes sense that these values should be far apart since it must be ensured that the final result v_K lies within these limits. Thereupon a new index L is defined. Now the mean value

$$(v_K)_{L=1} = \frac{(v_+)_0 + (v_-)_0}{2} \tag{C.01}$$

is formed and for the velocity calculated here the momentum is determined according to equation (6.84). Then the following definitions must be used:

$$(p_K + p_K')_{L=1} > m_{K-1} v_{K-1} \gamma_{K-1} \quad \Rightarrow \quad \begin{cases} (v_+)_1 = (v)_1 \\ (v_-)_1 = (v_-)_0 \end{cases} \tag{C.02}$$

$$(p_K + p_K')_{L=1} \leq m_{K-1} v_{K-1} \gamma_{K-1} \quad \Rightarrow \quad \begin{cases} (v_+)_1 = (v_+)_0 \\ (v_-)_1 = (v)_1 \end{cases} \tag{C.03}$$

This calculation is repeated with increasing index L until the results for v_+ and v_- are equal. Thus, the velocity of the rocket, whose mass is now reduced by Δm_{K-1}, is determined for this partial step. Subsequently, the next step is performed for $K = 2$ and so on.

The time that subjectively elapses inside the rocket between the emission of 2 signals is by definition Δt_0. For an external observer the view is different, and the value must be supplemented according to

$$\Delta t_K = \Delta t_0 \gamma_K \tag{C.04}$$

and the distance covered is

$$\Delta x_K = \Delta t_K v_K \tag{C.05}$$

After adding all N single values, the final result is

$$t_N = \sum_{K=1}^{N} \Delta t_0 \gamma_K \tag{C.06}$$

$$x_N = \sum_{K=1}^{N} \Delta t_0 v_K \tag{C.07}$$

At any arbitrary time t_K, a signal is sent back from the accelerated system S to the observers A and B. Observer A has moved with the same velocity as the rocket at the beginning of the experiment and continues its path without acceleration, while B measures a velocity v_0 with respect to A. From B's point of view, A is either moving in direction to S or in the opposite way during signal propagation. In case of $v_0 \neq 0$ the values for acceleration a_K and velocity v_0 can each be positive or negative, so different arrangements must be made for performing the calculations. This was already done in a similar form in Chap. 6.4.1 with the equations Eq. (6.60) to (6.74), but there the acceleration of the rocket was kept constant over the entire course of the experiment. In contrast, here the exit direction of the propulsion gas v' represents the effect of precondition. If $v' > 0$ then the acceleration is negative, at $v' < 0$ it is positive. The equations used in section 6.4.1 must therefore be modified with respect to the boundary conditions and read as follows here

$$v' < 0 \quad (a_S > 0): \qquad t_{K,R} = \frac{x_K - v_0 t_K}{c\left(1 + \frac{v_0}{c}\right)} \tag{C.08}$$

$$v' > 0 \quad (a_S < 0): \qquad t_{K,R} = \frac{|x_K - v_0 t_K|}{c\left(1 - \frac{v_0}{c}\right)} \tag{C.09}$$

Thus, for the limiting case applies

$$v_0 = 0: \qquad t_{K,R} = \frac{|x_K|}{c} \tag{C.10}$$

Generally follows

$$t_T(K) = \frac{t_K + t_{K,R}}{\gamma(v_0)} \tag{C.11}$$

In addition, for the determined final velocity v_N, the following is specified for different system velocities v_0 for better comparability of the calculations

$$v_T = v_N - v_0 \tag{C.12}$$

C.2 Specific specifications for the calculation

When defining the boundary conditions for the calculation, the ratio of outflowing mass per time interval is relevant. In order to simplify the representation, here the outflow mass of the rocket is normalized to 1 and the standard time interval, valid subjectively inside the rocket, is set to $\Delta t_0 = 1\text{s}$. From this it follows, for example, for the case when 0.5% of the rocket mass flows out per second for propulsion, that when 50% of the mass is ejected, a total of 100 iteration steps have been performed. This case can be defined for the calculations using the form

$$\Delta m_0 = \Delta t_0 \cdot 0{,}5\% \qquad\qquad \mathrm{N}/\Delta t_0 = 100 \tag{C.13}$$

If, for example, the number of iteration steps is then increased by a factor of 10, the time interval and the outflowing supporting mass are reduced by the same factor for the subsequent calculations.

The initial values of the velocities $(v_+)_{L=0}$ and $(v_-)_{L=0}$ for the bisection should be chosen far apart, but the mean value must be non-zero, otherwise there will be disturbances during the calculation; $(v_+)_{L=0} = 0{,}9c$ and $(v_-)_{L=0} = -0{,}8c$ were chosen in this case.

C.3 Flowchart and VBA program code of the process

A flow chart (Fig. C.1) shows how the running program is designed. It is a process with two nested iteration loops; the running indices have been labeled K and L. The representation of the VBA program code (Fig. C.2) follows the flowchart representation. The VBA codes used for the formula characters are shown in the following listing.

Symbol	VBA-Code	Symbol	VBA-Code	Symbol	VBA-Code
v_0	v0	v_0'	v0g	Δt_0	dt0
$(v_+)_L$	vmax	$(v_-)_L$	vmin	$(v_+)_{L=0}$	vmax0
$(v_-)_{L=0}$	vmin0	t_K	tK	t_{K-1}	tKm1
t_T	tT	x_K	xK	$t_{K,R}$	tKR
$(v_K)_L$	vL	v_{K-1}	vKm1	v_K	vK
m_K	mK	Δm_0	dm0	Δm_K	dmK
p_{K-1}	pKm1	$(p_K + p_K')_L$	pL	v_K'	vKg
$(v_K)_{L-1}$	vLm1	$(v_K')_L$	vLg	v_T	vT
a_K	aK	γ^3	Ga3	$\gamma^3 a_K$	aKGa3

Tab. C.1: Formula symbols and referring VBA-Codes

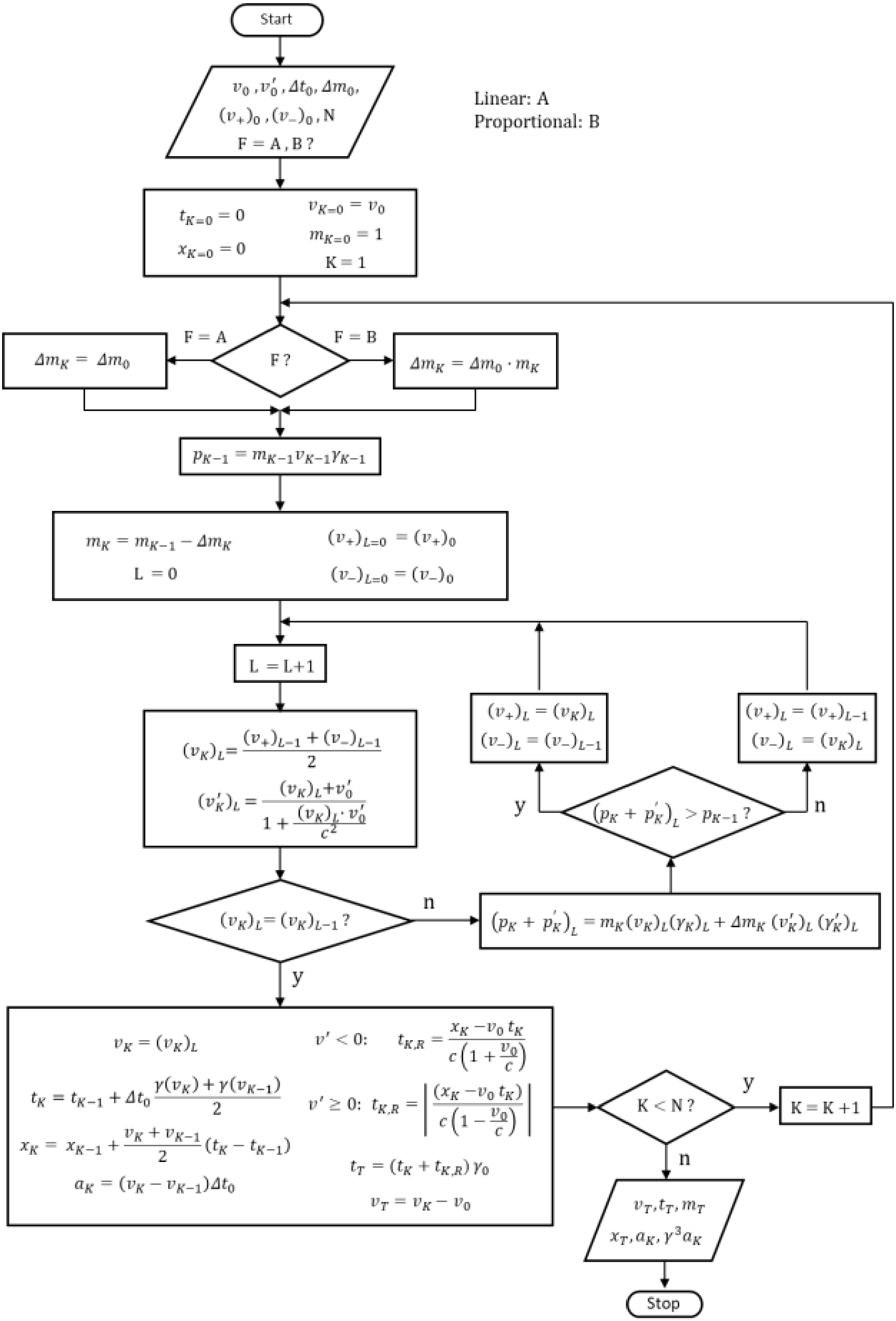

Fig. C.1: Flowchart of the calculation process

```
Sub C()
Dim v0, v0g, tS, dtS, dm0, mF, vmax0, vmin0, vmax, vmin, mK, tK As Double
Dim tKm1, tKR, tT, xK, vK, vKm1, dmK, pKm1, pL, vL, vLm1 As Double
Dim N, K, L, vKg, vT, vLg, c, aK, Ga3, aKGa3 As Double
Dim F, A1, A2, B1, B2 As String
'Input
    F = "B1"                'Define A1, A2, B1 or B2
                            'A: Linear mass reduction, B: Prop. mass reduction
                            '1: Def. number of iteration steps, 2: Def. end mass
    v0 = 0                  'Initial velocity in km/s
    v0g = -4                'Initial velocity gas in km/s
    dm0 = 0.25 / 100        'Initial output mass in %/s
'Specific input Def. 1
    tS = 400                'Time until a signal is emitted
    N = 1000                'Number of iteration steps
'Specific input Def. 2
    dtS = 1                 'Iteration time in s
    mF = 10 / 100           'Mass at end of trial in %
'Start Calculation
    If F = "A1" Or F = "A2" Or F = "B1" Or F = "B2" Then
        GoTo Calc:
        Else
        Debug.Print "Input error: Chose A1, A2, B1, or B2"
        GoTo Out1:
        End If
Calc:
    If F = "A1" Or F = "B1" Then
        dtS = tS / N
        End If
    mK = 1                   'Initial value mass
    vmax0 = 0.9              'Initial value max. for calculation (in rel. to c)
    vmin0 = -0.8             'Initial value min. for calculation (in rel. to c)
    c = 299792.458           'speed of light in km/s
    tK = 0
    xK = 0
    vK = v0 / c
    v0g = v0g / c
Mainloop:
        K = K + 1
        If F = "A1" Or F = "A2" Then
            dmK = dm0 * dtS
            Else
            dmK = dm0 * dtS * mK
            End If
        pKm1 = mK * vK / (1 - vK ^ 2) ^ 0.5     'Momentum rocket for K - 1
        mK = mK - dmK                           'Rest rocket mass for K
        If mK <= 0 Then
            K = K - 1
            mK = mK + dmK
            Debug.Print "Rocket mass zero"
            GoTo Out2:
            End If
        vmax = vmax0
        vmin = vmin0                            'Req.: vmin0 unequal -vmax0
        L = 0
Do
        L = L + 1
        vLm1 = vL
        vL = (vmax + vmin) / 2
        vLg = (vL + v0g) / (1 + vL * v0g)
        pL = mK * vL / (1 - vL ^ 2) ^ 0.5 + dmK * vLg / (1 - vLg ^ 2) ^ 0.5
```

```
If pL > pKm1 Then
            vmax = vL
            Else: vmin = vL
            End If
Loop Until vLm1 = vL
    vKm1 = vK
    vK = vL
    vKg = vLg
    tKm1 = tK
    tK = tK + dtS * (1 / (1 - vK ^ 2) ^ 0.5 + 1 / (1 - vKm1 ^ 2) ^ 0.5) / 2
    xK = xK + (vK + vKm1) / 2 * (tK - tKm1) * c
    aK = (vK - vKm1) / (dtS / (1 - ((vK + vKm1) / 2) ^ 2) ^ 0.5) * c * 1000
    Ga3 = (1 / (1 - ((vK + vKm1) / 2) ^ 2)) ^ 1.5
    If v0g > 0 Then
            tKR = Abs(xK - v0 * tK) / c / (1 - v0 / c)
            Else: tKR = (xK - v0 * tK) / c / (1 + v0 / c)
            End If
        tT = (tK + tKR) * (1 - (v0 / c) ^ 2) ^ 0.5
        vT = (vK * c - v0)
        aKGa3 = aK * Ga3
    If F = "A1" Or F = "B1" Then
        If K < N Then
            GoTo Mainloop:
            End If
        End If
    If F = "A2" Or F = "B2" Then
        If mK > mF Then
            GoTo Mainloop:
            End If
        End If
Out2:
Results in view of an observer moving with v0 at beginning of trial
Debug.Print "vT =", vT 'velocity when signal is emitted in km/s
Debug.Print "tN =", tK 'Total time until a signal is emitted in s
Debug.Print "tT =", tT 'Total time for transmission of signal in s
Debug.Print "mN =", mK 'Rocket mass at emission in relation to 1
Debug.Print "xN =", xK 'Distance covered at emission of signal in km
Debug.Print "aN =", aK 'Acceleration in m/s²
Debug.Print "aNGa3 =", aKGa3     'Acceleration * Gamma ^ 3 in m/s²
Out1:
End Sub
```

Fig. C2: VBA Program-Code for the calculation process presented in Fig. C1

In the following tables Tab. C.2, C.3 and C.4 supplementary calculations are shown according to Tab. 6.4 from Chap. 6.4.2. Instead of using the program "A1", the variant "A2" could also have been selected. In this case, the desired final value of the rocket mass and the iteration time are specified, and the number of iteration steps results from the calculation. Example from Tab C2: Parameters "A1" $t_S = 100s$, $N = 1000$ correspond to "A2" $m_F = 50\%$ and $\Delta t_S = 0{,}1s$. The calculated value for K is then $N = 1001$. The results are very similar, but not completely identical. Since in this case the influence of the number of iteration steps was in the foreground, calculation "A1" was chosen.

The values of t_T are of particular interest for comparisons, since they would be accessible for experimental testing due to the simple use of precision clocks. The results of t_T obtained here are shown separately in Tab. 6.6, Tab. 6.7 and Fig. 6.4, but do not show any systematic differences, so that the principle of relativity is also observed here as in all other cases.

N	v_T	t_T	m_N	x_N
10	2,67508561278727	100,000397329364	0,500000000000000	119,116010675216
10^2	2,76261372200990	100,000408141269	0,500000000000000	122,357320955608
10^3	2,77158897232187	100,000409292747	0,500000000000055	122,702523336750
10^4	2,77248872482278	100,000409408634	0,500000000000055	122,737265091767
10^5	2,77257872237194	100,000409420246	0,499999999996724	122,740741494222
10^6	2,77258772224753	100,000409422400	0,500000000041133	122,741089155569
10^7	2,77258862465211	100,000409440862	0,499999999708066	122,741124020357
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	8,9931	$1,4064 \cdot 10^{-3}$		$3,4698 \cdot 10^{2}$
10^2	$8,7528 \cdot 10^{-2}$	$1,0812 \cdot 10^{-5}$	0	3,2413
10^3	$8,9753 \cdot 10^{-3}$	$1,1515 \cdot 10^{-6}$	$5,4956 \cdot 10^{-14}$	$3,4520 \cdot 10^{-1}$
10^4	$8,9975 \cdot 10^{-4}$	$1,1589 \cdot 10^{-7}$	0	$3,4742 \cdot 10^{-2}$
10^5	$8,9998 \cdot 10^{-5}$	$1,1612 \cdot 10^{-8}$	$-3,3309 \cdot 10^{-12}$	$3,4764 \cdot 10^{-3}$
10^6	$8,9998 \cdot 10^{-6}$	$2,1540 \cdot 10^{-9}$	$4,4409 \cdot 10^{-11}$	$3,4766 \cdot 10^{-4}$
10^7	$9,0240 \cdot 10^{-7}$	$1,8462 \cdot 10^{-8}$	$-3,3307 \cdot 10^{-10}$	$3,4865 \cdot 10^{-3}$
10^8	$8,9931 \cdot 10^{-8}$	$1,4069 \cdot 10^{-11}$		$3,4698 \cdot 10^{-6}$
10^9	$8,9931 \cdot 10^{-9}$	$1,4069 \cdot 10^{-12}$		$3,4698 \cdot 10^{-7}$
10^{10}	$8,9931 \cdot 10^{-10}$	$1,4211 \cdot 10^{-13}$		$3,4698 \cdot 10^{-8}$
10^{11}	$8,9931 \cdot 10^{-11}$	0		$3,4698 \cdot 10^{-9}$
10^{12}	$8,9933 \cdot 10^{-12}$	0		$3,4699 \cdot 10^{-10}$
10^{13}	$8,9928 \cdot 10^{-13}$	0		$3,4703 \cdot 10^{-11}$
10^{14}	$9,0150 \cdot 10^{-14}$	0		$3,4674 \cdot 10^{-12}$
10^{15}	$8,8818 \cdot 10^{-15}$	0		$3,4106 \cdot 10^{-13}$
10^{16}	0	0		0
	v_T	t_T		x_N
10^7	2,77258862155768	100,000409422541		122,741123853607
10^8	2,77258871148869	100,000409422555		122,741127323411
10^9	2,77258872048179	100,000409422556		122,741127670391
10^{10}	2,77258872138110	100,000409422556		122,741127705089
10^{11}	2,77258872147103	100,000409422556		122,741127708559
10^{12}	2,77258872148003	100,000409422556		122,741127708906
10^{13}	2,77258872148093	100,000409422556		122,741127708941
10^{14}	2,77258872148102	100,000409422556		122,741127708944
10^{15}	2,77258872148102	100,000409422556		122,741127708945
10^{16}	2,77258872148102	100,000409422556		122,741127708945

a)

N	v_T	t_T	m_N	x_N
10	2,67508151022224	100,000397329361	0,500000000000000	37019,1440520908
10^2	2,76260948280941	100,000408141266	0,500000000000000	37022,3853647753
10^3	2,77158471909860	100,000409292744	0,500000000000055	37022,7305674122
10^4	2,77248447045912	100,000409408630	0,500000000000055	37022,7653092072
10^5	2,77257447227356	100,000409420227	0,499999999996724	37022,7687858303
10^6	2,77258347644647	100,000409421377	0,500000000041133	37022,7691339111
10^7	2,77258481950150	100,000409421716	0,499999999708066	37022,7691906506
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	8,9943	$1,1550 \cdot 10^{-3}$		$3,4709 \cdot 10^{2}$
10^2	$8,7528 \cdot 10^{-2}$	$1,0812 \cdot 10^{-5}$	0	3,2413
10^3	$8,9752 \cdot 10^{-3}$	$1,1515 \cdot 10^{-6}$	$5,4956 \cdot 10^{-14}$	$3,4520 \cdot 10^{-1}$
10^4	$8,9975 \cdot 10^{-4}$	$1,1589 \cdot 10^{-7}$	0	$3,4742 \cdot 10^{-2}$
10^5	$9,0002 \cdot 10^{-5}$	$1,1597 \cdot 10^{-8}$	$-3,3309 \cdot 10^{-12}$	$3,4766 \cdot 10^{-3}$
10^6	$9,0042 \cdot 10^{-6}$	$1,1500 \cdot 10^{-9}$	$4,4409 \cdot 10^{-11}$	$3,4808 \cdot 10^{-4}$
10^7	$1,3431 \cdot 10^{-6}$	$3,3900 \cdot 10^{-10}$	$-3,3307 \cdot 10^{-10}$	$5,6740 \cdot 10^{-5}$
10^8	$8,9943 \cdot 10^{-8}$	$1,1553 \cdot 10^{-11}$		$3,4709 \cdot 10^{-6}$
10^9	$8,9943 \cdot 10^{-9}$	$1,1511 \cdot 10^{-12}$		$3,4709 \cdot 10^{-7}$
10^{10}	$8,9943 \cdot 10^{-10}$	$1,1369 \cdot 10^{-13}$		$3,4706 \cdot 10^{-8}$
10^{11}	$8,9943 \cdot 10^{-11}$	0		$3,4706 \cdot 10^{-9}$
10^{12}	$8,9941 \cdot 10^{-12}$	0		$3,4925 \cdot 10^{-10}$
10^{13}	$8,9928 \cdot 10^{-13}$	0		0
10^{14}	$9,0150 \cdot 10^{-14}$	0		0
10^{15}	$8,8818 \cdot 10^{-15}$	0		0
10^{16}	0	0		0
	v_T	t_T		x_N
10^7	2,77258437587408	100,000409421493		37022,7691686202
10^8	2,77258446581684	100,000409421504		37022,7691720911
10^9	2,77258447481111	100,000409421505		37022,7691724381
10^{10}	2,77258447571054	100,000409421505		37022,7691724729
10^{11}	2,77258447580048	100,000409421505		37022,7691724764
10^{12}	2,77258447580948	100,000409421505		37022,7691724767
10^{13}	2,77258447581038	100,000409421505		37022,7691724768
10^{14}	2,77258447581047	100,000409421505		37022,7691724768
10^{15}	2,77258447581048	100,000409421505		37022,7691724768
10^{16}	2,77258447581048	100,000409421505		37022,7691724768

b)

N	v_T	t_T	m_N	x_N
10	2,67496628539311	100,000397329348	0,500000000000000	200123,569407464
10^2	2,76249047724150	100,000408141251	0,500000000000000	200126,810789598
10^3	2,77146532579354	100,000409292729	0,500000000000055	200127,155999630
10^4	2,77236503944528	100,000409408616	0,500000000000055	200127,190742224
10^5	2,77245505647284	100,000409420216	0,499999999996724	200127,194219809
10^6	2,77246407688722	100,000409421380	0,500000000041133	200127,194569079
10^7	2,77246734040796	100,000409422016	0,499999999708066	200127,194715347
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	8,9985	$1,1586 \cdot 10^{-3}$		$3,4742 \cdot 10^{2}$
10^2	$8,7524 \cdot 10^{-2}$	$1,0812 \cdot 10^{-5}$	0	3,2414
10^3	$8,9748 \cdot 10^{-3}$	$1,1515 \cdot 10^{-6}$	$5,4956 \cdot 10^{-14}$	$3,4521 \cdot 10^{-1}$
10^4	$8,9971 \cdot 10^{-4}$	$1,1589 \cdot 10^{-7}$	0	$3,4743 \cdot 10^{-2}$
10^5	$9,0017 \cdot 10^{-5}$	$1,1600 \cdot 10^{-8}$	$-3,3309 \cdot 10^{-12}$	$3,4776 \cdot 10^{-3}$
10^6	$9,0204 \cdot 10^{-6}$	$1,1640 \cdot 10^{-9}$	$4,4409 \cdot 10^{-11}$	$3,4927 \cdot 10^{-4}$
10^7	$3,2635 \cdot 10^{-6}$	$6,3601 \cdot 10^{-10}$	$-3,3307 \cdot 10^{-10}$	$1,4627 \cdot 10^{-4}$
10^8	$8,9985 \cdot 10^{-8}$	$1,1582 \cdot 10^{-11}$		$3,4742 \cdot 10^{-6}$
10^9	$8,9985 \cdot 10^{-9}$	$1,1653 \cdot 10^{-12}$		$3,4741 \cdot 10^{-7}$
10^{10}	$8,9985 \cdot 10^{-10}$	$1,1369 \cdot 10^{-13}$		$3,4750 \cdot 10^{-8}$
10^{11}	$8,9985 \cdot 10^{-11}$	0		$3,4634 \cdot 10^{-9}$
10^{12}	$8,9986 \cdot 10^{-12}$	0		$3,4925 \cdot 10^{-10}$
10^{13}	$8,9972 \cdot 10^{-13}$	0		0
10^{14}	$9,0150 \cdot 10^{-14}$	0		0
10^{15}	$8,8818 \cdot 10^{-15}$	0		0
10^{16}	0	0		0
	v_T	t_T		x_N
10^7	2,77246497673978	100,000409421496		200127,194603821
10^8	2,77246506672503	100,000409421507		200127,194607295
10^9	2,77246507572356	100,000409421509		200127,194607642
10^{10}	2,77246507662341	100,000409421509		200127,194607677
10^{11}	2,77246507671339	100,000409421509		200127,194607680
10^{12}	2,77246507672239	100,000409421509		200127,194607681
10^{13}	2,77246507672329	100,000409421509		200127,194607681
10^{14}	2,77246507672338	100,000409421509		200127,194607681
10^{15}	2,77246507672339	100,000409421509		200127,194607681
10^{16}	2,77246507672339	100,000409421509		200127,194607681

c)

N	v_T	t_T	m_N	x_N
10	2,67210782993243	100,000397329281	0,500000000000000	1000675,97202456
10^2	2,75953844190371	100,000408141179	0,500000000000000	1000679,21513818
10^3	2,76850369436397	100,000409292656	0,500000000000055	1000679,56053261
10^4	2,76940245165497	100,000409408544	0,500000000000055	1000679,59529404
10^5	2,76949246576442	100,000409420158	0,499999999996724	1000679,59877786
10^6	2,76950155149825	100,000409421342	0,500000000041133	1000679,59913297
10^7	2,76951415107033	100,000409423363	0,499999999708066	1000679,59971782
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	9,0100	$1,1639 \cdot 10^{-3}$		$3,4913 \cdot 10^{2}$
10^2	$8,7431 \cdot 10^{-2}$	$1,0812 \cdot 10^{-5}$	0	3,2431
10^3	$8,9653 \cdot 10^{-3}$	$1,1515 \cdot 10^{-6}$	$5,4956E \cdot 10^{-14}$	$3,4539 \cdot 10^{-1}$
10^4	$8,9876 \cdot 10^{-4}$	$1,1589 \cdot 10^{-7}$	0	$3,4761 \cdot 10^{-2}$
10^5	$9,0014 \cdot 10^{-5}$	$1,1614 \cdot 10^{-8}$	$-3,3309 \cdot 10^{-12}$	$3,4838 \cdot 10^{-3}$
10^6	$9,0857 \cdot 10^{-6}$	$1,1840 \cdot 10^{-9}$	$4,4409 \cdot 10^{-11}$	$3,5511 \cdot 10^{-4}$
10^7	$1,2600 \cdot 10^{-5}$	$2,0210 \cdot 10^{-9}$	$-3,3307 \cdot 10^{-10}$	$5,8485 \cdot 10^{-4}$
10^8	$9,0100 \cdot 10^{-8}$	$1,1639 \cdot 10^{-11}$		$3,4913 \cdot 10^{-6}$
10^9	$9,0100 \cdot 10^{-9}$	$1,1653 \cdot 10^{-12}$		$3,4913 \cdot 10^{-7}$
10^{10}	$9,0100 \cdot 10^{-10}$	$1,1369 \cdot 10^{-13}$		$3,4925 \cdot 10^{-8}$
10^{11}	$9,0100 \cdot 10^{-11}$	0		$3,4925 \cdot 10^{-9}$
10^{12}	$9,0101 \cdot 10^{-12}$	0		0
10^{13}	$9,0106 \cdot 10^{-13}$	0		0
10^{14}	$9,0150 \cdot 10^{-14}$	0		0
10^{15}	$8,8818 \cdot 10^{-15}$	0		0
10^{16}	0	0		0
	v_T	t_T		x_N
10^7	2,76950245249750	100,000409421458		1000679,59916788
10^8	2,76950254259743	100,000409421470		1000679,59917137
10^9	2,76950255160742	100,000409421471		1000679,59917172
10^{10}	2,76950255250842	100,000409421471		1000679,59917176
10^{11}	2,76950255259852	100,000409421471		1000679,59917176
10^{12}	2,76950255260753	100,000409421471		1000679,59917176
10^{13}	2,76950255260843	100,000409421471		1000679,59917176
10^{14}	2,76950255260852	100,000409421471		1000679,59917176
10^{15}	2,76950255260853	100,000409421471		1000679,59917176
10^{16}	2,76950255260853	100,000409421471		1000679,59917176

d)

Tab: C.2: Calculation of relativistic rocket velocity according to program
Type: “A1”, $v_0' = -4\ \mathrm{km/s}$, $\Delta m_0 = 0.5\%$, $t_0 = 100s$
a) $v_0 = 0$, b) $v_0 = 369\ \mathrm{km/s}$, c) $v_0 = 2000\ \mathrm{km/s}$, d) $v_0 = 10000\ \mathrm{km/s}$

N	v_T	t_T	m_N	x_N
10	7,84085772014717	1000,00912888441	0,100000000000000	2736,750700612000
10^{2}	9,05101119073233	1000,00983714046	0,099999999999999	2949,077033897790
10^{3}	9,19416709875657	1000,00991970498	0,099999999999989	2973,828794295530
10^{4}	9,20872063626419	1000,00992810957	0,099999999999856	2976,348382091860
10^{5}	9,21017837171215	1000,00992895150	0,099999999998368	2976,600796650620
10^{6}	9,21032416890996	1000,00992904517	0,100000000009585	2976,626042686920
10^{7}	9,21033875458106	1000,00992885138	0,099999999922464	2976,628567836020
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	$1,4507 \cdot 10^{2}$	$8,6118 \cdot 10^{-2}$		$2,5109 \cdot 10^{4}$
10^{2}	1,2102	$7,0826 \cdot 10^{-4}$	$-8,0491 \cdot 10^{-16}$	$2,1233 \cdot 10^{2}$
10^{3}	$1,4316 \cdot 10^{-1}$	$8,2565 \cdot 10^{-5}$	$-1,0700 \cdot 10^{-14}$	$2,4752 \cdot 10^{1}$
10^{4}	$1,4554 \cdot 10^{-2}$	$8,4046 \cdot 10^{-6}$	$-1,3300 \cdot 10^{-13}$	2,5196
10^{5}	$1,4577 \cdot 10^{-3}$	$8,4193 \cdot 10^{-7}$	$-1,4880 \cdot 10^{-12}$	$2,5241 \cdot 10^{-1}$
10^{6}	$1,4580 \cdot 10^{-4}$	$9,3670 \cdot 10^{-8}$	$1,1217 \cdot 10^{-11}$	$2,5246 \cdot 10^{-2}$
10^{7}	$1,4586 \cdot 10^{-5}$	$-1,9379 \cdot 10^{-7}$	$-8,7121 \cdot 10^{-11}$	$2,5251 \cdot 10^{-3}$
10^{8}	$1,4507 \cdot 10^{-6}$	$8,6118 \cdot 10^{-10}$		$2,5109 \cdot 10^{-4}$
10^{9}	$1,4507 \cdot 10^{-7}$	$8,6175 \cdot 10^{-11}$		$2,5109 \cdot 10^{-5}$
10^{10}	$1,4507 \cdot 10^{-8}$	$8,6402 \cdot 10^{-12}$		$2,5109 \cdot 10^{-6}$
10^{11}	$1,4507 \cdot 10^{-9}$	$9,0949 \cdot 10^{-13}$		$2,5109 \cdot 10^{-7}$
10^{12}	$1,4507 \cdot 10^{-10}$	0		$2,5109 \cdot 10^{-8}$
10^{13}	$1,4506 \cdot 10^{-11}$	0		$2,5107 \cdot 10^{-9}$
10^{14}	$1,4513 \cdot 10^{-12}$	0		$2,5102 \cdot 10^{-10}$
10^{15}	$1,4566 \cdot 10^{-13}$	0		$2,5011 \cdot 10^{-11}$
10^{16}	$1,4211 \cdot 10^{-14}$	0		0
	v_T	t_T		x_N
10^{7}	9,21033867546060	1000,00992905378		2976,628553565180
10^{8}	9,21034012611567	1000,00992905464		2976,628804653010
10^{9}	9,21034027118117	1000,00992905473		2976,628829761790
10^{10}	9,21034028568772	1000,00992905474		2976,628832272670
10^{11}	9,21034028713838	1000,00992905474		2976,628832523760
10^{12}	9,21034028728345	1000,00992905474		2976,628832548870
10^{13}	9,21034028729795	1000,00992905474		2976,628832551380
10^{14}	9,21034028729940	1000,00992905474		2976,628832551630
10^{15}	9,21034028729955	1000,00992905474	*a)*	2976,628832551650
10^{16}	9,21034028729956	1000,00992905474		2976,628832551660

N	v_T	t_T	m_N	x_N
10	7,84084554146182	1000,00912888434	0,100000000000000	371737,0322956670
10^{2}	9,05099707036669	1000,00983714038	0,099999999999999	371949,3587899240
10^{3}	9,19415274738003	1000,00991970490	0,099999999999989	371974,1105690590
10^{4}	9,20870626170580	1000,00992810949	0,099999999999856	371976,6301589470
10^{5}	9,21016399816938	1000,00992895148	0,099999999998368	371976,8825756170
10^{6}	9,21030981978799	1000,00992903570	0,100000000009585	371976,9078311300
10^{7}	9,21032474295856	1000,00992904906	0,099999999922464	371976,9105254130
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	$1,4507 \cdot 10^{2}$	$8,3757 \cdot 10^{-2}$		$2,5111 \cdot 10^{4}$
10^{2}	1,2102	$7,0826 \cdot 10^{-4}$	$-8,0491 \cdot 10^{-16}$	$2,1233 \cdot 10^{2}$
10^{3}	$1,4316 \cdot 10^{-1}$	$8,2565 \cdot 10^{-5}$	$-1,0700 \cdot 10^{-14}$	$2,4752 \cdot 10^{1}$
10^{4}	$1,4554 \cdot 10^{-2}$	$8,4046 \cdot 10^{-6}$	$-1,3300 \cdot 10^{-13}$	2,5196
10^{5}	$1,4577 \cdot 10^{-3}$	$8,4199 \cdot 10^{-7}$	$-1,4880 \cdot 10^{-12}$	$2,5242 \cdot 10^{-1}$
10^{6}	$1,4582 \cdot 10^{-4}$	$8,4220 \cdot 10^{-8}$	$1,1217 \cdot 10^{-11}$	$2,5256 \cdot 10^{-2}$
10^{7}	$1,4923 \cdot 10^{-5}$	$1,3360 \cdot 10^{-8}$	$-8,7121 \cdot 10^{-11}$	$2,6943 \cdot 10^{-3}$
10^{8}	$1,4507 \cdot 10^{-6}$	$8,3753 \cdot 10^{-10}$		$2,5111 \cdot 10^{-4}$
10^{9}	$1,4507 \cdot 10^{-7}$	$8,3787 \cdot 10^{-11}$		$2,5111 \cdot 10^{-5}$
10^{10}	$1,4507 \cdot 10^{-8}$	$8,4128 \cdot 10^{-12}$		$2,5111 \cdot 10^{-6}$
10^{11}	$1,4507 \cdot 10^{-9}$	0		$2,5111 \cdot 10^{-7}$
10^{12}	$1,4507 \cdot 10^{-10}$	0		$2,5088 \cdot 10^{-8}$
10^{13}	$1,4508 \cdot 10^{-11}$	0		$2,5029 \cdot 10^{-9}$
10^{14}	$1,4513 \cdot 10^{-12}$	0		0
10^{15}	$1,4566 \cdot 10^{-13}$	0		0
10^{16}	$1,4211 \cdot 10^{-14}$	0		0
	v_T	t_T		x_N
10^{7}	9,21032432694012	1000,00992904408		371976,9103422510
10^{8}	9,21032577765533	1000,00992904491		371976,9105933640
10^{9}	9,21032592272685	1000,00992904500		371976,9106184750
10^{10}	9,21032593723401	1000,00992904501		371976,9106209860
10^{11}	9,21032593868472	1000,00992904501		371976,9106212370
10^{12}	9,21032593882979	1000,00992904501		371976,9106212620
10^{13}	9,21032593884430	1000,00992904501		371976,9106212650
10^{14}	9,21032593884575	1000,00992904501		371976,9106212650
10^{15}	9,21032593884590	1000,00992904501	*b)*	371976,9106212650
10^{16}	9,21032593884591	1000,00992904501		371976,9106212650

N	v_T	t_T	m_N	x_N
10	7,84050712993712	1000,00912888406	0,100000000000000	2002781,31911852
10^{2}	9,05060615444427	1000,00983714003	0,099999999999999	2002993,65017758
10^{3}	9,19375561457377	1000,00991970454	0,099999999999989	2003018,40248872
10^{4}	9,20830849818162	1000,00992810913	0,099999999999856	2003020,92213354
10^{5}	9,20976618582245	1000,00992895115	0,099999999998368	2003021,17456373
10^{6}	9,20991210652369	1000,00992903552	0,100000000009585	2003021,19985815
10^{7}	9,20992848275819	1000,00992904651	0,099999999922464	2003021,20324950
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	$1,4509 \cdot 10^{2}$	$8,3796 \cdot 10^{-2}$		$2,5122 \cdot 10^{4}$
10^{2}	1,2101	$7,0826 \cdot 10^{-4}$	$-8,0491 \cdot 10^{-16}$	$2,1233 \cdot 10^{2}$
10^{3}	$1,4315 \cdot 10^{-1}$	$8,2565 \cdot 10^{-5}$	$-1,0700 \cdot 10^{-14}$	$2,4752 \cdot 10^{1}$
10^{4}	$1,4553 \cdot 10^{-2}$	$8,4046 \cdot 10^{-6}$	$-1,3300 \cdot 10^{-13}$	2,5196
10^{5}	$1,4577 \cdot 10^{-3}$	$8,4202 \cdot 10^{-7}$	$-1,4880 \cdot 10^{-12}$	$2,5243 \cdot 10^{-1}$
10^{6}	$1,4592 \cdot 10^{-4}$	$8,4370 \cdot 10^{-8}$	$1,1217 \cdot 10^{-11}$	$2,5294 \cdot 10^{-2}$
10^{7}	$1,6376 \cdot 10^{-5}$	$1,0990 \cdot 10^{-8}$	$-8,7121 \cdot 10^{-11}$	$3,3913 \cdot 10^{-3}$
10^{8}	$1,4509 \cdot 10^{-6}$	$8,3799 \cdot 10^{-10}$		$2,5122 \cdot 10^{-4}$
10^{9}	$1,4509 \cdot 10^{-7}$	$8,3787 \cdot 10^{-11}$		$2,5121 \cdot 10^{-5}$
10^{10}	$1,4509 \cdot 10^{-8}$	$8,4128 \cdot 10^{-12}$		$2,5122 \cdot 10^{-6}$
10^{11}	$1,4509 \cdot 10^{-9}$	0		$2,5122 \cdot 10^{-7}$
10^{12}	$1,4509 \cdot 10^{-10}$	0		$2,5146 \cdot 10^{-8}$
10^{13}	$1,4509 \cdot 10^{-11}$	0		$2,5611 \cdot 10^{-9}$
10^{14}	$1,4513 \cdot 10^{-12}$	0		0
10^{15}	$1,4566 \cdot 10^{-13}$	0		0
10^{16}	$1,4211 \cdot 10^{-14}$	0		0
	v_T	t_T		x_N
10^{7}	9,20992661571773	1000,00992904390		2003021,20237030
10^{8}	9,20992806663713	1000,00992904474		2003021,20262152
10^{9}	9,20992821172907	1000,00992904482		2003021,20264664
10^{10}	9,20992822623827	1000,00992904483		2003021,20264915
10^{11}	9,20992822768919	1000,00992904483		2003021,20264941
10^{12}	9,20992822783428	1000,00992904483		2003021,20264943
10^{13}	9,20992822784879	1000,00992904483		2003021,20264943
10^{14}	9,20992822785024	1000,00992904483		2003021,20264943
10^{15}	9,20992822785039	1000,00992904483	*c)*	2003021,20264943
10^{16}	9,20992822785040	1000,00992904483		2003021,20264943

N	v_T	t_T	m_N	x_N
10	7,83212547315452	1000,00912888264	0,100000000000000	10008306,1716827
10^{2}	9,04092953464169	1000,00983713831	0,099999999999999	10008518,6162407
10^{3}	9,18392577772829	1000,00991970277	0,099999999999989	10008543,3817823
10^{4}	9,19846309125933	1000,00992810738	0,099999999999856	10008545,9027777
10^{5}	9,19991928923446	1000,00992894953	0,099999999998368	10008546,1553823
10^{6}	9,20006556392400	1000,00992903452	0,100000000009585	10008546,1808763
10^{7}	9,20008899787535	1000,00992905726	0,099999999922464	10008546,1876819
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	$1,4507 \cdot 10^{2}$	$8,3954 \cdot 10^{-2}$		$2,5182 \cdot 10^{2}$
10^{2}	1,2088	$7,0826 \cdot 10^{-4}$	$-8,0491 \cdot 10^{-16}$	$2,1244 \cdot 10^{2}$
10^{3}	$1,4300 \cdot 10^{-1}$	$8,2564 \cdot 10^{-5}$	$-1,0700 \cdot 10^{-14}$	$2,4766 \cdot 10^{1}$
10^{4}	$1,4537 \cdot 10^{-2}$	$8,4046 \cdot 10^{-6}$	$-1,3300 \cdot 10^{-13}$	2,5210
10^{5}	$1,4562 \cdot 10^{-3}$	$8,4215 \cdot 10^{-7}$	$-1,4880 \cdot 10^{-12}$	$2,5260 \cdot 10^{-1}$
10^{6}	$1,4627 \cdot 10^{-4}$	$8,4990 \cdot 10^{-8}$	$1,1217 \cdot 10^{-11}$	$2,5494 \cdot 10^{-2}$
10^{7}	$2,3434 \cdot 10^{-5}$	$2,2740 \cdot 10^{-8}$	$-8,7121 \cdot 10^{-11}$	$6,8056 \cdot 10^{-3}$
10^{8}	$1,4507 \cdot 10^{-6}$	$8,3958 \cdot 10^{-10}$		$2,5182 \cdot 10^{-4}$
10^{9}	$1,4507 \cdot 10^{-7}$	$8,3901 \cdot 10^{-11}$		$2,5183 \cdot 10^{-5}$
10^{10}	$1,4507 \cdot 10^{-8}$	$8,4128 \cdot 10^{-12}$		$2,5183 \cdot 10^{-6}$
10^{11}	$1,4507 \cdot 10^{-9}$	0		$2,5146 \cdot 10^{-7}$
10^{12}	$1,4507 \cdot 10^{-10}$	0		$2,6077 \cdot 10^{-8}$
10^{13}	$1,4506 \cdot 10^{-11}$	0		0
10^{14}	$1,4513 \cdot 10^{-12}$	0		0
10^{15}	$1,4566 \cdot 10^{-13}$	0		0
10^{16}	$1,4211 \cdot 10^{-14}$	0		0
	v_T	t_T		x_N
10^{7}	9,20008007052064	1000,00992904292		10008546,1833945
10^{8}	9,20008152118030	1000,00992904376		10008546,1836464
10^{9}	9,20008166624627	1000,00992904384		10008546,1836716
10^{10}	9,20008168075286	1000,00992904385		10008546,1836741
10^{11}	9,20008168220352	1000,00992904385		10008546,1836743
10^{12}	9,20008168234859	1000,00992904385		10008546,1836744
10^{13}	9,20008168236309	1000,00992904385		10008546,1836744
10^{14}	9,20008168236455	1000,00992904385		10008546,1836744
10^{15}	9,20008168236469	1000,00992904385	*d)*	10008546,1836744
10^{16}	9,20008168236471	1000,00992904385		10008546,1836744

Tab. C.3: Calculation of relativistic rocket velocity according to program
Type: “A1”, $v_0' = -4\,\mathrm{km/s}$, $\Delta m_0 = 0.09\%$, $t_0 = 1000s$
a) $v_0 = 0$, b) $v_0 = 369\,\mathrm{km/s}$, c) $v_0 = 2000\,\mathrm{km/s}$, d) $v_0 = 10000\,\mathrm{km/s}$

a)

N	v_T	t_T	m_N	x_N
10	196,021417688228	10002,2826214110	0,100000000000000	684187,711109235
10^2	226,275235660056	10002,4597552575	0,099999999999999	737269,299206788
10^3	229,854130642931	10002,4804055720	0,099999999999989	743457,239721413
10^4	230,217968797944	10002,4825076705	0,099999999999856	744087,136710344
10^5	230,254412155672	10002,4827182607	0,099999999998368	744150,240353761
10^6	230,258057081830	10002,4827393235	0,100000000009585	744156,551855839
10^7	230,258421713536	10002,4827414384	0,099999999922464	744157,183283767
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	$3,6266 \cdot 10^{2}$	$2,0948 \cdot 10^{1}$		$6,2771 \cdot 10^{6}$
10^2	$3,0254 \cdot 10^{1}$	$1,7713 \cdot 10^{-1}$	$-8,0491 \cdot 10^{-16}$	$5,3082 \cdot 10^{4}$
10^3	3,5789	$2,0650 \cdot 10^{-2}$	$-1,0700 \cdot 10^{-14}$	$6,1879 \cdot 10^{3}$
10^4	$3,6384 \cdot 10^{-1}$	$2,1021 \cdot 10^{-3}$	$-1,3300 \cdot 10^{-13}$	$6,2990 \cdot 10^{2}$
10^5	$3,6443 \cdot 10^{-2}$	$2,1059 \cdot 10^{-4}$	$-1,4880 \cdot 10^{-12}$	$6,3104 \cdot 10^{1}$
10^6	$3,6449 \cdot 10^{-3}$	$2,1063 \cdot 10^{-5}$	$1,1217 \cdot 10^{-11}$	6,3115
10^7	$3,6463 \cdot 10^{-4}$	$2,1149 \cdot 10^{-6}$	$-8,7121 \cdot 10^{-11}$	$6,3143 \cdot 10^{-1}$
10^8	$3,6266 \cdot 10^{-5}$	$2,0948 \cdot 10^{-7}$		$6,2772 \cdot 10^{-2}$
10^9	$3,6266 \cdot 10^{-6}$	$2,0947 \cdot 10^{-8}$		$6,2772 \cdot 10^{-3}$
10^{10}	$3,6266 \cdot 10^{-7}$	$2,0955 \cdot 10^{-9}$		$6,2772 \cdot 10^{-4}$
10^{11}	$3,6266 \cdot 10^{-8}$	$2,0918 \cdot 10^{-10}$		$6,2772 \cdot 10^{-5}$
10^{12}	$3,6266 \cdot 10^{-9}$	$2,1828 \cdot 10^{-11}$		$6,2772 \cdot 10^{-6}$
10^{13}	$3,6266 \cdot 10^{-10}$	0		$6,2771 \cdot 10^{-7}$
10^{14}	$3,6266 \cdot 10^{-11}$	0		$6,2748 \cdot 10^{-8}$
10^{15}	$3,6380 \cdot 10^{-12}$	0		$6,2864 \cdot 10^{-9}$
10^{16}	$3,6948 \cdot 10^{-13}$	0		0
	v_T	t_T		x_N
10^7	230,258419745292	10002,4827414183		744157,179575260
10^8	230,258456011638	10002,4827416278		744157,242347202
10^9	230,258459638272	10002,4827416488		744157,248624396
10^{10}	230,258460000936	10002,4827416509		744157,249252115
10^{11}	230,258460037202	10002,4827416511		744157,249314887
10^{12}	230,258460040829	10002,4827416511		744157,249321164
10^{13}	230,258460041192	10002,4827416511		744157,249321792
10^{14}	230,258460041228	10002,4827416511		744157,249321855
10^{15}	230,258460041231	10002,4827416511		744157,249321861
10^{16}	230,258460041232	10002,4827416511		744157,249321862

b)

N	v_T	t_T	m_N	x_N
10	196,020933328363	10002,2826210019	0,100000000000000	4374191,05576218
10^2	226,274637774396	10002,4597547615	0,099999999999999	4427272,68489423
10^3	229,853518480855	10002,4804050636	0,099999999999989	4433460,62991467
10^4	230,217355174200	10002,4825071609	0,099999999999856	4434090,52736042
10^5	230,253798388818	10002,4827177511	0,099999999998368	4434153,63106899
10^6	230,257443325741	10002,4827388142	0,100000000009585	4434159,94267449
10^7	230,257808300206	10002,4827409268	0,099999999922464	4434160,57580526
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	$3,6266 \cdot 10^{2}$	$2,0948 \cdot 10^{1}$		$6,2772 \cdot 10^{6}$
10^2	$3,0254 \cdot 10^{1}$	$1,7713 \cdot 10^{-1}$	$-8,0491 \cdot 10^{-16}$	$5,3082 \cdot 10^{4}$
10^3	3,5789	$2,0650 \cdot 10^{-2}$	$-1,0700 \cdot 10^{-14}$	$6,1879 \cdot 10^{3}$
10^4	$3,6384 \cdot 10^{-1}$	$2,1021 \cdot 10^{-3}$	$-1,3300 \cdot 10^{-13}$	$6,2990 \cdot 10^{2}$
10^5	$3,6443 \cdot 10^{-2}$	$2,1059 \cdot 10^{-4}$	$-1,4880 \cdot 10^{-12}$	$6,3104 \cdot 10^{1}$
10^6	$3,6449 \cdot 10^{-3}$	$2,1063 \cdot 10^{-5}$	$1,1217 \cdot 10^{-11}$	6,3116
10^7	$3,6497 \cdot 10^{-4}$	$2,1126 \cdot 10^{-6}$	$-8,7121 \cdot 10^{-11}$	$6,3313 \cdot 10^{-1}$
10^8	$3,6266 \cdot 10^{-5}$	$2,0948 \cdot 10^{-7}$		$6,2772 \cdot 10^{-2}$
10^9	$3,6266 \cdot 10^{-6}$	$2,0947 \cdot 10^{-8}$		$6,2772 \cdot 10^{-3}$
10^{10}	$3,6266 \cdot 10^{-7}$	$2,0955 \cdot 10^{-9}$		$6,2772 \cdot 10^{-4}$
10^{11}	$3,6266 \cdot 10^{-8}$	$2,0918 \cdot 10^{-10}$		$6,2772 \cdot 10^{-5}$
10^{12}	$3,6266 \cdot 10^{-9}$	$2,1828 \cdot 10^{-11}$		$6,2771 \cdot 10^{-6}$
10^{13}	$3,6266 \cdot 10^{-10}$	0		$6,2771 \cdot 10^{-7}$
10^{14}	$3,6266 \cdot 10^{-11}$	0		$6,2399 \cdot 10^{-8}$
10^{15}	$3,6380 \cdot 10^{-12}$	0		0
10^{16}	$3,6948 \cdot 10^{-13}$	0		0
	v_T	t_T		x_N
10^7	230,257805988392	10002,4827409090		4434160,57039689
10^8	230,257842254657	10002,4827411185		4434160,63316912
10^9	230,257845881283	10002,4827411395		4434160,63944635
10^{10}	230,257846243946	10002,4827411416		4434160,64007407
10^{11}	230,257846280212	10002,4827411418		4434160,64013684
10^{12}	230,257846283839	10002,4827411418		4434160,64014312
10^{13}	230,257846284201	10002,4827411418		4434160,64014375
10^{14}	230,257846284238	10002,4827411418		4434160,64014381
10^{15}	230,257846284241	10002,4827411418		4434160,64014382
10^{16}	230,257846284242	10002,4827411418		4434160,64014382

c)

N	v_T	t_T	m_N	x_N
10	196,011677951838	10002,2826191942	0,100000000000000	20684648,1809011
10^2	226,263782574663	10002,4597525694	0,099999999999999	20737730,9547342
10^3	229,842470286295	10002,4804028168	0,099999999999989	20743919,0319696
10^4	230,206287313833	10002,4825049083	0,099999999999856	20744548,9428655
10^5	230,242728572615	10002,4827154982	0,099999999998368	20744612,0480015
10^6	230,246373417548	10002,4827365626	0,100000000009585	20744618,3601214
10^7	230,246739825310	10002,4827386978	0,099999999922464	20744619,0002277
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	$3,6264 \cdot 10^{2}$	$2,0949 \cdot 10^{1}$		$6,2775 \cdot 10^{6}$
10^2	$3,0252 \cdot 10^{1}$	$1,7713 \cdot 10^{-1}$	$-8,0491 \cdot 10^{-16}$	$5,3083 \cdot 10^{4}$
10^3	3,5787	$2,0650 \cdot 10^{-2}$	$-1,0700 \cdot 10^{-14}$	$6,1881 \cdot 10^{3}$
10^4	$3,6382 \cdot 10^{-1}$	$2,1021 \cdot 10^{-3}$	$-1,3300 \cdot 10^{-13}$	$6,2991 \cdot 10^{2}$
10^5	$3,6441 \cdot 10^{-2}$	$2,1059 \cdot 10^{-4}$	$-1,4880 \cdot 10^{-12}$	$6,3105 \cdot 10^{1}$
10^6	$3,6448 \cdot 10^{-3}$	$2,1064 \cdot 10^{-5}$	$1,1217 \cdot 10^{-11}$	6,3121
10^7	$3,6641 \cdot 10^{-4}$	$2,1352 \cdot 10^{-6}$	$-8,7121 \cdot 10^{-11}$	$6,4011 \cdot 10^{-1}$
10^8	$3,6265 \cdot 10^{-5}$	$2,0949 \cdot 10^{-7}$		$6,2775 \cdot 10^{-2}$
10^9	$3,6265 \cdot 10^{-6}$	$2,0949 \cdot 10^{-8}$		$6,2775 \cdot 10^{-3}$
10^{10}	$3,6265 \cdot 10^{-7}$	$2,0955 \cdot 10^{-9}$		$6,2774 \cdot 10^{-4}$
10^{11}	$3,6265 \cdot 10^{-8}$	$2,0918 \cdot 10^{-10}$		$6,2775 \cdot 10^{-5}$
10^{12}	$3,6265 \cdot 10^{-9}$	$2,1828 \cdot 10^{-11}$		$6,2771 \cdot 10^{-6}$
10^{13}	$3,6263 \cdot 10^{-10}$	0		$6,2957 \cdot 10^{-7}$
10^{14}	$3,6266 \cdot 10^{-11}$	0		$6,3330 \cdot 10^{-8}$
10^{15}	$3,6380 \cdot 10^{-12}$	0		0
10^{16}	$3,6948 \cdot 10^{-13}$	0		0
	v_T	t_T		x_N
10^7	230,246736063268	10002,4827386575		20744618,9878669
10^8	230,246772327840	10002,4827388670		20744619,0506414
10^9	230,246775954297	10002,4827388879		20744619,0569189
10^{10}	230,246776316943	10002,4827388900		20744619,0575466
10^{11}	230,246776353207	10002,4827388902		20744619,0576094
10^{12}	230,246776356834	10002,4827388902		20744619,0576157
10^{13}	230,246776357197	10002,4827388902		20744619,0576163
10^{14}	230,246776357233	10002,4827388902		20744619,0576164
10^{15}	230,246776357236	10002,4827388902		20744619,0576164
10^{16}	230,246776357237	10002,4827388902		20744619,0576164

d)

N	v_T	t_T	m_N	x_N
10	195,798241945411	10002,2826103399	0,100000000000000	100740248,476771
10^2	226,016565669348	10002,4597418176	0,099999999999999	100793359,642360
10^3	229,591238915968	10002,4803917961	0,099999999999989	100799551,023330
10^4	229,954647601144	10002,4824938598	0,099999999999856	100800181,270485
10^5	229,991048026237	10002,4827044481	0,099999999998368	100800244,409694
10^6	229,994689292966	10002,4827255184	0,100000000009585	100800250,727051
10^7	229,995062373247	10002,4827277682	0,099999999922464	100800251,401665
	δv_T	δt_T	δm_N	δx_N
$\bar{x}$	$3,6225 \cdot 10^{2}$	$2,0950 \cdot 10^{1}$		$6,2812 \cdot 10^{6}$
10^2	$3,0218 \cdot 10^{1}$	$1,7713 \cdot 10^{-1}$	$-8,0491 \cdot 10^{-16}$	$5,3111 \cdot 10^{4}$
10^3	3,5747	$2,0650 \cdot 10^{-2}$	$-1,0700 \cdot 10^{-14}$	$6,1914 \cdot 10^{3}$
10^4	$3,6341 \cdot 10^{-1}$	$2,1021 \cdot 10^{-3}$	$-1,3300 \cdot 10^{-13}$	$6,3025 \cdot 10^{2}$
10^5	$3,6400 \cdot 10^{-2}$	$2,1059 \cdot 10^{-4}$	$-1,4880 \cdot 10^{-12}$	$6,3139 \cdot 10^{1}$
10^6	$3,6413 \cdot 10^{-3}$	$2,1070 \cdot 10^{-5}$	$1,1217 \cdot 10^{-11}$	6,3174
10^7	$3,7308 \cdot 10^{-4}$	$2,2498 \cdot 10^{-6}$	$-8,7121 \cdot 10^{-11}$	$6,7461 \cdot 10^{-1}$
10^8	$3,6225 \cdot 10^{-5}$	$2,0950 \cdot 10^{-7}$		$6,2813 \cdot 10^{-2}$
10^9	$3,6225 \cdot 10^{-6}$	$2,0949 \cdot 10^{-8}$		$6,2813 \cdot 10^{-3}$
10^{10}	$3,6225 \cdot 10^{-7}$	$2,0955 \cdot 10^{-9}$		$6,2813 \cdot 10^{-4}$
10^{11}	$3,6225 \cdot 10^{-8}$	$2,0918 \cdot 10^{-10}$		$6,2808 \cdot 10^{-5}$
10^{12}	$3,6225 \cdot 10^{-9}$	$2,1828 \cdot 10^{-11}$		$6,2883 \cdot 10^{-6}$
10^{13}	$3,6226 \cdot 10^{-10}$	0		$6,2585 \cdot 10^{-7}$
10^{14}	$3,6238 \cdot 10^{-11}$	0		0
10^{15}	$3,6096 \cdot 10^{-12}$	0		0
10^{16}	$3,6948 \cdot 10^{-13}$	0		0
	v_T	t_T		x_N
10^7	229,994689292966	10002,4827276134		100800251,355179
10^8	229,994725518139	10002,4827278229		100800251,417992
10^9	229,994729140657	10002,4827278438		100800251,424273
10^{10}	229,994729502908	10002,4827278459		100800251,424902
10^{11}	229,994729539134	10002,4827278461		100800251,424964
10^{12}	229,994729542756	10002,4827278462		100800251,424971
10^{13}	229,994729543118	10002,4827278462		100800251,424971
10^{14}	229,994729543155	10002,4827278462		100800251,424971
10^{15}	229,994729543158	10002,4827278462		100800251,424971
10^{16}	229,994729543159	10002,4827278462		100800251,424971

Tab. C.4: Calculation of relativistic rocket velocity according to program
Type: “A1”, $v_0' = -100$ km/s, $\Delta m_0 = 0.009\%$, $t_0 = 10000s$
a) $v_0 = 0$, b) $v_0 = 369$ km/s, c) $v_0 = 2000$ km/s, d) $v_0 = 10000$ km/s

C.4 Relativistic rocket equation according to J. Akeret

Since 1946 there is an analytical solution for the relativistic rocket equation by J. Akeret [90]. For this not only the momentum theorem and the relativistic velocity addition are necessary (as with the numerical derivation presented so far) but additionally the energy conservation theorem is used.

For the derivation of the equations, formula symbols are used which differ from the original text but are consistent with the representations used so far in this presentation. Functions related to the outflowing gas used for causing thrust are denoted by f'; relations referring to the moving rocket, on the other hand, are shown without this label. The actual mass of the rocket is m, and dm' is the fraction of the propellant gas. This gives rise to the equations shown below.

a) The energy theorem provides:

$$d\left\{\frac{mc^2}{\sqrt{1-v^2/c^2}}\right\} = -\frac{dm'\cdot c^2}{\sqrt{1-v'^2/c^2}} \tag{C.21}$$

b) the relation for momentum:

$$d\left\{\frac{mv}{\sqrt{1-v^2/c^2}}\right\} = \frac{dm'\cdot v'}{\sqrt{1-v'^2/c^2}} \tag{C.22}$$

c) the relativistic addition theorem:

$$v' = \frac{v_0'-v}{1-\frac{v\cdot v_0'}{c^2}} \tag{C.23}$$

where v_0' has the meaning of the (constant) exit velocity of the gas relative to the rocket. The equations (C.21) and (C.22) can be further developed to

$$dm\frac{c^2}{\sqrt{1-v^2/c^2}} + mc^2\cdot d\left\{\frac{1}{\sqrt{1-v^2/c^2}}\right\} = -dm'\frac{c^2}{\sqrt{1-v'^2/c^2}} \tag{C.24}$$

$$dm\frac{v}{\sqrt{1-v^2/c^2}} + m\frac{dv}{\sqrt{1-v^2/c^2}} + mv\cdot d\left\{\frac{1}{\sqrt{1-v^2/c^2}}\right\} = dm'\frac{v'}{\sqrt{1-v'^2/c^2}} \tag{C.25}$$

For the solution, the values of v' and dm' must be eliminated. To do this, first in equation (C.24) in the term on the right-hand side the value for v' from equation (C.23) is inserted

$$\frac{c^2}{\sqrt{1-\frac{v'^2}{c^2}}} = \frac{c^2}{\sqrt{1-\frac{\left(\frac{v_0'-v}{1-v\cdot v_0'/c^2}\right)^2}{c^2}}}$$

$$= \frac{c^2-v_0'v}{\sqrt{1-\frac{v^2}{c^2}-\frac{v_0'^2}{c^2}+\frac{v^2v_0'^2}{c^4}}} = \frac{c^2-v_0'v}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{v_0'^2}{c^2}}} \tag{C.26}$$

In the same way follows

$$\frac{v'}{\sqrt{1-\frac{v'^2}{c^2}}}=\frac{v_0'-v}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{v_0'^2}{c^2}}} \tag{C.27}$$

Equations (C.26) and (C.27) are substituted into Eq. (C.24) and (C.25), respectively, and these are resolved to dm' and equated. The result is:

$$m\left\{\frac{c^2-vv_0'}{\sqrt{1-v^2/c^2}}\right\}dv+mv_0'(c^2-v^2)\cdot d\left\{\frac{1}{\sqrt{1-v^2/c^2}}\right\}+dm\frac{v_0'(c^2-v^2)}{\sqrt{1-v^2/c^2}}=0 \tag{C.28}$$

The two differentials with the dependence on v must be unified and using the differential chain rule it follows

$$d\left\{\frac{1}{\sqrt{1-v^2/c^2}}\right\}=\frac{v}{c^2\left\{1-\frac{v^2}{c^2}\right\}^{3/2}}\,dv \tag{C.29}$$

After substituting in eq. (C.28) and separating the terms for mass and velocity, the final result is

$$\frac{dm}{m}=-\frac{dv}{v_0'(1-v^2/c^2)} \tag{C.30}$$

The integration results in

$$ln(m)=-\frac{c}{2v_0'}\,ln\left\{\frac{c+v}{c-v}\right\}+C \tag{C.31}$$

With the initial value for mass m_0 and the final value m the relativistic rocket equation according to J. Akeret arises

$$\frac{m}{m_0}=\left\{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right\}^{c/2v_0'} \tag{C.32}$$

or

$$\frac{v}{c}=\frac{1-\left(\frac{m}{m_0}\right)^{2v_0'/c}}{1+\left(\frac{m}{m_0}\right)^{2v_0'/c}} \tag{C.33}$$

In Section 6.4.2, calculations from this equation are contrasted with the classical rocket formula of K. E. Tsiolkovsky and the numerical relations derived in this annex.